

Sheet 3

Topic: Extended Kalman Filter

Due: November 22, 2021

Do not use Octave or other software to solve either of the following exercises.

Exercise 1: Bayes Filter and Extended Kalman Filter

- Describe briefly the two main steps of the Bayes filter in your own words.
- Describe briefly the meaning of the following probability density functions: $p(x_t | u_t, x_{t-1})$, $p(z_t | x_t)$, and $\text{bel}(x_t)$, which are processed by the Bayes filter.
- Specify the distributions that correspond to the above mentioned three terms in the EKF.
- Explain in a few sentences all of the components of the EKF algorithm, i. e., μ_t , Σ_t , $\bar{\mu}_t$, $\bar{\Sigma}_t$, g , G_t^x , R_t , h , H_t^x , Q_t , K_t and why they are needed. Specify the dimensionality of these components.

Exercise 2: Jacobians

- Derive the Jacobian matrix G_t^x of the noise-free motion function g with respect to the pose of the robot. Use the odometry motion model as in exercise sheet 1:

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta_{trans} \cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans} \sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix}.$$

- Derive the Jacobian matrix ${}^{\text{low}}H_t^i$ of the noise-free sensor function h corresponding to the i^{th} landmark:

$$h(\bar{\mu}_t, i) = z_t^i = \begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(\bar{\mu}_{i,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{i,y} - \bar{\mu}_{t,y})^2} \\ \text{atan2}(\bar{\mu}_{i,y} - \bar{\mu}_{t,y}, \bar{\mu}_{i,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix},$$

where $(\bar{\mu}_{i,x}, \bar{\mu}_{i,y})^T$ is the position of the landmark, $(\bar{\mu}_{t,x}, \bar{\mu}_{t,y}, \bar{\mu}_{t,\theta})^T$ is the pose of the robot at time t , and r_t^i and ϕ_t^i are the observed range and bearing of the landmark, respectively.

Hint: Use $\frac{\partial}{\partial x} \text{atan2}(y, x) = \frac{-y}{x^2+y^2}$, and $\frac{\partial}{\partial y} \text{atan2}(y, x) = \frac{x}{x^2+y^2}$.