Theoretical Computer Science (Bridging Course)

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Exercise Sheet 11

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Exercise 11.1 (CNF, DNF)

(a) Convert $\phi := \neg(p \to q) \vee ((r \vee s) \to (q \vee t)) \vee (\neg p \to \neg v)$ into Conjunctive Normal Form.

Solution:

$$
\begin{aligned}
&\neg(p \to q) \vee ((r \wedge s) \to (q \vee t)) \vee (\neg p \to \neg v) \equiv \\
&\equiv \neg(\neg p \vee q) \vee (\neg(r \vee s) \vee (q \vee t)) \vee (p \vee \neg v) \equiv \\
&\equiv (p \wedge \neg q) \vee (\neg r \wedge \neg s) \vee q \vee t \vee p \vee \neg v \equiv \\
&\equiv (p \wedge \neg q) \vee ((\neg r \vee q \vee t \vee p \vee \neg v) \wedge (\neg s \vee q \vee t \vee p \vee \neg v)) \equiv \\
&\equiv (\neg r \vee p \vee q \vee t \vee \neg v) \wedge (\neg s \vee p \vee q \vee t \vee \neg v) \wedge (\neg r \vee p \vee t \vee \neg v \vee \top) \wedge (\neg s \vee p \vee t \vee \neg v \vee \top) \equiv \\
&\equiv (\neg r \vee p \vee q \vee t \vee \neg v) \wedge (\neg s \vee p \vee q \vee t \vee \neg v).
\end{aligned}
$$

(b) Convert $\phi := \bigvee_{i=1}^{n} (p_i \leftrightarrow q_i)$ into Disjunctive Normal Form.

Solution:

$$
\begin{aligned}
\bigvee_{i=1}^{n} (p_i \leftrightarrow q_i) &\equiv \bigvee_{i=1}^{n} ((p_i \to q_i) \wedge (q_i \to p_i)) \equiv \\
&\equiv \bigvee_{i=1}^{n} ((\neg p_i \vee q_i) \wedge (\neg q_i \vee p_i)) \equiv \\
&\equiv \bigvee_{i=1}^{n} ((\neg p_i \wedge q_i) \vee (\neg p_i \wedge p_i) \vee (q_i \wedge \neg q_i) \vee (q_i \wedge p_i)) \equiv \\
&\equiv \bigvee_{i=1}^{n} ((\neg p_i \wedge \neg q_i) \vee (q_i \wedge p_i)) \equiv \\
&\equiv \bigvee_{i=1}^{n} (\neg p_i \wedge \neg q_i) \vee \bigvee_{i=1}^{n} (q_i \wedge p_i).
\end{aligned}
$$

Exercise 11.2 (Derivation, 3 marks)
Give a derivation of $\phi = B \wedge C$ from the knowledge base

$$KB = \{A, B, A \vee C, K \wedge E \leftrightarrow A \wedge B, \neg C \to D, E \vee F \to \neg D\},$$

using the inference rules for propositional logic.

Solution:

1. A (KB)
2. B (KB)
3. $A \wedge B$ (1, 2, and introduction)
4. $K \wedge E \leftrightarrow A \wedge B$ (KB)
5. $A \wedge B \rightarrow K \wedge E$ (4, $\leftrightarrow$ elimination)
6. $K \wedge E$ (3, 5, modus ponens)
7. E (6, and elimination)
8. $E \vee F$ (7, or introduction)
9. $E \vee F \rightarrow \neg D$ (KB)
10. $\neg D$ (8, 9, modus ponens)
11. $\neg C \rightarrow D$ (KB)
12. C (10, 11, modus tolens)
13. $B \wedge C$ (2, 12, and introduction)

Exercise 11.3 (Contradiction Theorem)

Prove the contradiction theorem: $KB \cup \{\varphi\}$ is unsatisfiable iff $KB \models \neg\varphi$.
Hint: *Deduction Theorem* can be useful here.

Solution:

$$
\begin{aligned}
& KB \cup \{\varphi\} \text{ is unsatisfiable} \Leftrightarrow \\
\Leftrightarrow\ & KB \cup \{\varphi\} \models \bot \text{ (immediate)} \Leftrightarrow \\
\Leftrightarrow\ & KB \models \varphi \rightarrow \bot \text{ (deduction theorem)} \Leftrightarrow \\
\Leftrightarrow\ & KB \models (\neg\varphi \vee \bot) \text{ (logical equivalence).} \Leftrightarrow \\
\Leftrightarrow\ & KB \models \neg\varphi \text{ (logical equivalence).}
\end{aligned}
$$