Theoretical Computer Science (Bridging Course)

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Exercise Sheet 10

Due: 22nd January 2015

Exercise 10.1 (Propositional Logic)

Determine the validity or invalidity of the following argument:

"If Alice is elected class-president, then either Betty is elected vice-president, or Carol is elected treasurer. Betty is elected vice-president. Therefore if Alice is elected class-president, then Carol is not elected treasurer."

Please explain every formal step.

Exercise 10.2 (Propositional Logic)

(a) Consider the following logical formula:

$$\phi = (A \leftrightarrow \neg B) \wedge \neg(C \vee B \rightarrow A)$$

Show that $\phi \equiv \neg A \wedge B$ by using the equivalences from the lectures (see slide 17, `08.pdf`) and the equivalences $\psi \wedge \neg\psi \equiv \bot$ and $\psi \vee \bot \equiv \psi \equiv \bot \vee \psi$. Apply in each step only one of the equivalences with the exception that you *may* implicitly use associativity.

(b) Consider a vocabulary with only four atomic propositions A, B, C, D. How many models are there for the following formulae? Explain.

i) $(A \wedge B) \vee (B \wedge C)$

ii) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 10.3 (Propositional Logic)

Show that the following formula is *valid*:

$$(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A).$$

The implication $\neg B \rightarrow \neg A$ is sometimes called *contrapositive* or *counternominal* implication of $A \rightarrow B$.