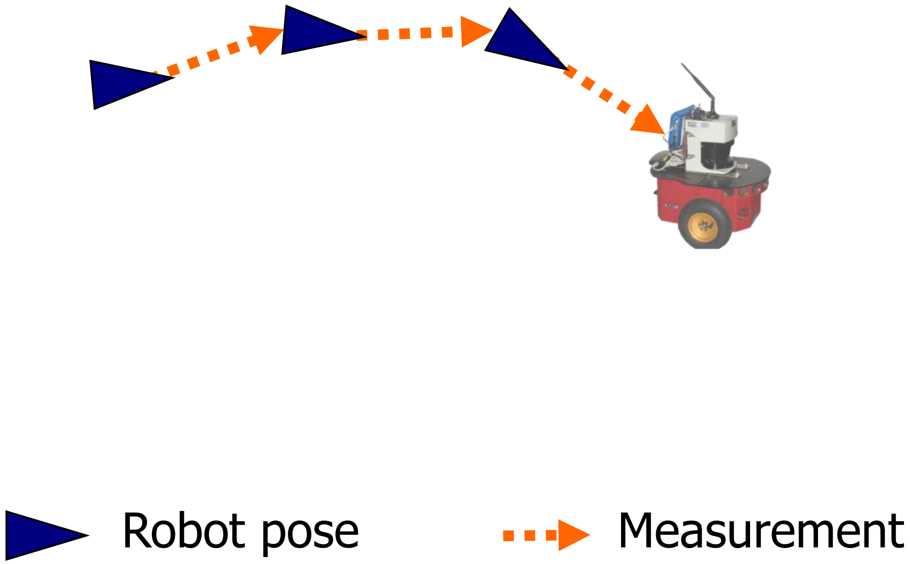
Robot Mapping

SLAM Front-Ends

Gian Diego Tipaldi, Wolfram Burgard

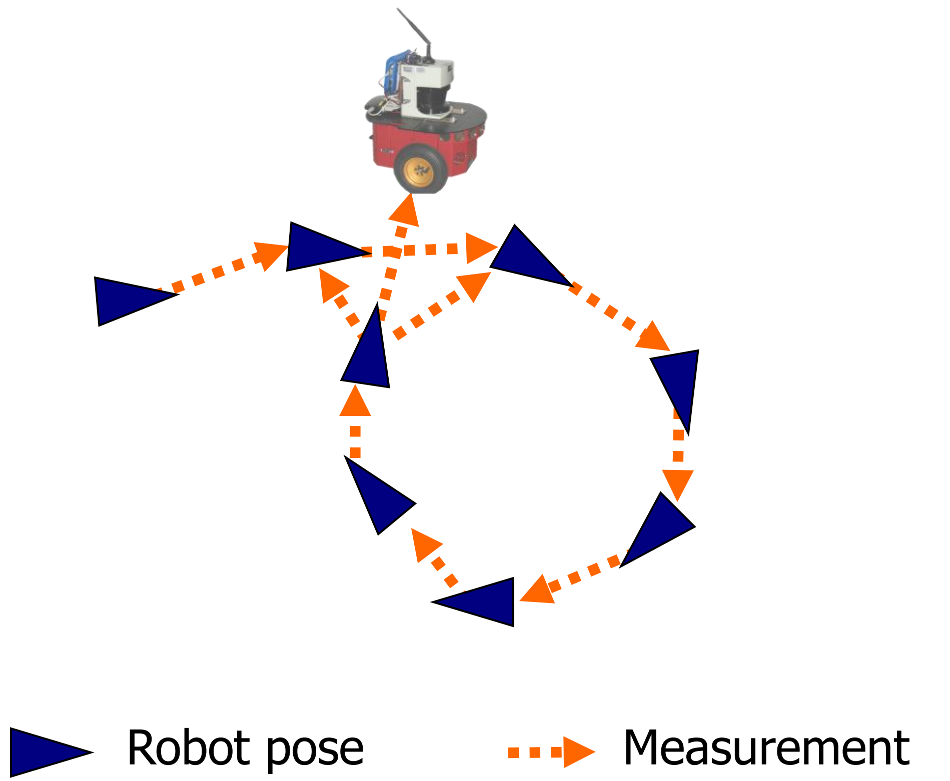
Graph-Based SLAM

- Measurements connect the nodes through odometry and observations

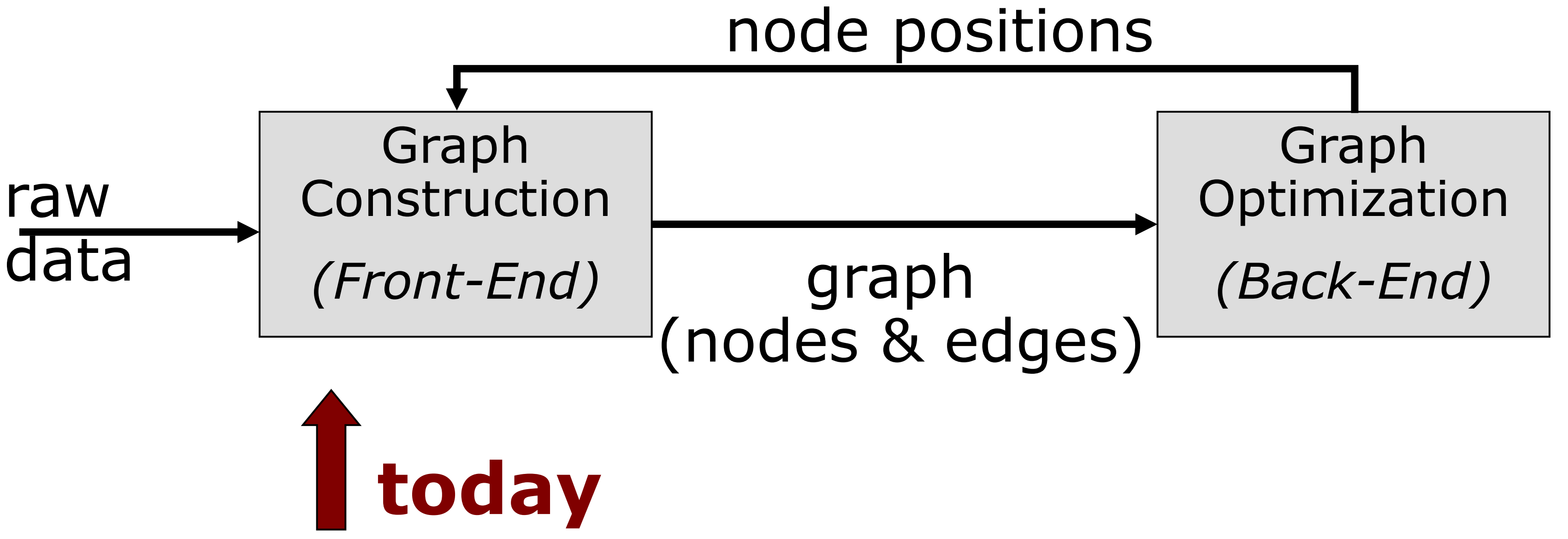


Graph-Based SLAM

- Measurements connect the nodes through odometry and observations
- How to obtain the measurements?



Interplay between Front-End and Back-End



Measurements From Matching

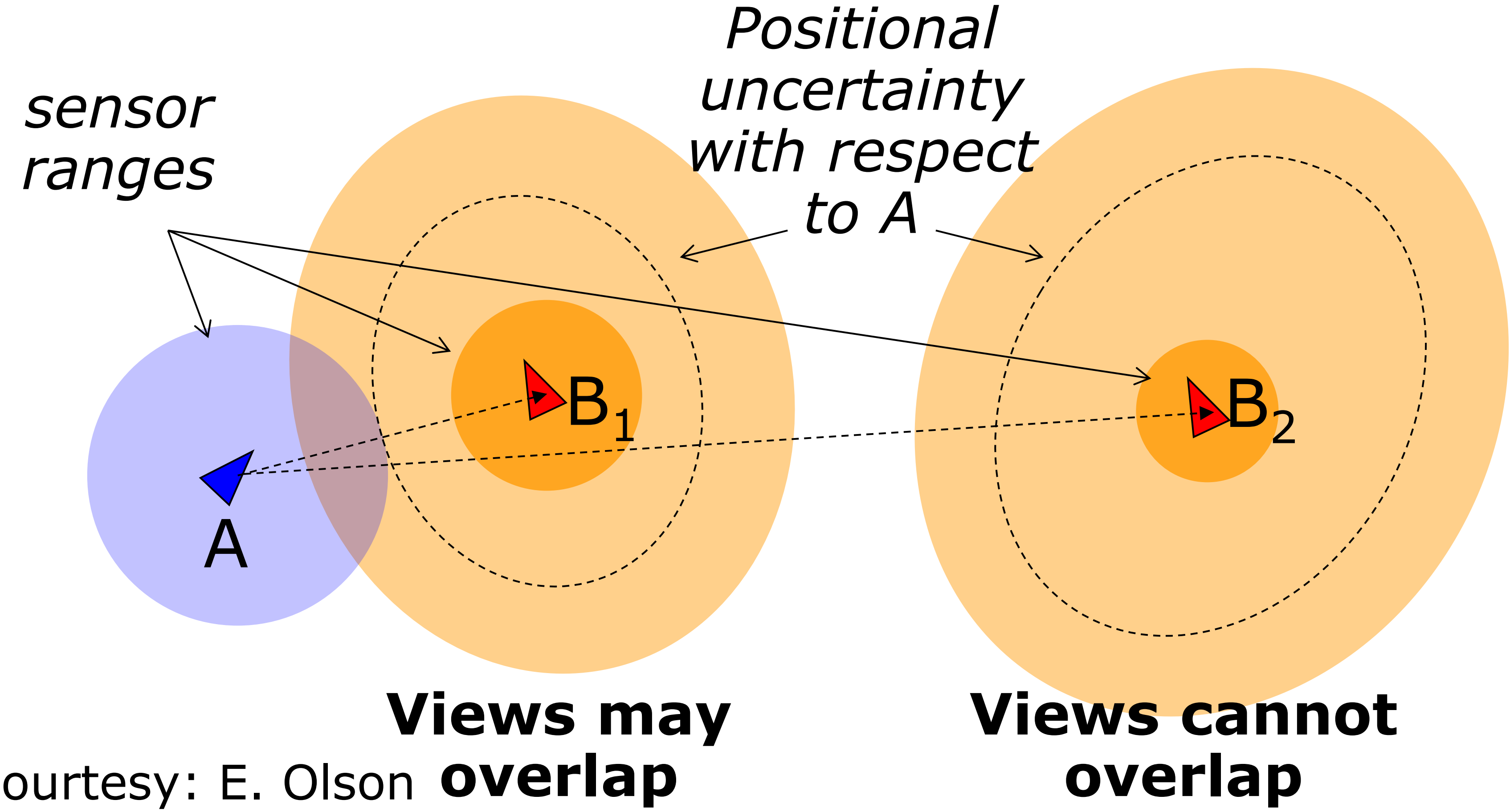
- Measurements can be obtained by matching observations

Popular approaches

- Dense matching
- Point-to-point matching
- Feature-based matching

Where to Search for Matches?

- Consider uncertainty of the nodes with respect to the current one



Courtesy: E. Olson

Note on the Uncertainty

- In graph-based SLAM, computing the uncertainty relative to A requires inverting the Hessian **H**
- Fast approximation by Dijkstra expansion (“propagate uncertainty along the shortest path in the graph”)
- Conservative estimate

Do you Recall Scan Matching?

Maximize the likelihood of the **current** pose relative to the **previous** pose and map

$$x_t^* = \underset{x_t}{\operatorname{argmax}} \left\{ p(z_t \mid x_t, m_{t-1}) \, p(x_t \mid u_{t-1}, x_{t-1}^*) \right\}$$

current measurement

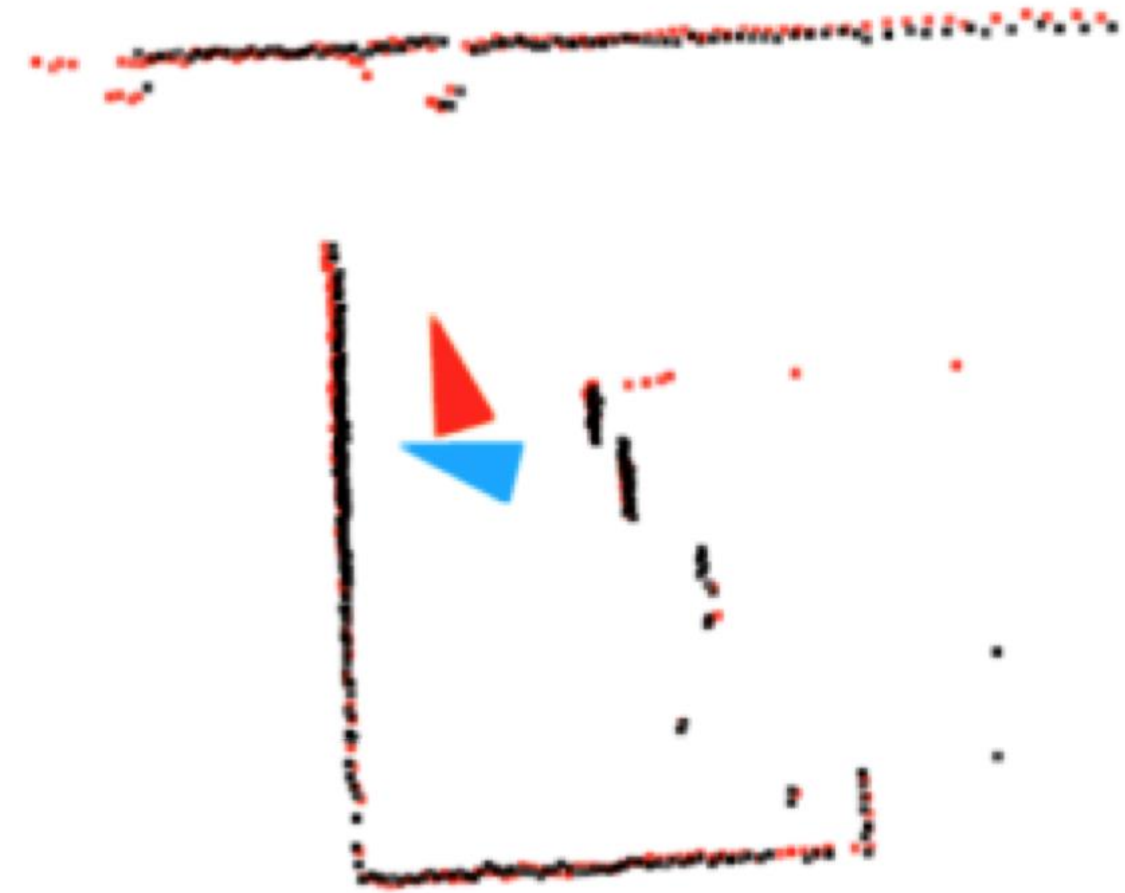
map constructed so far

robot motion

Sensor Matching as Front-End

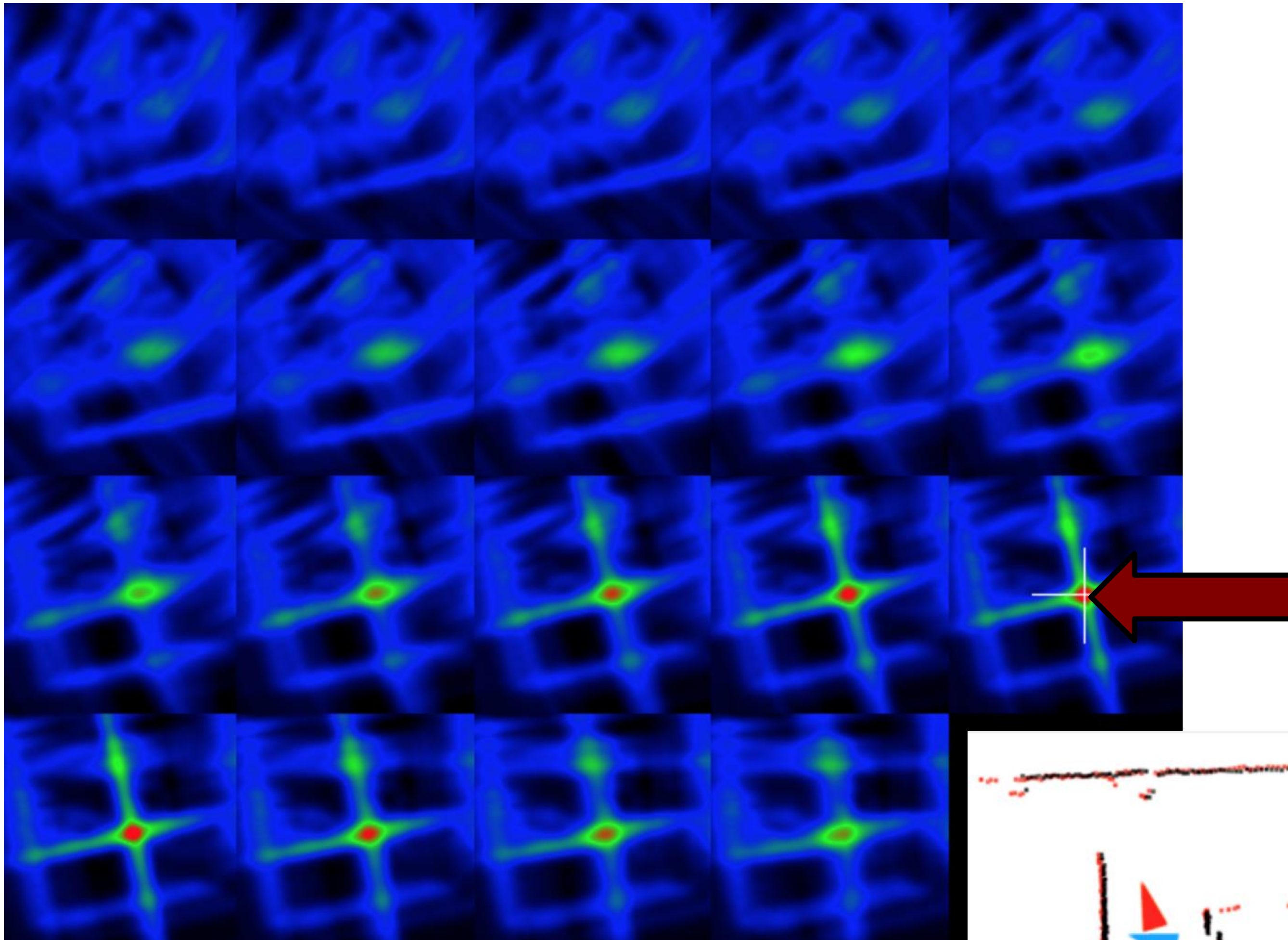
- Estimate uncertainty of nodes relative to the current pose
- Get previous observations in the relevant area
- Match the current observations with the previous ones
- Evaluate match
- Accept match based on a threshold

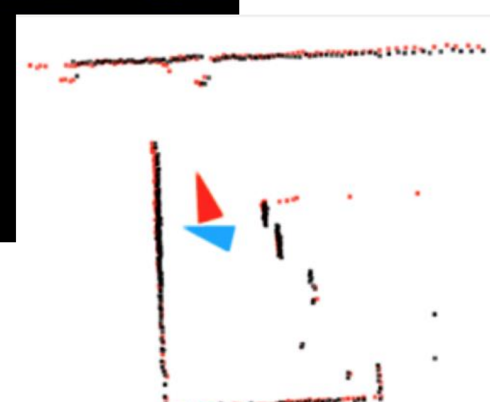
Correlative Matching



Courtesy: E. Olson

Correlative Matching





Courtesy: E. Olson

Problems

- Many matching to be performed
- Might be slow if many candidate locations
- Accuracy up to discretizations
- Uncertainties slow to compute

Point-to-Point Matching (ICP)

- Estimate uncertainty of nodes relative to the current pose
- Sample poses in relevant area
- Apply Iterative Closest Point algorithm
- Evaluate match
- Accept match based on a threshold

Point-to-Point Matching (ICP)

- Given two corresponding point sets:

$$X = \{x_1, ..., x_{N_x}\}$$

$$P = \{p_1, ..., p_{N_p}\}$$

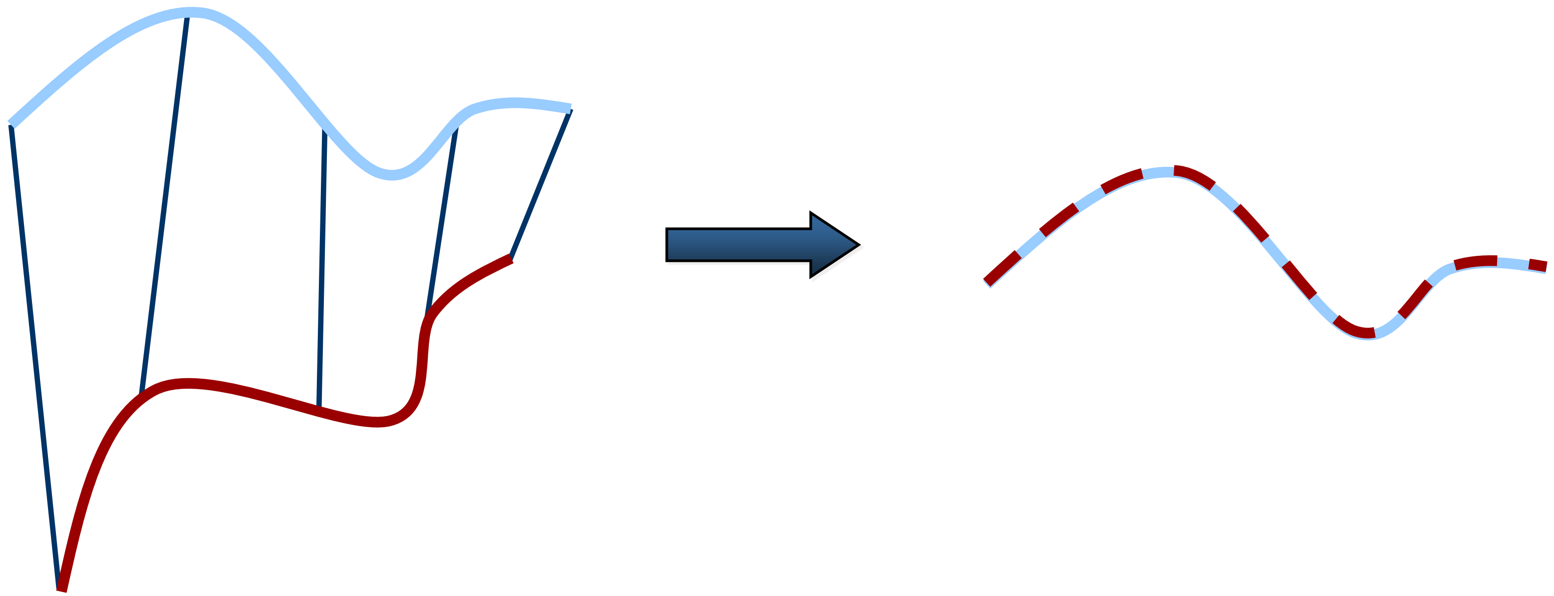
- Wanted: Translation *t* and rotation *R* that minimize:

$$E(R, t) = \frac{1}{N_p} \sum_{i=1}^{N_p} ||x_i - Rp_i - t||^2$$

Here, x_i and p_i are corresponding points

Key Idea

If the correct correspondences are known, the correct rotation/translation can be calculated in **closed form**



Center of Mass

$$\mu_x = \frac{1}{N_x}\sum_{i=1}^{N_x} x_i \quad \text{and} \quad \mu_p = \frac{1}{N_p}\sum_{i=1}^{N_p} p_i$$

are the centers of mass of the two sets

Idea:

Subtract the center of mass from every point in the two point sets

$$X' = \{x_i - \mu_x\} = \{x'_i\} \quad \text{and}$$
$$P' = \{p_i - \mu_p\} = \{p'_i\}$$

Singular Value Decomposition

Let $W = \sum_{i=1}^{N_p} x'_i p'^T_i$, we denote the singular value decomposition (SVD) of W by:

$$W = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V^T$$

Where $U, V \in \mathbb{R}^{3\times 3}$ are orthogonal, and $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the singular values

SVD

Theorem (without proof):

If rank(W) = 3, the optimal solution of E(R,t) is unique and is given by:

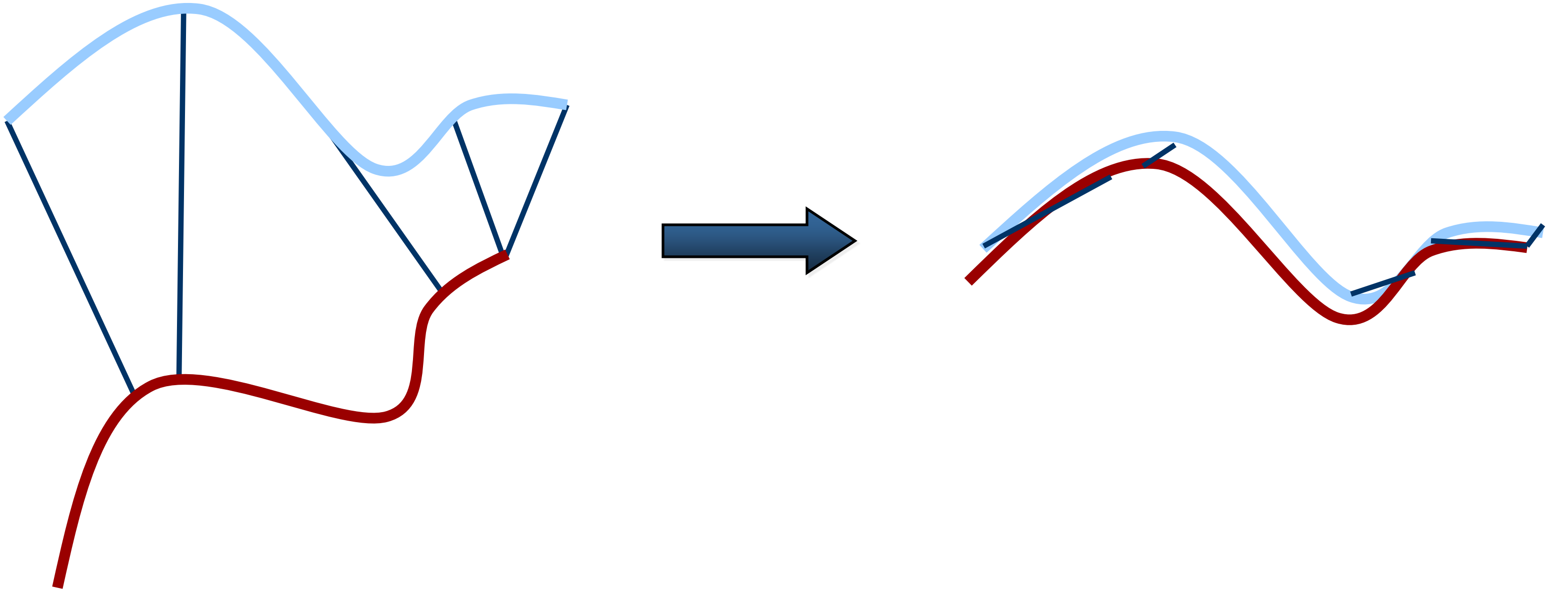
$$R = UV^T$$
$$t = \mu_x - R\mu_p$$

The minimal value of error function is:

$$E(R,t) = \sum_{i=1}^{N_p} (||x'_i||^2 + ||y'_i||^2) - 2(\sigma_1 + \sigma_2 + \sigma_3)$$

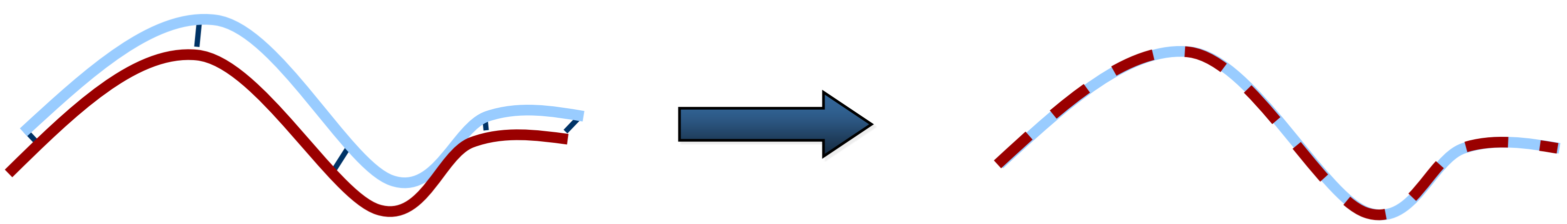
ICP with Unknown Data Association

If the correct correspondences are not known, it is generally impossible to determine the optimal relative rotation and translation in one step



Iterative Closest Point (ICP) Algorithm

- Idea: Iterate to find alignment
- Iterative Closest Points [Besl & McKay 92]
- Converges if starting positions are "close enough"



Basic ICP Algorithm

- Determine corresponding points
- Compute *R* and *t* via SVD
- Apply *R* and *t* to the points of the set to be registered
- Compute the error E(*R*,*t*)
- If error decreased and > threshold
  - Repeat these steps
  - Stop and output final alignment, otherwise

Problems

- ICP is sensitive to the initial guess
- Local minima
- Ambiguities in the environment

Feature-Based Matching

- Environment abstraction

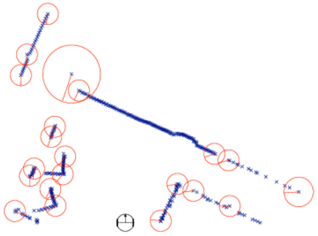
Indoor (fr-079)

[Courtesy of G. Grisetti]

Outdoor (Victoria park)

[Courtesy of M. Kaess]

- Sensor abstraction



Laser

Camera

[Courtesy of K. Mikolajczyk]

Feature-Based Matching

- Detect salient locations in the data
- Describe them with local information
- Match the set of features considering their appearance

- Features available
  - Laser: FLIRT, SHOT, NARF,…
  - Camera: SIFT, SURF, BRISK, FAST,…

Feature Matching (RANSAC)

Matching algorithm robust to outliers

Iteratively perform:

1. Sample a minimal solution set
2. Compute the transformation
3. Compute the inlier set
4. If inlier set > than previous, update

The number of iterations depends on the dimension of the minimal set

RANSAC Iterations

- Let q be the probability of an inlier

$$q = \frac{\binom{N_I}{k}}{\binom{N}{k}} = \frac{N_I!(N-k)!}{N!(N_I-k)!} = \prod_{i=0}^{k-1} \frac{N_I - i}{N - i}$$

RANSAC Iterations

- Let q be the probability of an inlier

$$q = \prod_{i=0}^{k-1} \frac{N_I - i}{N - i} \approx \left(\frac{N_I}{N}\right)^k$$

RANSAC Iterations

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- The probability of outliers in the MSS

$$(1 - q)^h$$

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$$(1-q)^h \leq \varepsilon$$

RANSAC Iterations

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- The probability of outliers in the MSS

$$(1-q)^h \leq \varepsilon$$

- The number of iterations is given by

$$h \geq \left\lceil \frac{\log \varepsilon}{\log (1-q)} \right\rceil$$

Problems

- Local minima
- Ambiguities in the environment

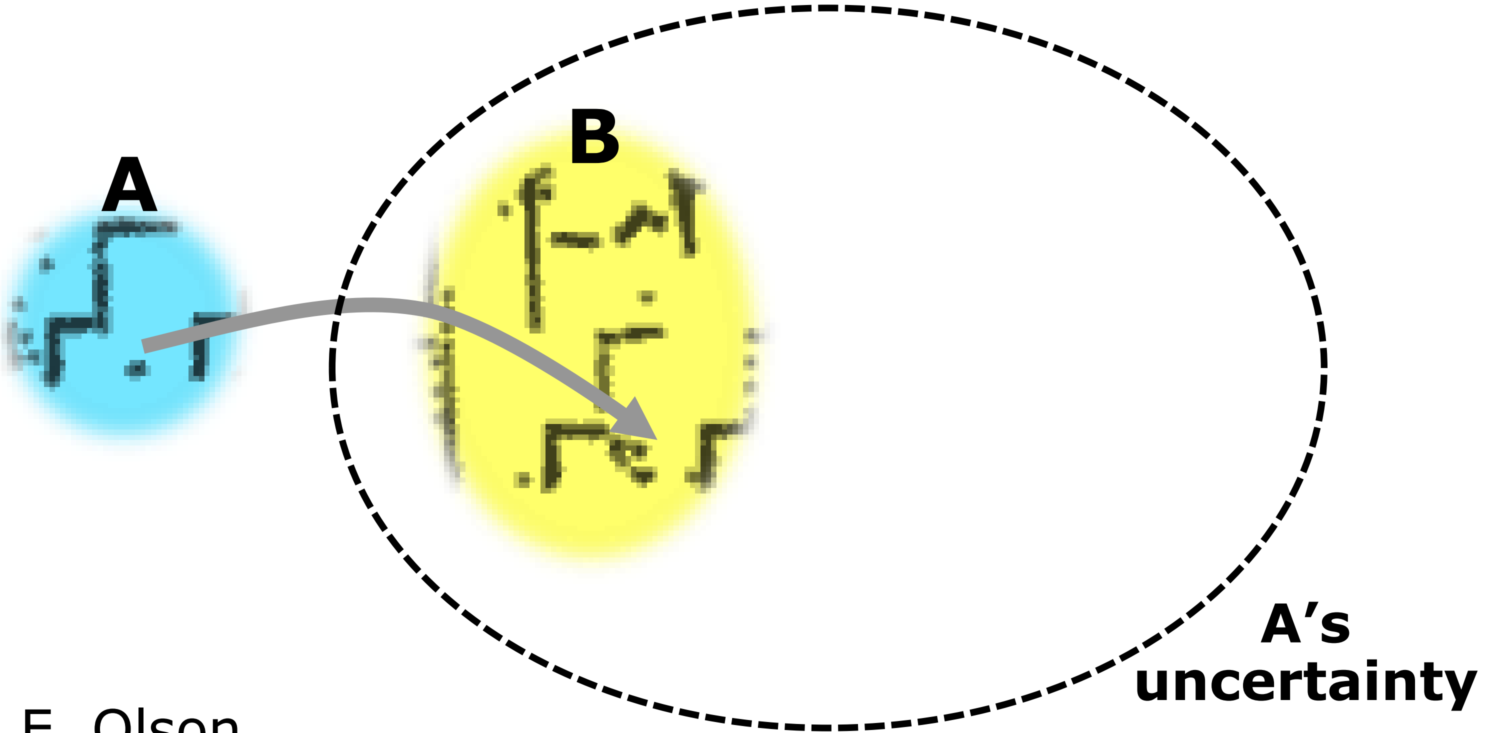
Problems

- Local minima
- **Ambiguities in the environment**

- Dealing with ambiguous areas in an environment is essential for robustly operating robots

Ambiguities - Global Ambiguity

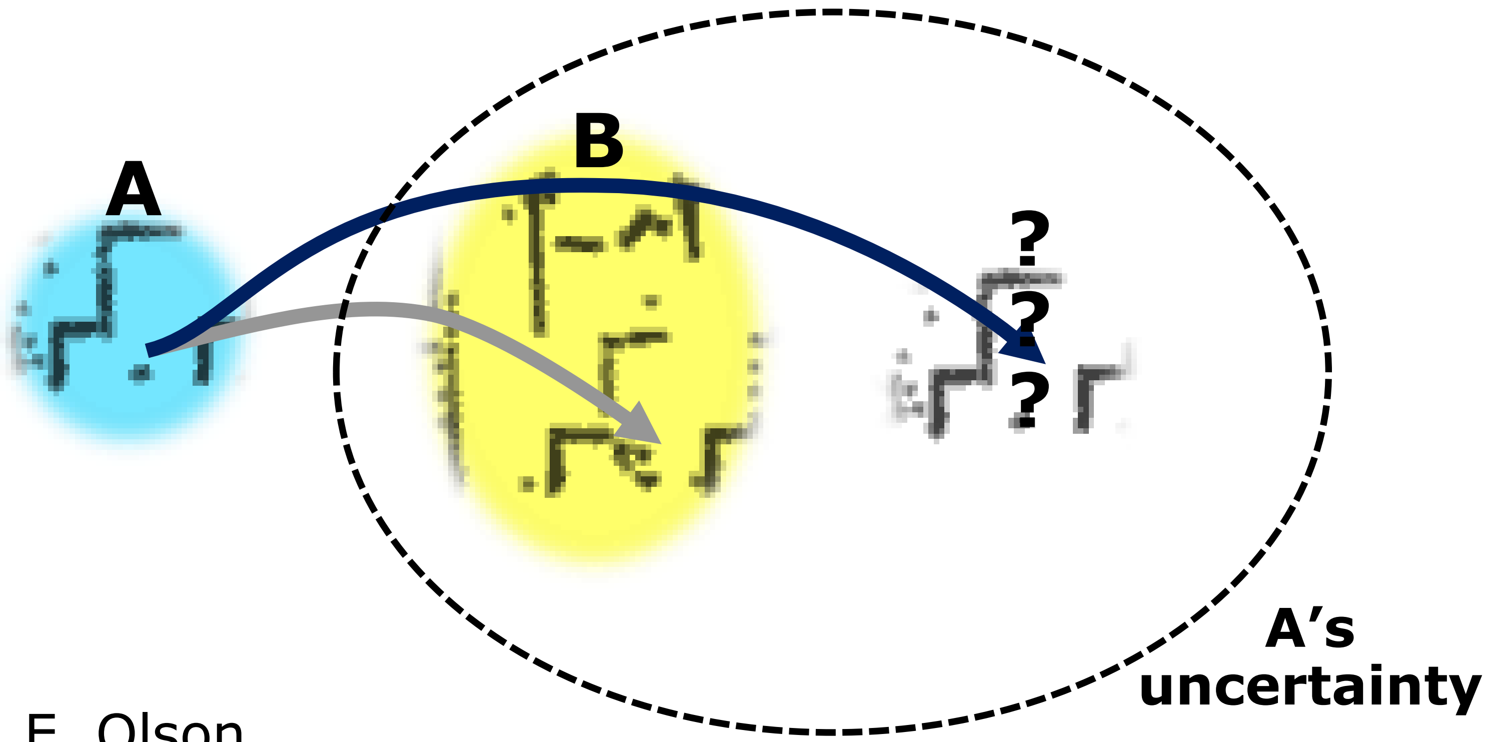
- B is inside the uncertainty ellipse of A
- Are A and B the same place?



Courtesy: E. Olson

Ambiguities - Global Ambiguity

- B is inside the uncertainty ellipse of A
- A and B might not be the same place



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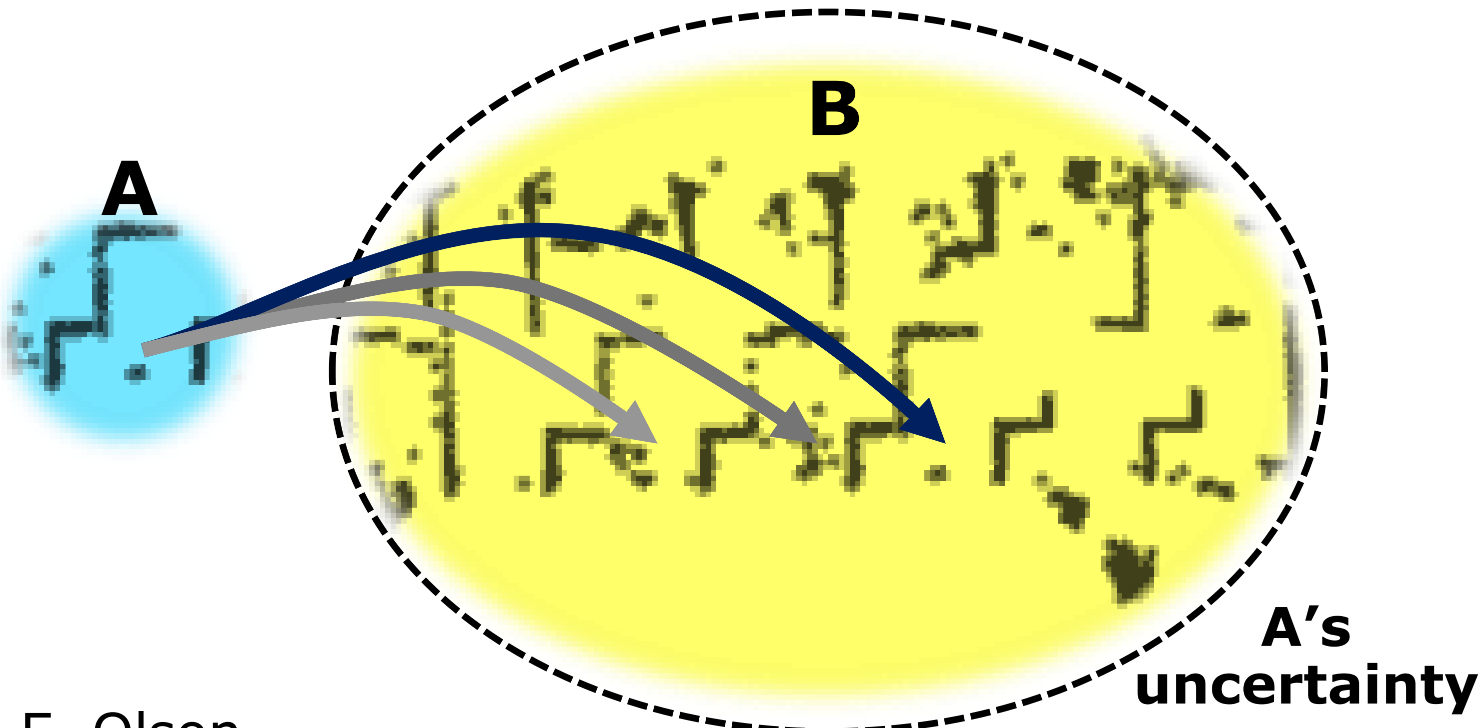
Ambiguities - Global Ambiguity

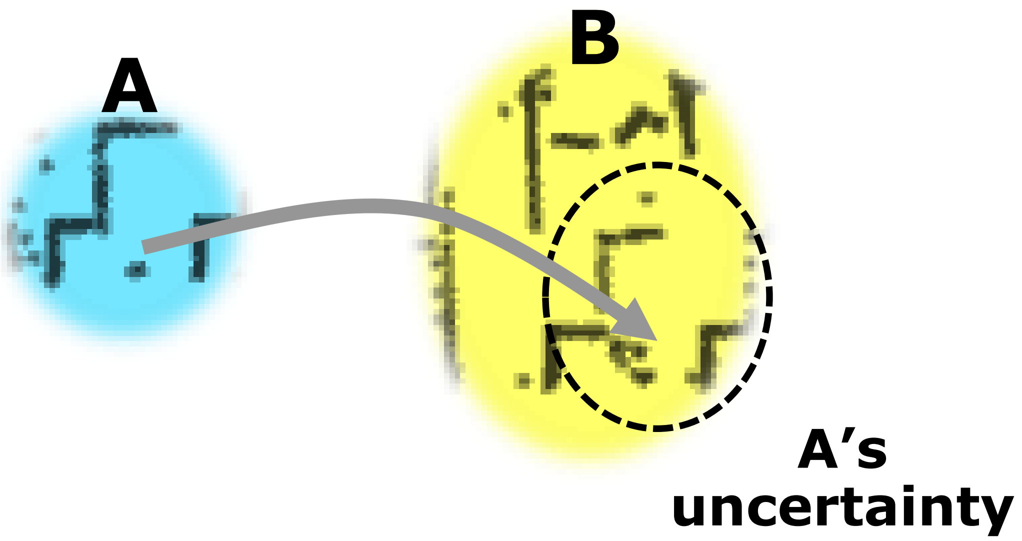
- B is inside the uncertainty ellipse of A
- A and B are not the same place



Courtesy: E. Olson

Ambiguities - Global Sufficiency

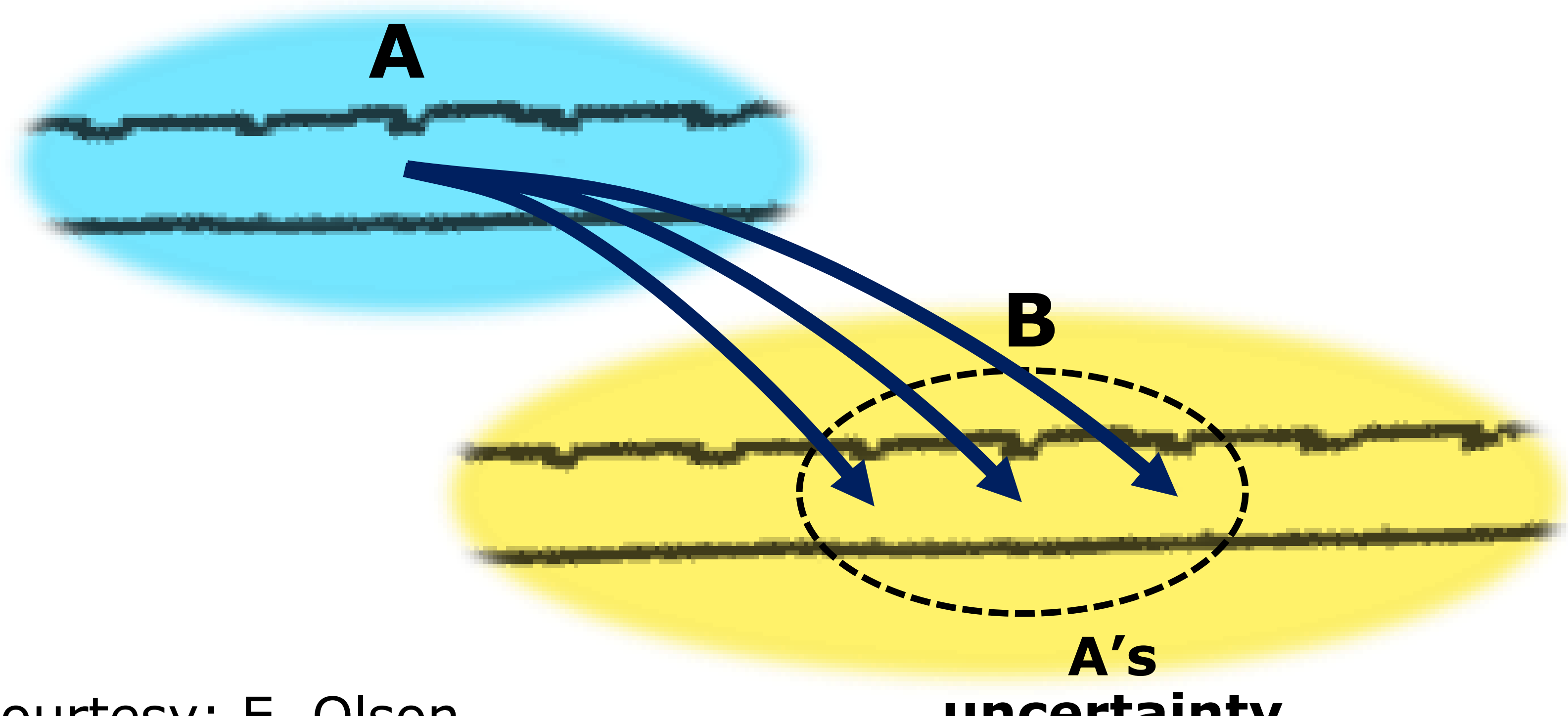
- B is inside the uncertainty ellipse of A
- The is no other possibility for a match



Courtesy: E. Olson

Ambiguities - Local Ambiguity

- "Picket Fence Problem": largely overlapping local matches

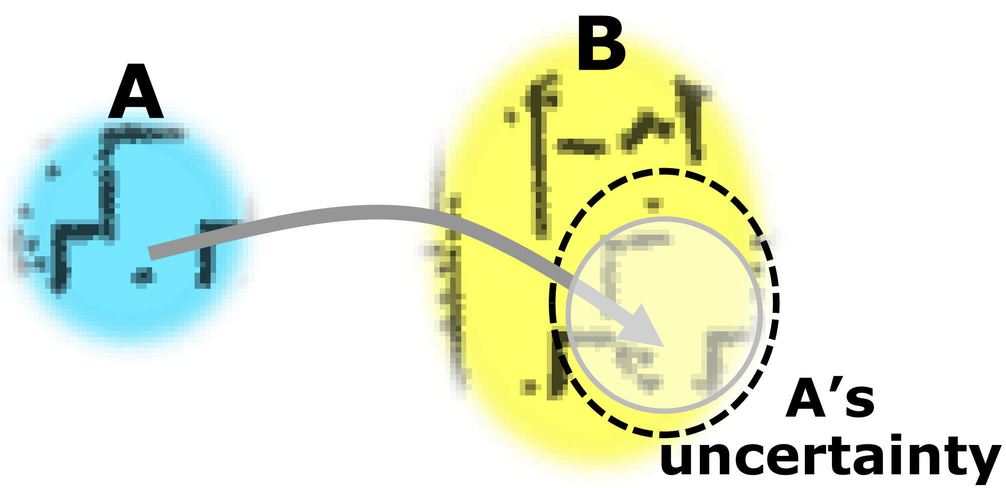


Courtesy: E. Olson

Global Match Criteria

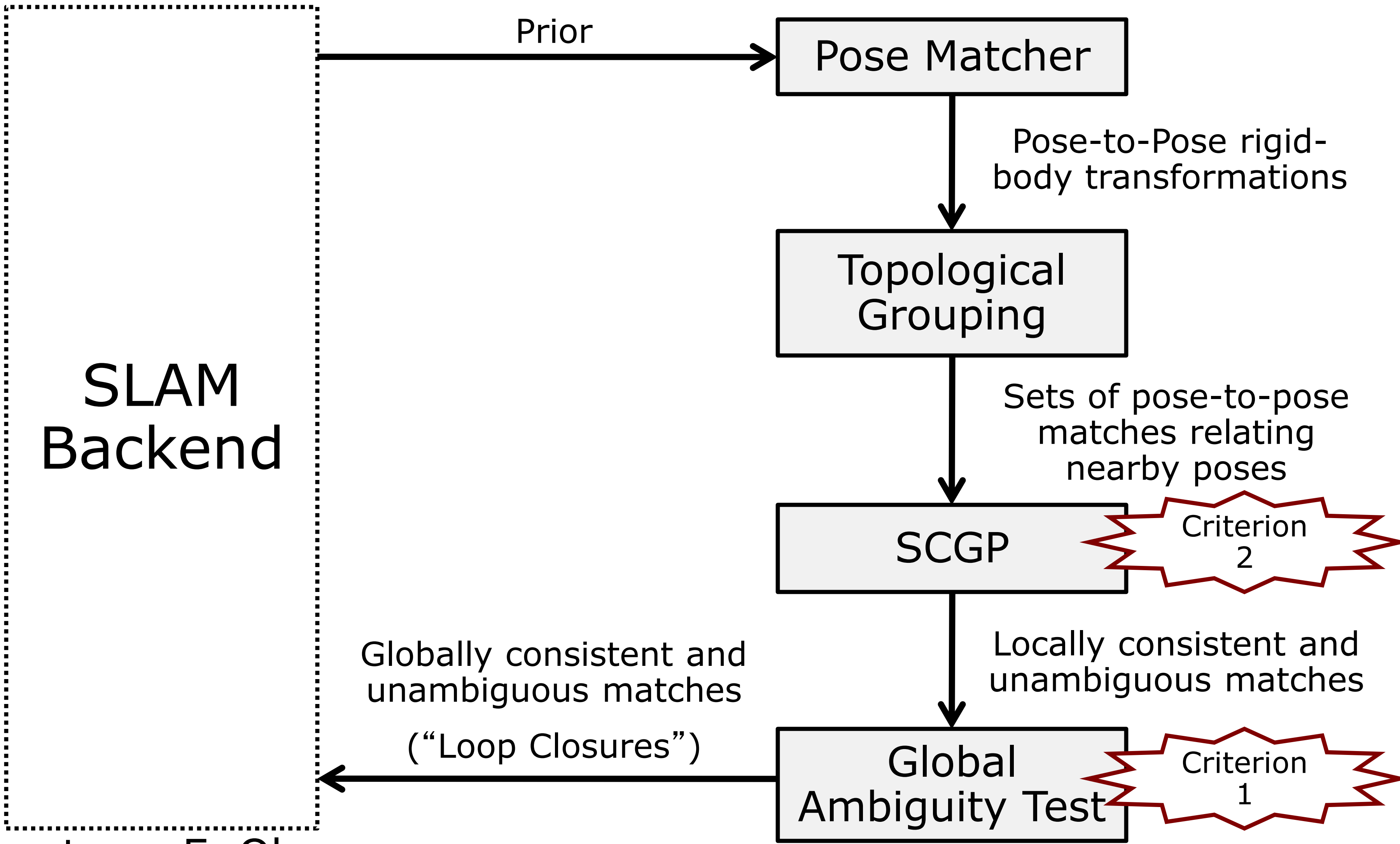
1. Global Sufficiency: There is no possible disjoint match (“A is not somewhere else entirely”)
2. Local unambiguity: There are no overlapping matches (“A is either here or somewhere else entirely”)

Both need to be satisfied for a match



Courtesy: E. Olson

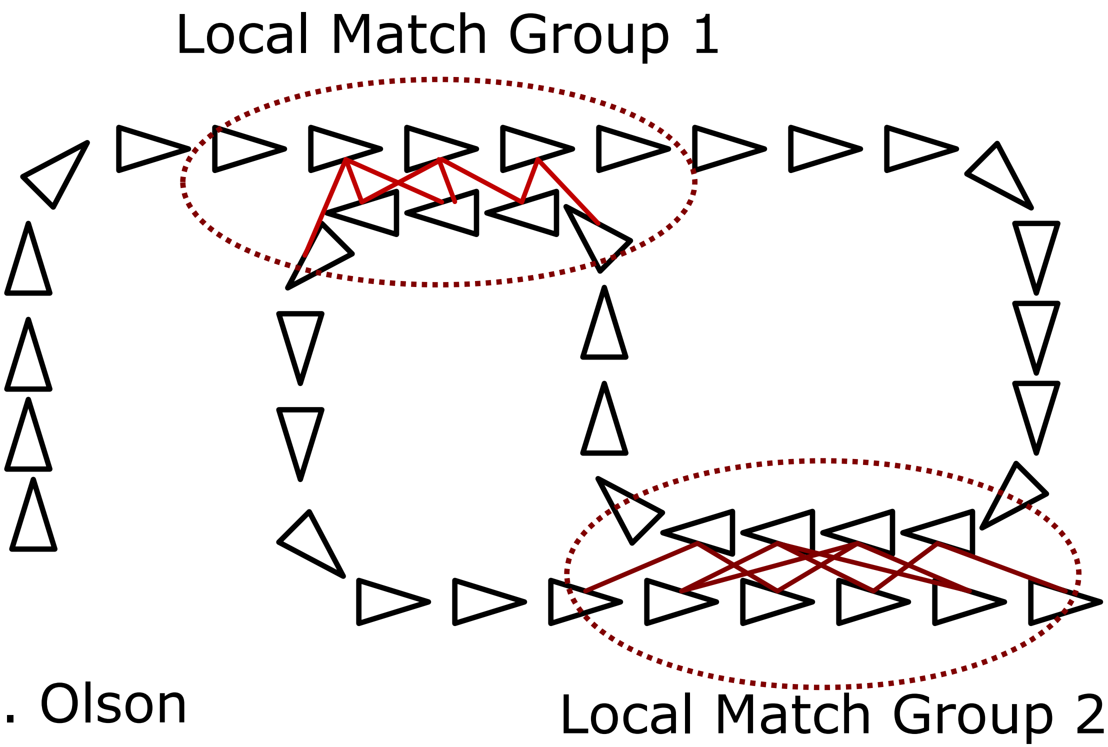
Olson’s Proposal



Courtesy: E. Olson

Topological Grouping

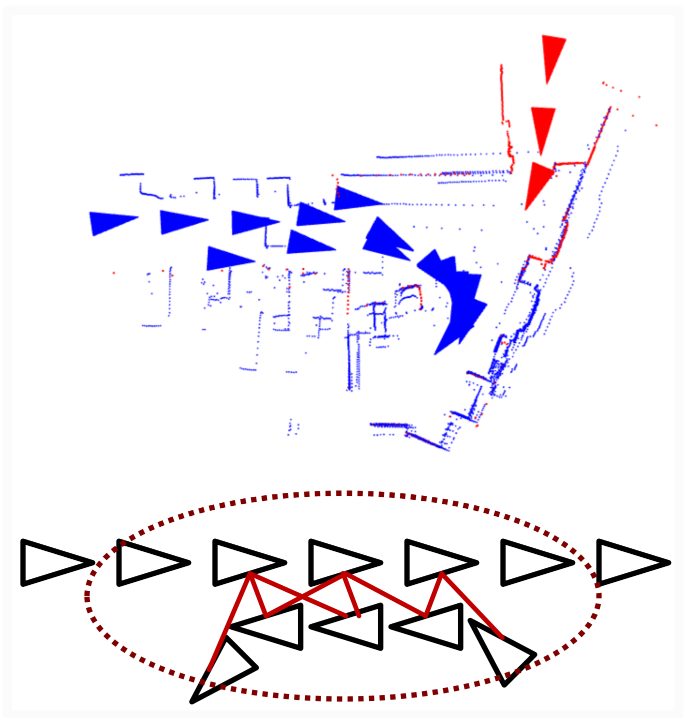
- Group together topologically-related pose-to-pose matches to form local matches
- Each group asks a “topological” question: Do two local maps match?



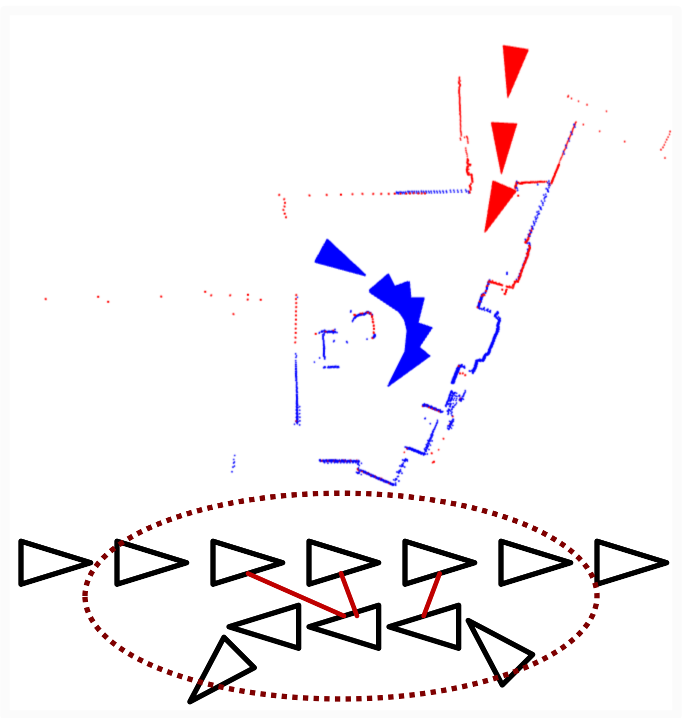
Courtesy: E. Olson

Locally Unambiguous Matches

Goal:



Unfiltered Local Match
(set of pose-to-pose matches)



Locally consistent and
unambiguous local match
(set of pose-to-pose matches)

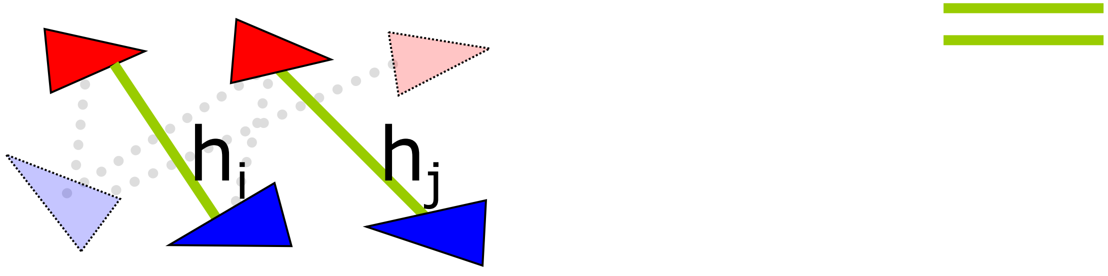
Courtesy: E. Olson

Locally Consistent Matches

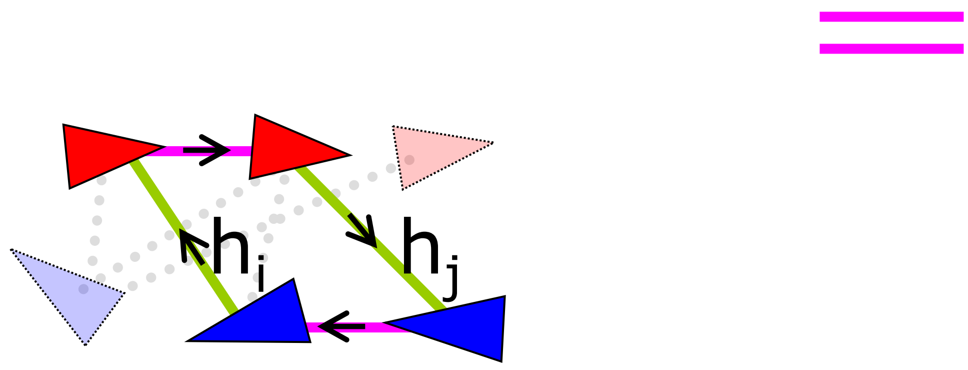
- Correct pose-to-pose hypotheses must agree with each other
- Incorrect pose-to-pose hypotheses tend to disagree with each other
- Find subset of self-consistent of hypotheses
- Multiple self-consistent subsets, are an indicator for a “picket fence”!

Do Two Hypotheses Agree?

- Consider two hypotheses **i** and **j** in the set:



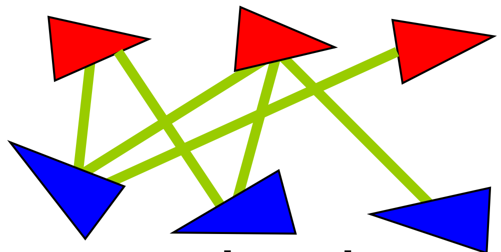
- Form a loop using edges from the prior graph



Rigid-body transformation around the loop should be the identity matrix

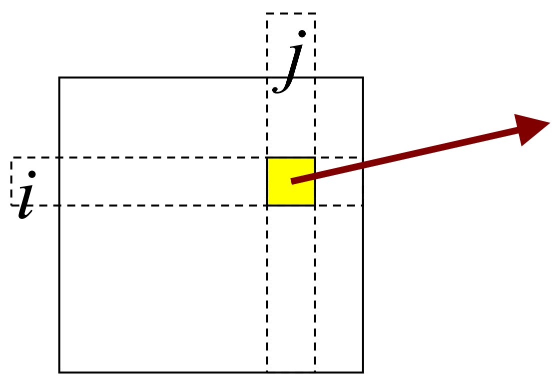
Idea of Olson's Method

- Form pair-wise consistency matrix **A**

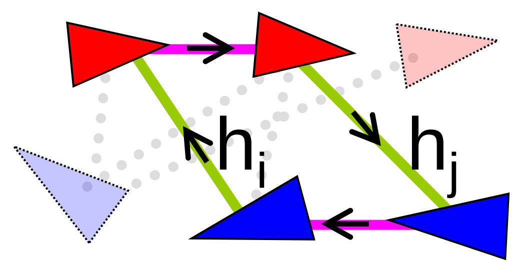


Hypothesis Set

A =



$$A_{ij} = P(\text{loop}(i,j) = I \mid h_i, h_j)$$

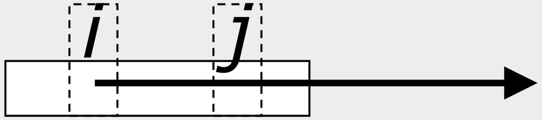


Courtesy: E. Olson

Single Cluster Graph Partitioning

- Idea: Identify the subset of consistent hypotheses
- Find the best **indicator vector** (represents a subset of the hypotheses)

Indicator vector v



v_i = 1 if h_i is correct,
0 if h_i is incorrect

Courtesy: E. Olson

Single Cluster Graph Partitioning

- Identify the subset of hypotheses that is maximally self-consistent
- Which subset **v** has the **greatest average pair-wise consistency** λ?

$$\lambda = \frac{\mathbf{v}^{\mathbf{T}}\mathbf{A}\mathbf{v}}{\mathbf{v}^{\mathbf{T}}\mathbf{v}}$$

Sum of all pair-wise consistencies between hypotheses in v

Number of hypotheses in v

Gallo et al 1989

- Densest subgraph problem

Courtesy: E. Olson

Consistent Local Matches

- We want find **v** that maximizes λ(**v**)

$$\lambda(\mathbf{v}) = \frac{\mathbf{v^T A v}}{\mathbf{v^T v}}$$

- Treat as continuous problem
- Derive and set to zero

$$\frac{\partial \lambda(\mathbf{v})}{\partial \mathbf{v}} = 0$$

- Which leads to (for symmetric A)

$$\frac{\partial \lambda(\mathbf{v})}{\partial \mathbf{v}} = 0 \quad \Longleftrightarrow \quad A\mathbf{v} = \lambda \mathbf{v}$$

Consistent Local Matches

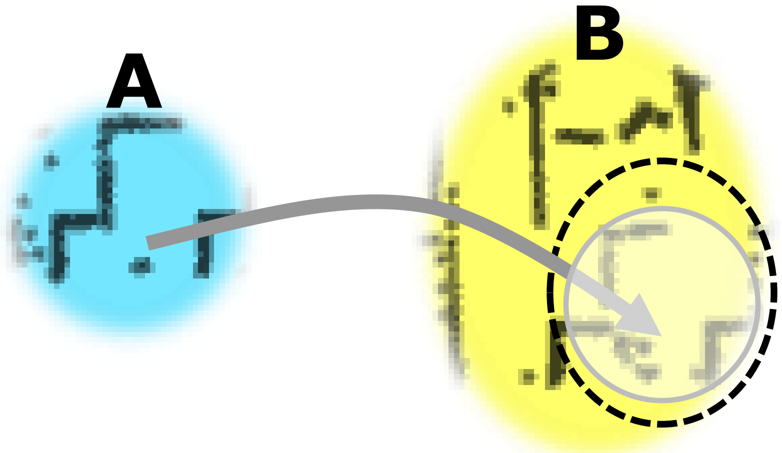
- $A\mathbf{v} = \lambda\mathbf{v}$: Eigenvalue/vector problem
- The dominant eigenvector $\mathbf{v_1}$ maximizes

$$\lambda(\mathbf{v}) = \frac{\mathbf{v}^{\mathbf{T}}\mathbf{A}\mathbf{v}}{\mathbf{v}^{\mathbf{T}}\mathbf{v}}$$

- The hypothesis represented by $\mathbf{v_1}$ is maximally self-consistent subset
- If λ_1/λ_2 is large (e.g., $\lambda_1/\lambda_2>2$) then $\mathbf{v_1}$ is regarded as locally unambiguous
- Discretize $\mathbf{v_1}$ after maximization

Global Consistency

- **Correct method:** Can two copies of A be arranged so that they both fit inside the covariance ellipse?
- **Approximation:** Is the dimension of A at least half the length of the dominant axis of the covariance ellipse?
- Potential failures for narrow local matches



Courtesy: E. Olson

Conclusions

- Matching local observations is used to generate pose-to-pose hypotheses
- Local matches assembled from pose-to-pose hypotheses
- Local ambiguity ("picket fence") can be resolved via SCGP's confidence metric
- Positional uncertainty: more uncertainty requires more evidence

Literature

FLIRT Features

- Tipaldi, Arras: “FLIRT -- Interest Regions for 2D Range Data”

Spectral Clustering

- Olson: “Recognizing Places using Spectrally Clustered Local Matches”

Slide Information

- These slides have been created by Cyrill Stachniss as part of the robot mapping course taught in 2012/13 and 2013/14. I created this set of slides partially extending existing material of Edwin Olson, Giorgio Grisetti, Bastian Steder, Rainer Kümmerle, Patrick Pfaff, and myself.
- I tried to acknowledge all people that contributed image or video material. In case I missed something, please let me know. If you adapt this course material, please make sure you keep the acknowledgements.
- Feel free to use and change the slides. If you use them, I would appreciate an acknowledgement as well. To satisfy my own curiosity, I appreciate a short email notice in case you use the material in your course.
- My video recordings are available through YouTube:
  http://www.youtube.com/playlist?list=PLgnQpQtFTOGQrZ4O5QzbIHgl3b1JHimN_&feature=g-list

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