# Robot Mapping

# Max-Mixture and Robust Least Squares for SLAM

**Cyrill Stachniss**

Courtesy for most images: Pratik Agarwal

# Least Squares in General

- Minimizes the **sum of the squared errors**
- Strong relation to ML estimation in the Gaussian case

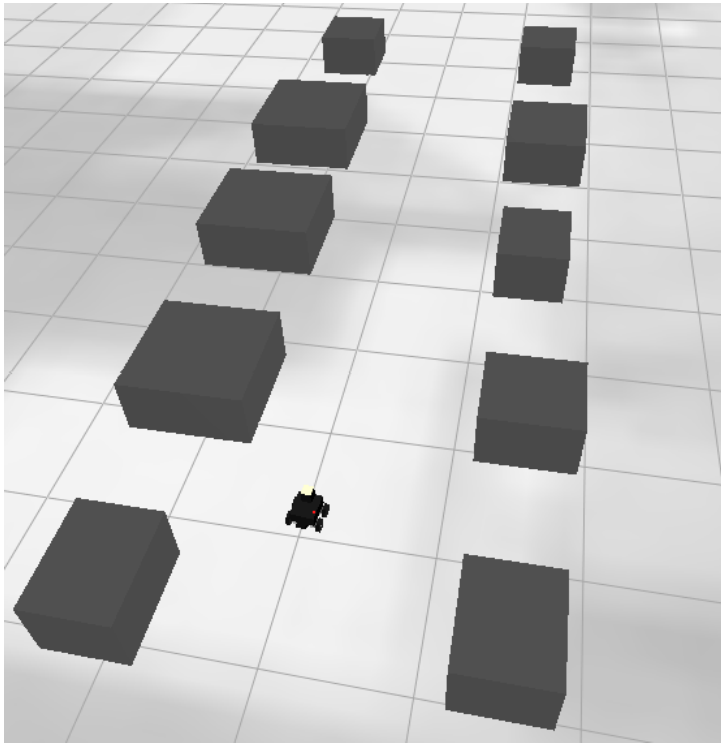
**Problems:**

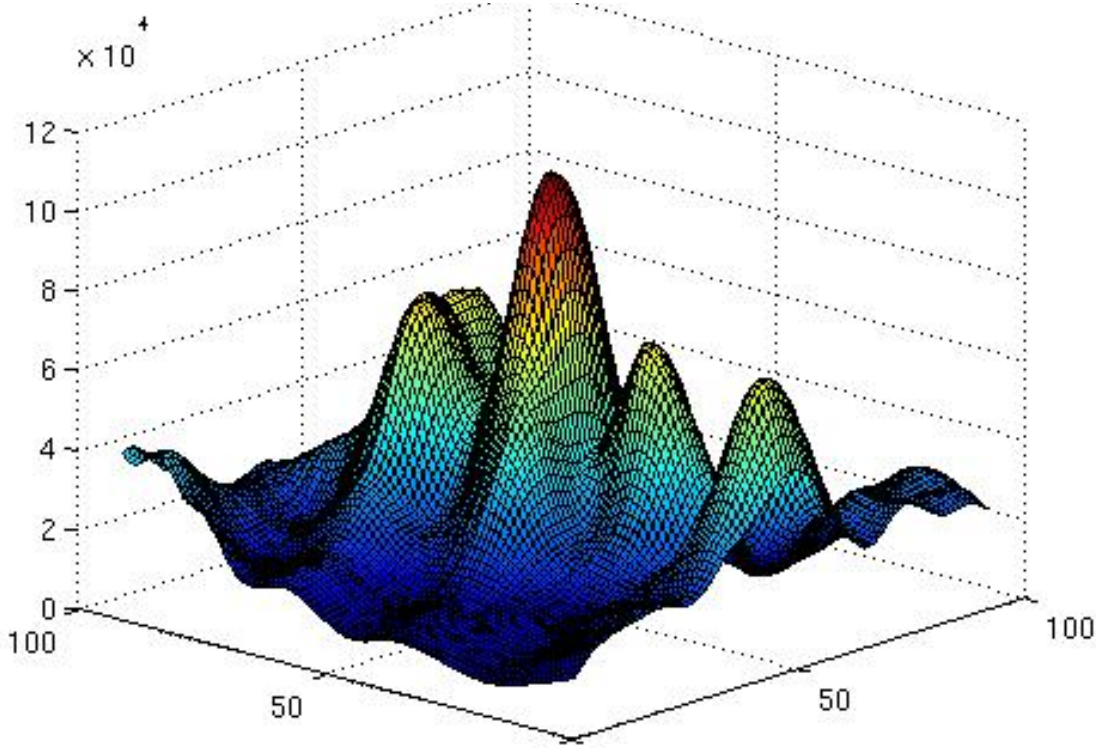
- **Sensitive to outliers**
- **Only Gaussians** (single modes)

# Data Association Is Ambiguous And Not Always Perfect

- Places that look identical
- Similar rooms in the same building
- Cluttered scenes
- GPS multi pass (signal reflections)
- …

# Example

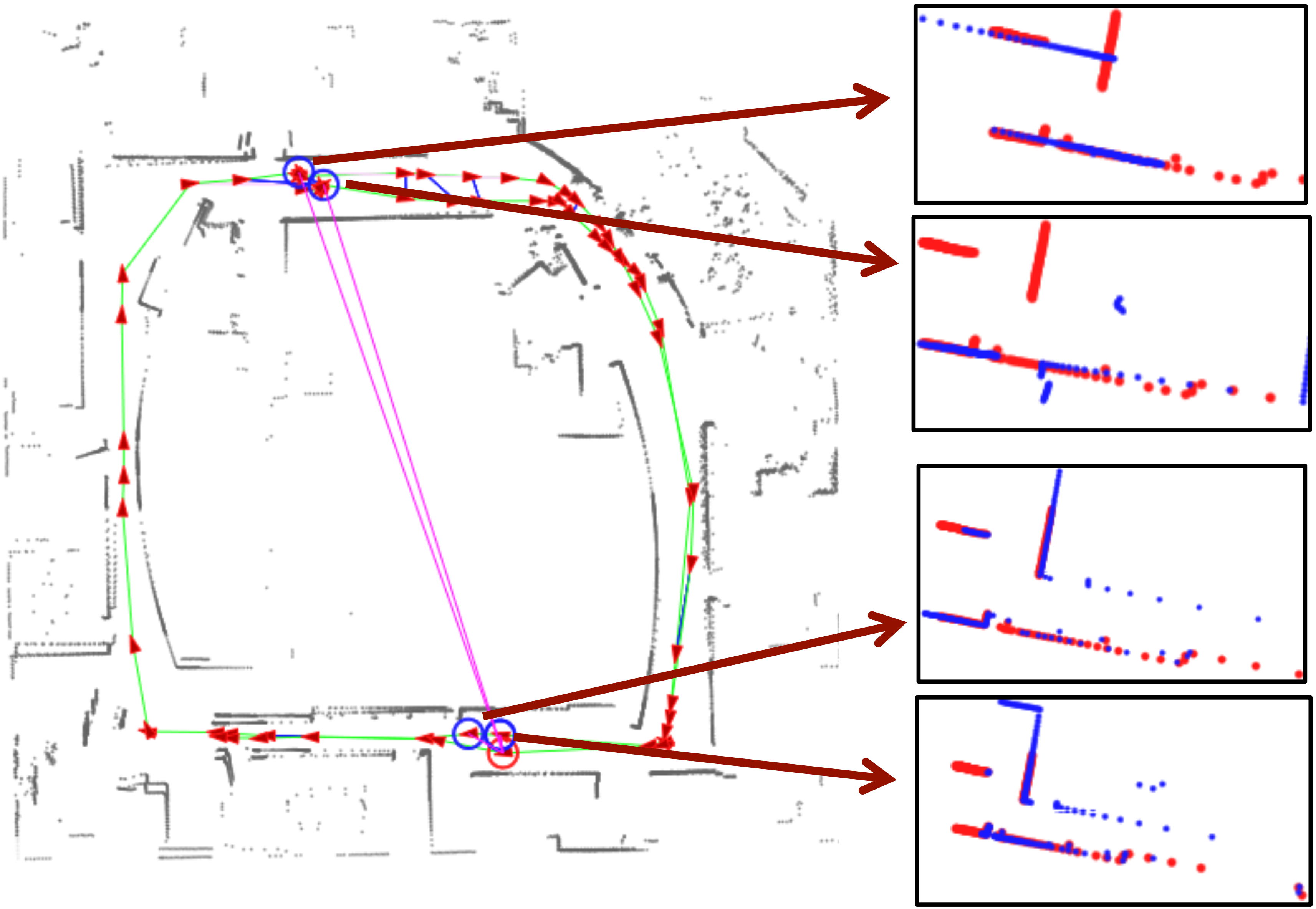




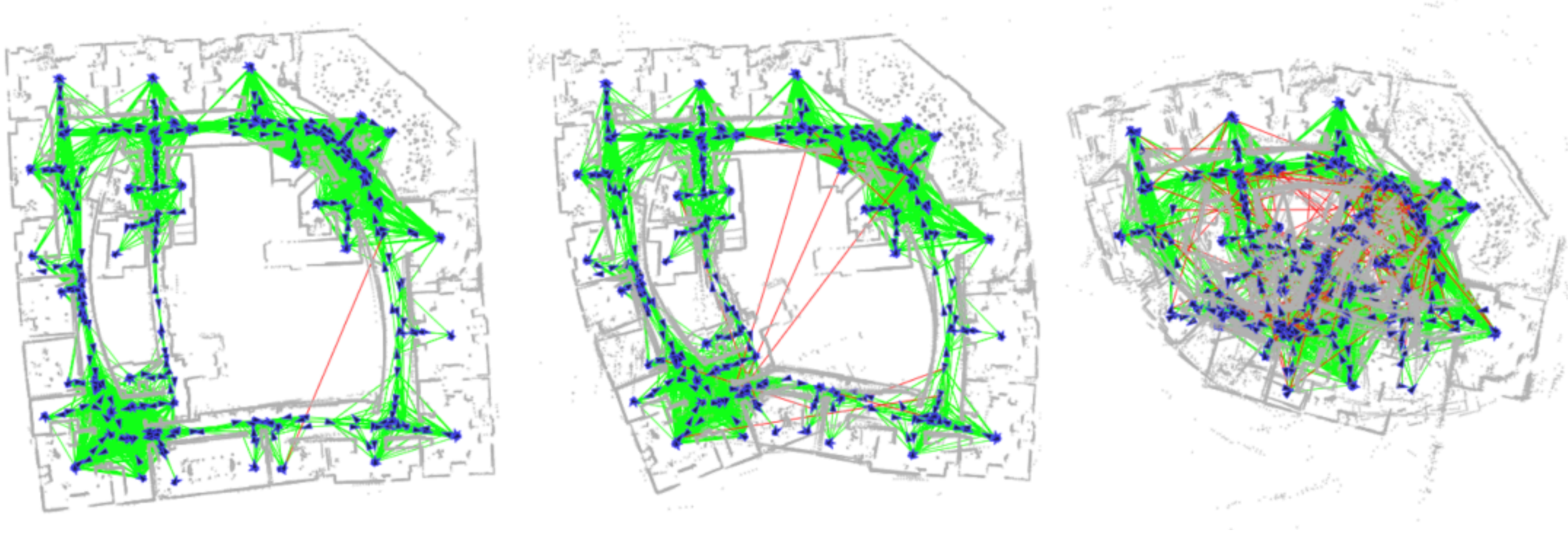

3D world

belief about the robot’s pose

# Such Situations Occur In Reality



# Committing To The Wrong Mode Can Lead to Mapping Failures



# Data Association Is Ambiguous And Not Always Perfect

- Places that look identical
- Similar rooms in the same building
- Cluttered scenes
- GPS multi pass (signal reflections)
- …

**How to incorporate that into graph-based SLAM?**

# Data Association Is Ambiguous And Not Always Perfect

- Places that look identical
- Similar rooms in the same building
- Cluttered scenes
- GPS multi pass (signal reflections)
- …

**How to incorporate that into graph-based SLAM?**

# Mathematical Model

- We can express a multi-modal belief by a sum of Gaussians

$$p(\mathbf{z} \mid \mathbf{x}) = \eta \exp(-\frac{1}{2}\mathbf{e}_{ij}^T \mathbf{\Omega}_{ij} \mathbf{e}_{ij})$$

$$\Downarrow$$

$$p(\mathbf{z} \mid \mathbf{x}) = \sum_k w_k \eta_k \exp(-\frac{1}{2}\mathbf{e}_{ij_k}^T \mathbf{\Omega}_{ij_k} \mathbf{e}_{ij_k})$$

**Sum of Gaussians with k modes**

# Problem

- During error minimization, we consider the negative log likelihood

$$-\log p(\mathbf{z} \mid \mathbf{x}) = \frac{1}{2}\mathbf{e}_{ij}^T \mathbf{\Omega}_{ij} \mathbf{e}_{ij} - \log \eta$$

$$\Downarrow$$

$$-\log p(\mathbf{z} \mid \mathbf{x}) = -\log \sum_k w_k \eta_k \exp(-\frac{1}{2}\mathbf{e}_{ij_k}^T \mathbf{\Omega}_{ij_k} \mathbf{e}_{ij_k})$$

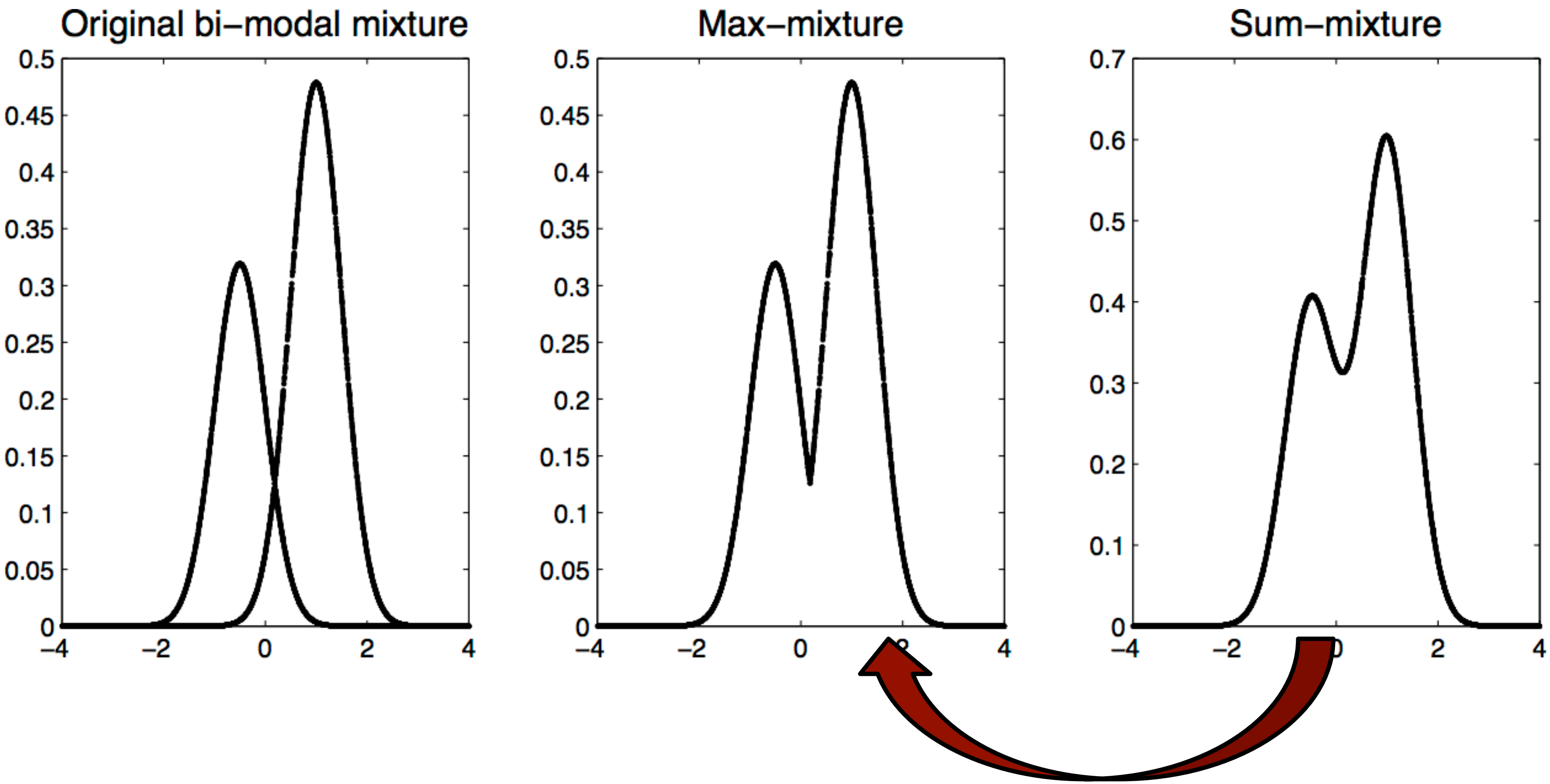
**The log cannot be moved inside the sum!**

# Max-Mixture Approximation

- Instead of computing the sum of Gaussians at $\mathrm{x}$, compute the maximum of the Gaussians

$$
\begin{aligned}
p(\mathbf{z} \mid \mathbf{x}) &= \sum_k w_k \eta_k \exp(-\frac{1}{2}\mathbf{e}_{ij_k}^T \mathbf{\Omega}_{ij_k} \mathbf{e}_{ij_k}) \\
&\simeq \max_k w_k \eta_k \exp(-\frac{1}{2}\mathbf{e}_{ij_k}^T \mathbf{\Omega}_{ij_k} \mathbf{e}_{ij_k})
\end{aligned}
$$

# Max-Mixture Approximation




**approximation error**

# Log Likelihood Of The Max-Mixture Formulation

- The log can be moved inside the max operator

$$p(\mathbf{z} \mid \mathbf{x}) \simeq \max_k w_k \eta_k \exp(-\frac{1}{2}\mathbf{e}_{ij_k}^T \mathbf{\Omega}_{ij_k} \mathbf{e}_{ij_k})$$

$$\log p(\mathbf{z} \mid \mathbf{x}) \simeq \max_k -\frac{1}{2}\mathbf{e}_{ij_k}^T \mathbf{\Omega}_{ij_k} \mathbf{e}_{ij_k} + \log(w_k \eta_k)$$

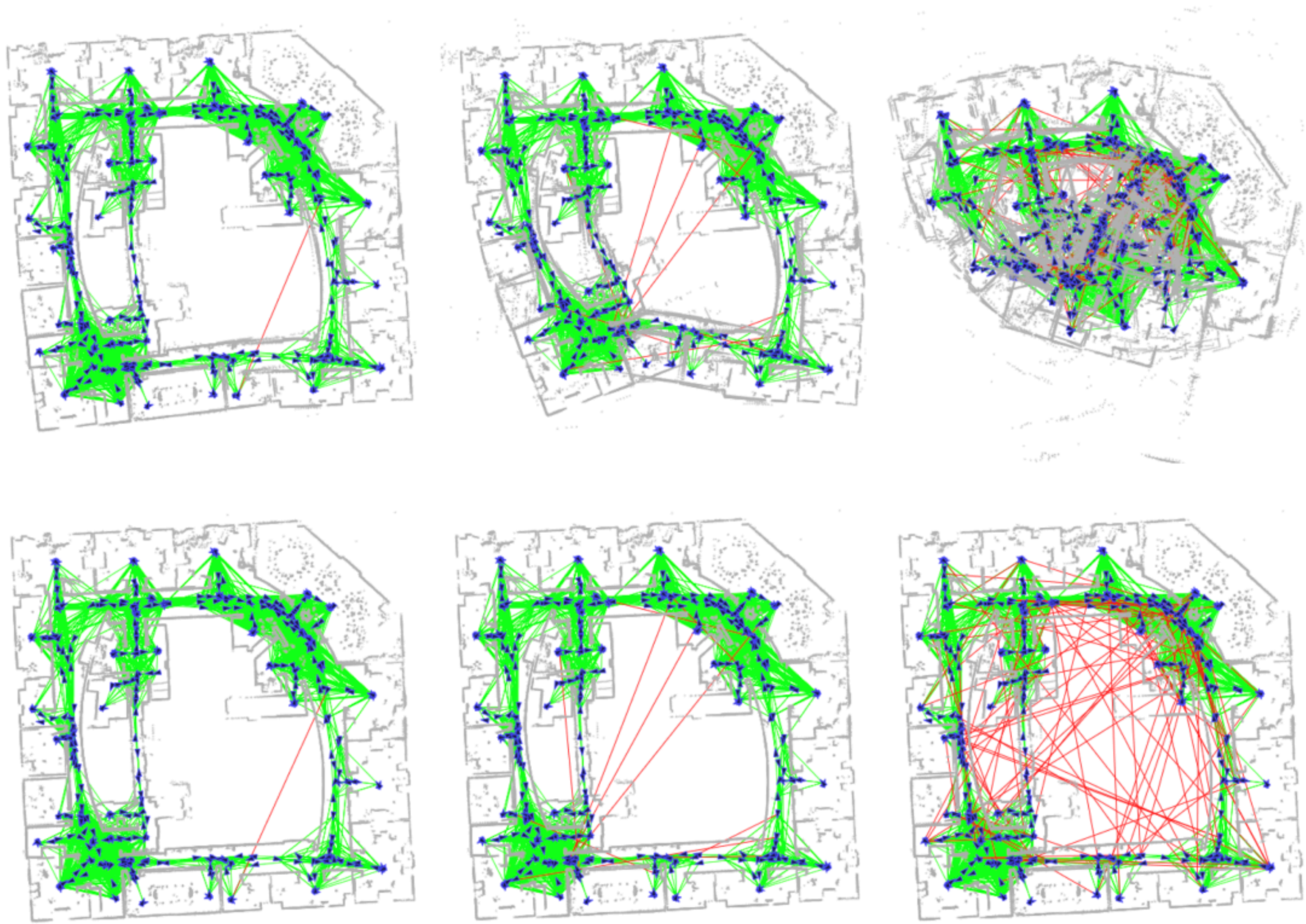
or: $$-\log p(\mathbf{z} \mid \mathbf{x}) \simeq \min_k \frac{1}{2}\mathbf{e}_{ij_k}^T \mathbf{\Omega}_{ij_k} \mathbf{e}_{ij_k} - \log(w_k \eta_k)$$

# Integration

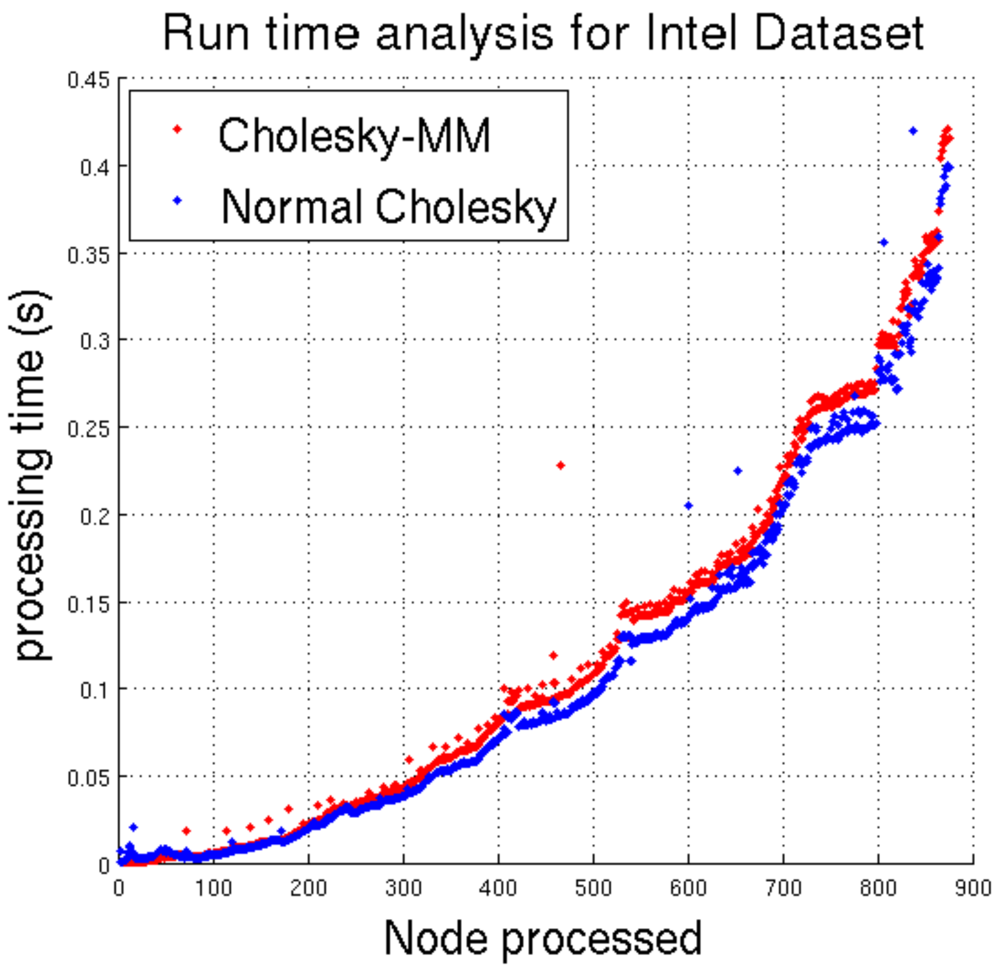
- With the max-mixture formulation, the log likelihood again results in local quadratic forms
- Easy to integrate in the optimizer:

1. Evaluate all k components
2. Select the component with the maximum log likelihood
3. Perform the optimization as before using only the max components (as a single Gaussian)

# Performance (Gauss vs. MM)

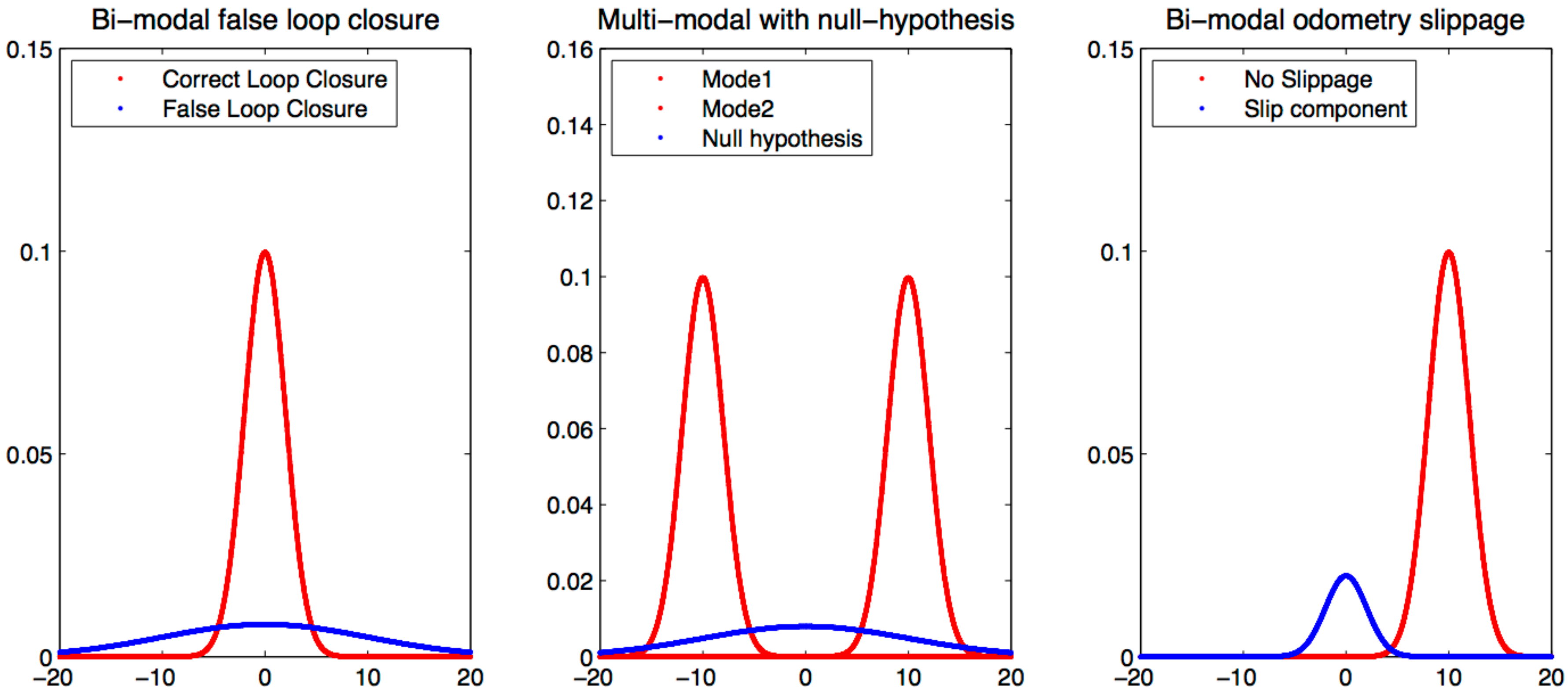


# Runtime



Run time analysis for Intel Dataset
Cholesky-MM
Normal Cholesky
processing time (s)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
100
200
300
400
500
600
700
800
900
Node processed

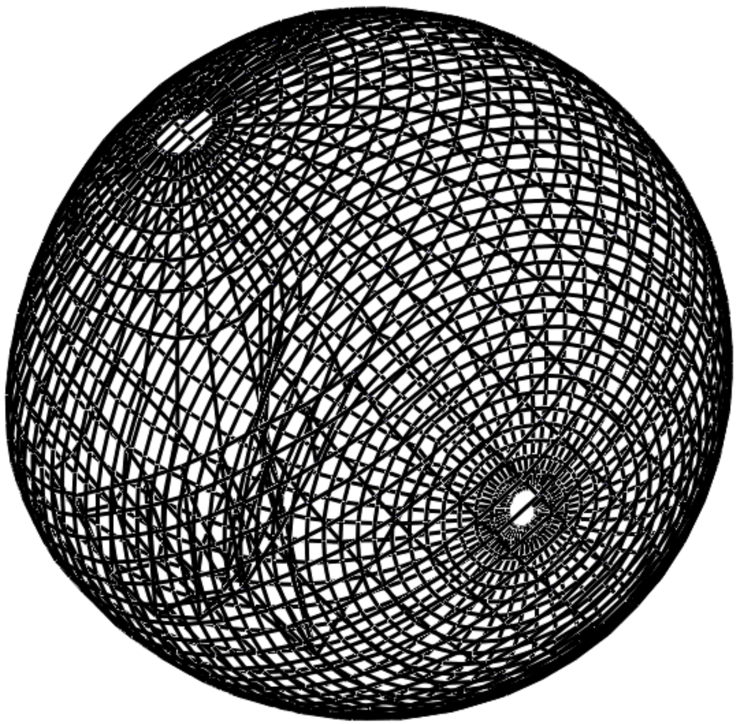
# MM For Outlier Rejection



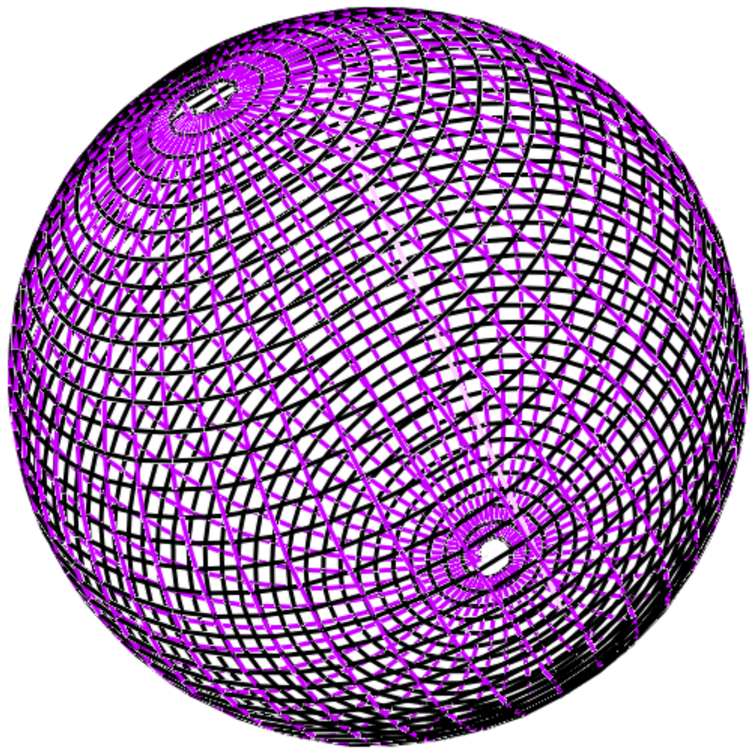
# Max-Mixture and Outliers

- MM formulation is useful for multi-model constraints (D.A. ambiguities)
- MM is also a handy tool outliers (D.A. failures)
- Here, one mode represents the edge and a second model uses a  flat Gaussian for the outlier hypothesis

# Performance (1 outlier)

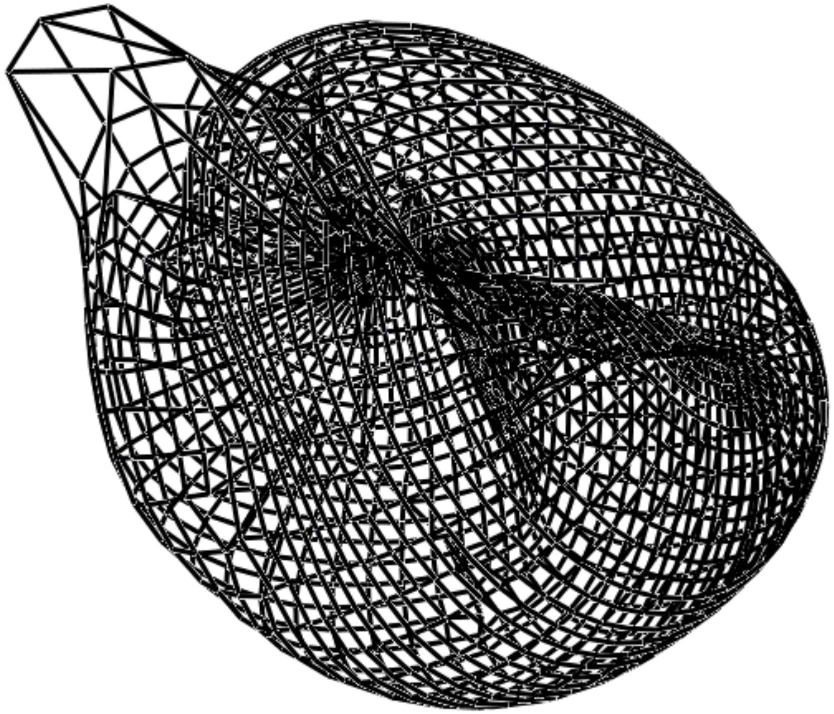


Gauss-Newton

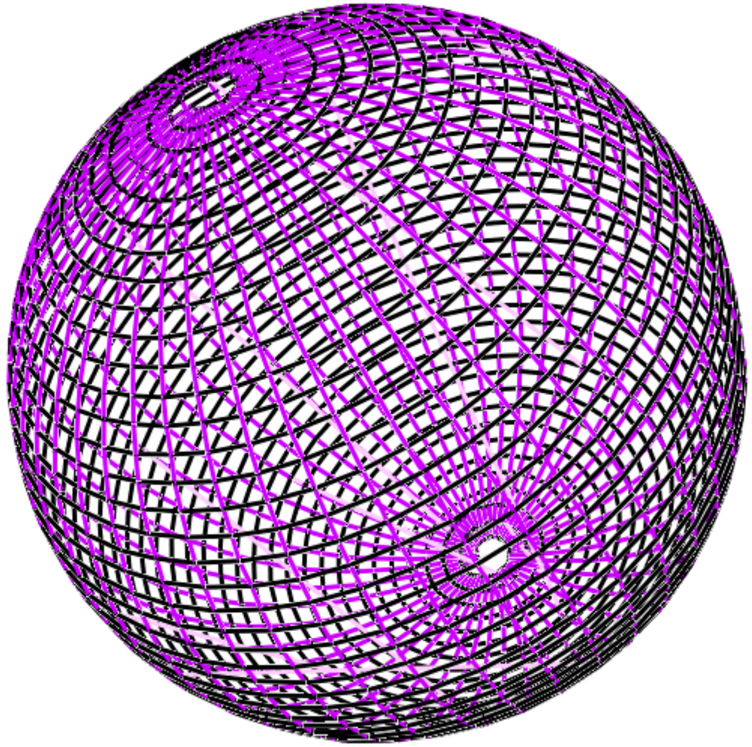


MM Gauss-Newton

# Performance (10 outliers)

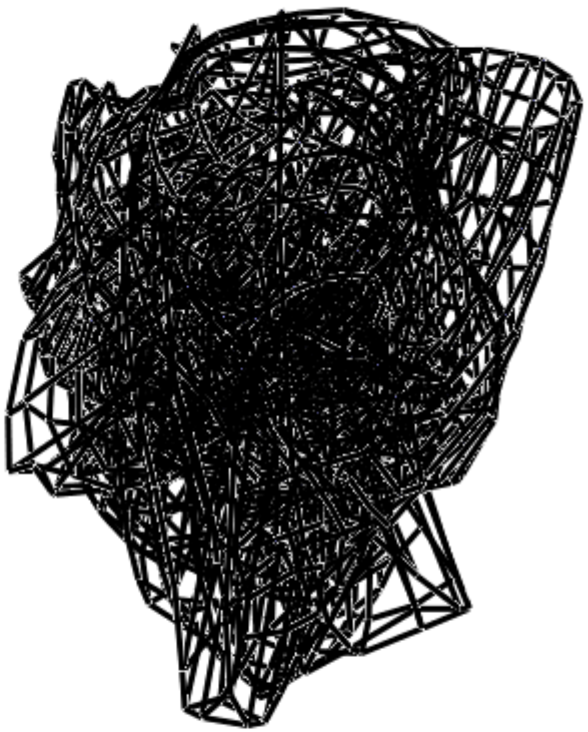


Gauss-Newton

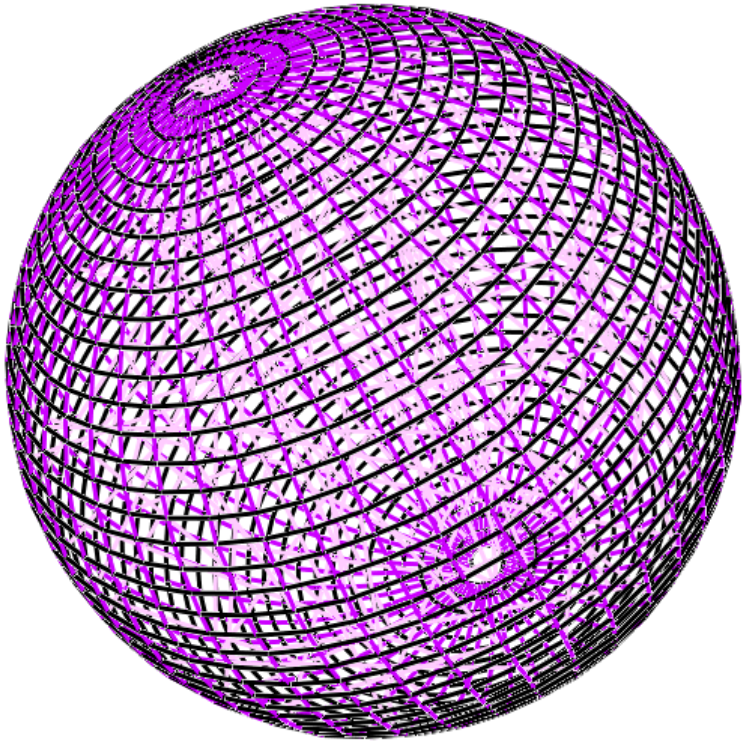


MM Gauss-Newton

# Performance (100 outliers)



Gauss-Newton



MM Gauss-Newton

# Standard Gaussian Least Squares

$$X^* = \underset{X}{\operatorname{argmin}} \sum_{ij} \underbrace{\mathbf{e}_{ij}(X)^T \Omega_{ij} \mathbf{e}_{ij}(X)}_{\chi^2_{ij}}$$

# Dynamic Covariance Scaling

$$X^* = \operatorname*{argmin}_X \sum_{ij} \underbrace{\mathbf{e}_{ij}(X)^T \Omega_{ij} \mathbf{e}_{ij}(X)}_{\chi^2_{ij}}$$

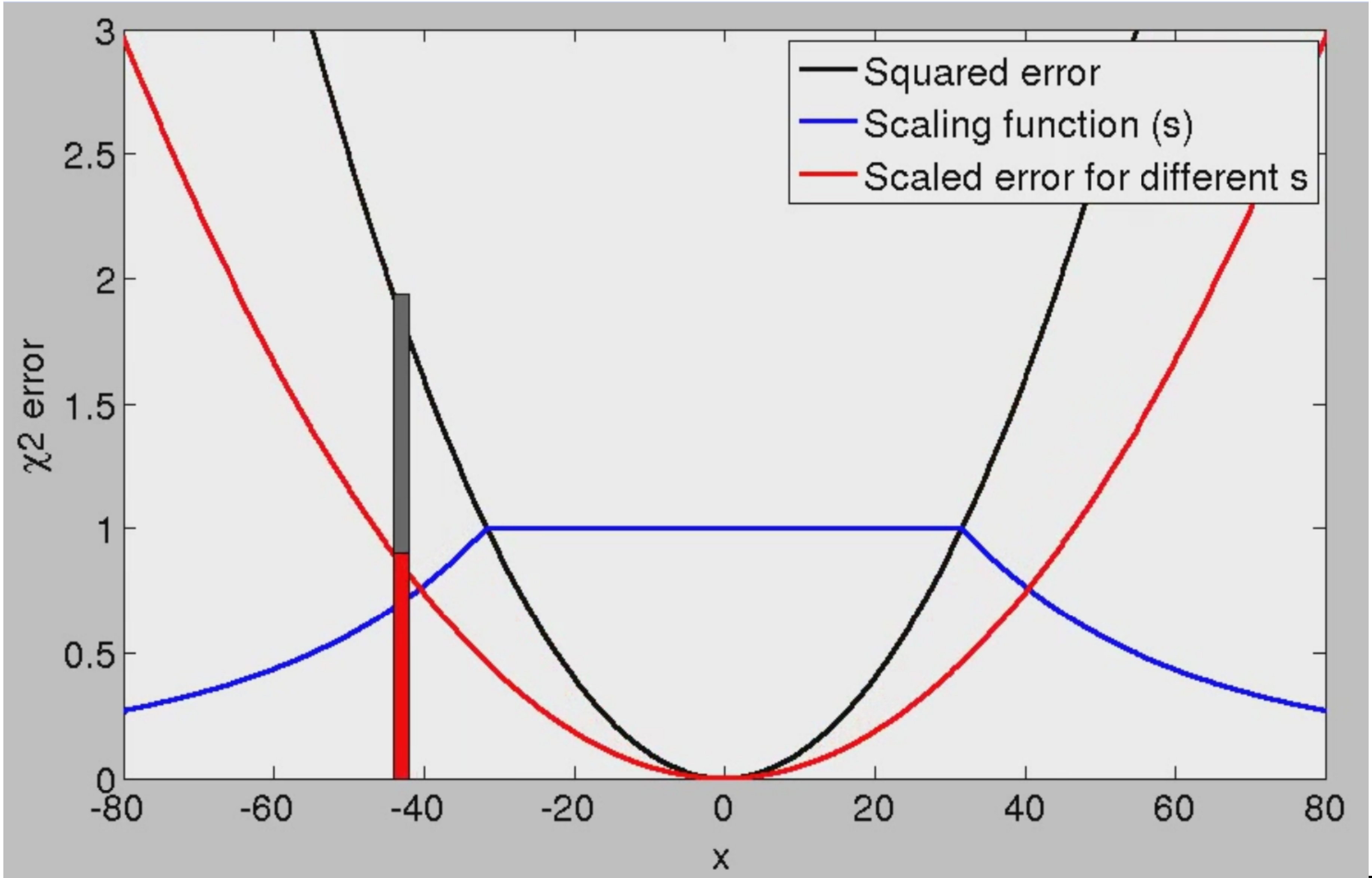
$$X^* = \operatorname*{argmin}_X \sum_{ij} \mathbf{e}_{ij}(X)^T \left(s_{ij}^2 \Omega_{ij}\right) \mathbf{e}_{ij}(X)$$

# Scaling Parameter

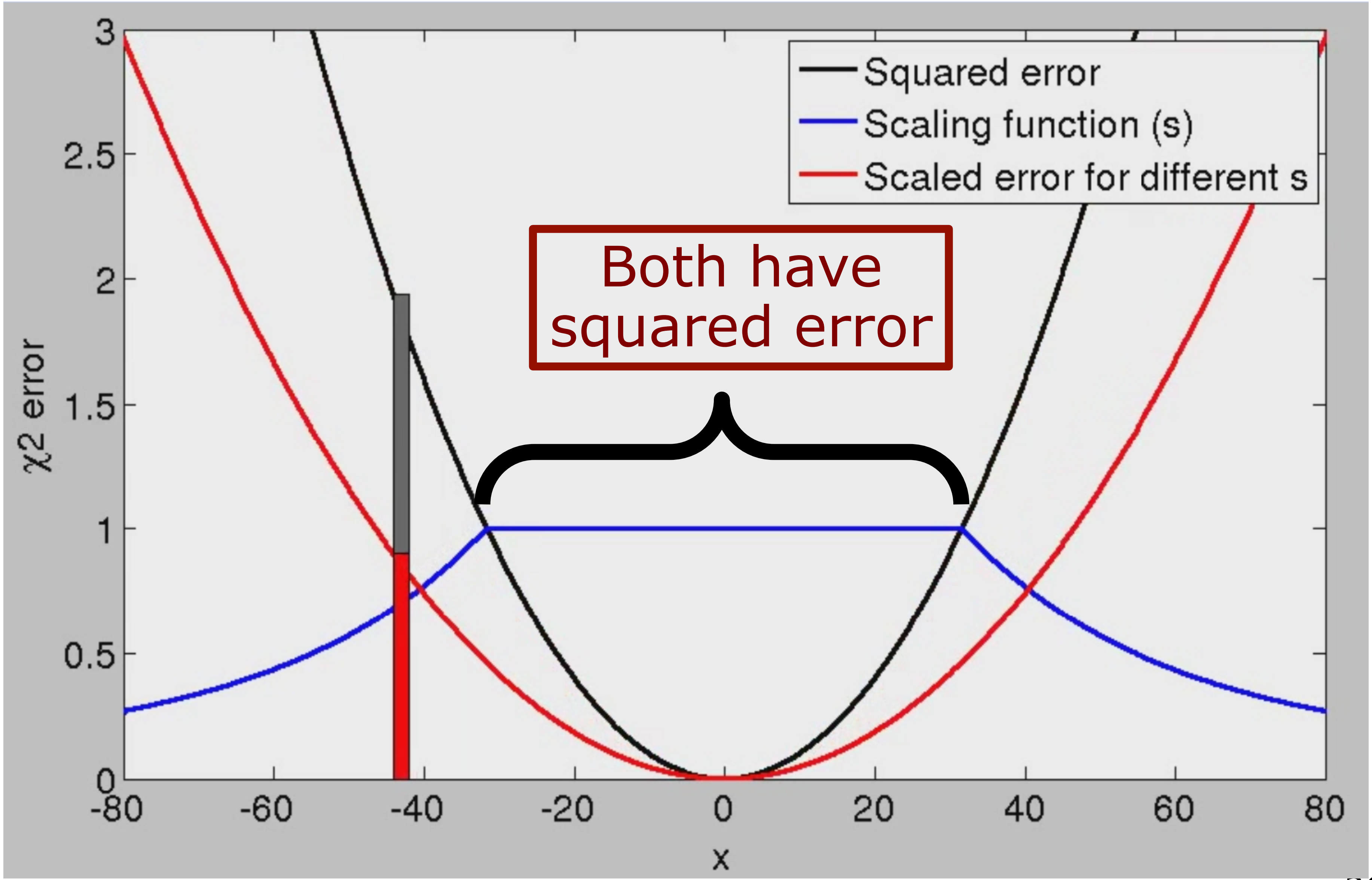
$$X^* = \underset{X}{\operatorname{argmin}} \sum_{ij} \mathbf{e}_{ij}(X)^T \left(s_{ij}^2 \Omega_{ij}\right) \mathbf{e}_{ij}(X)$$

$$s_{ij} = \min\left(1, \frac{2\Phi}{\Phi + \chi_{ij}^2}\right)$$

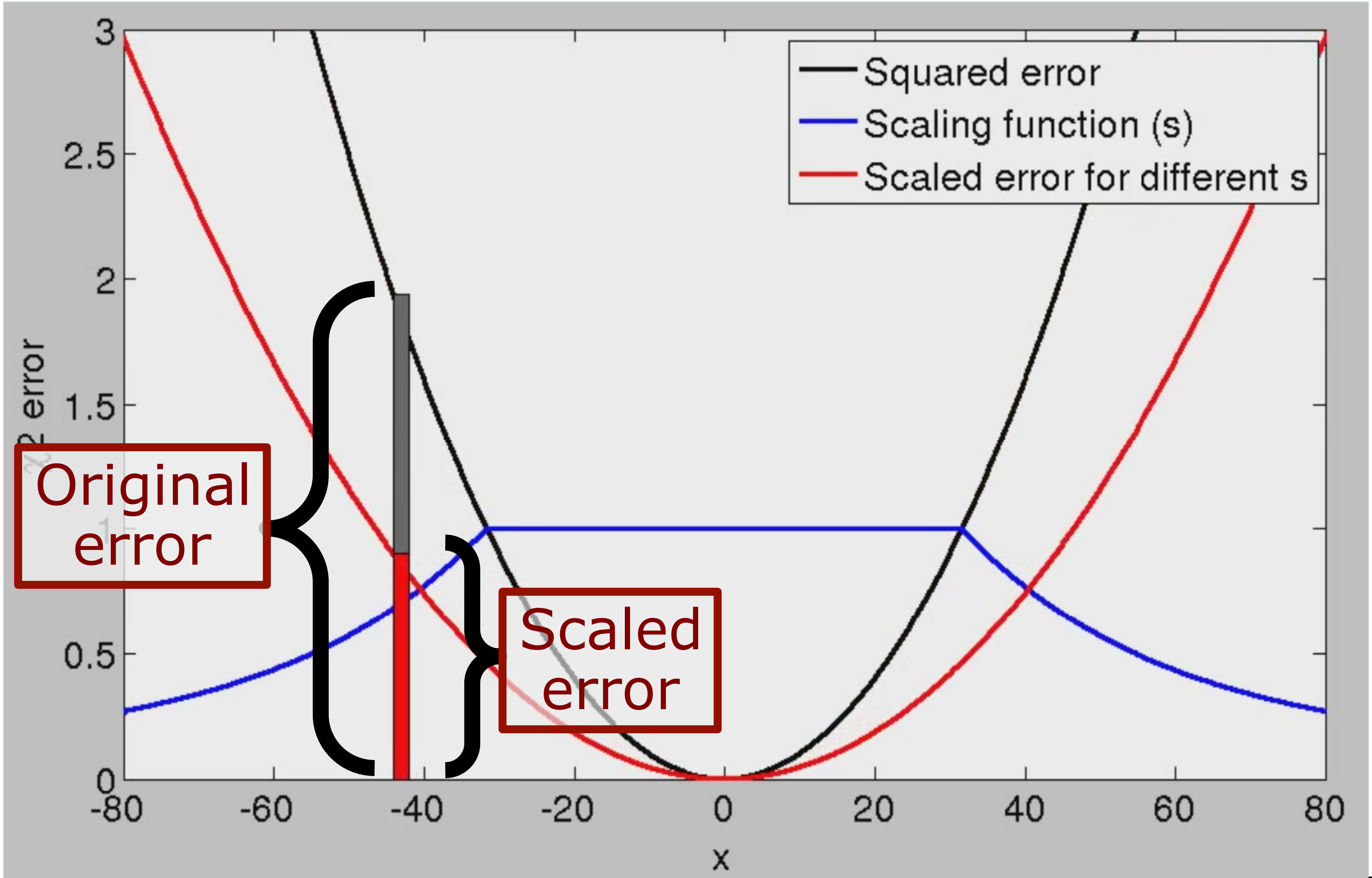
# Dynamic Covariance Scaling



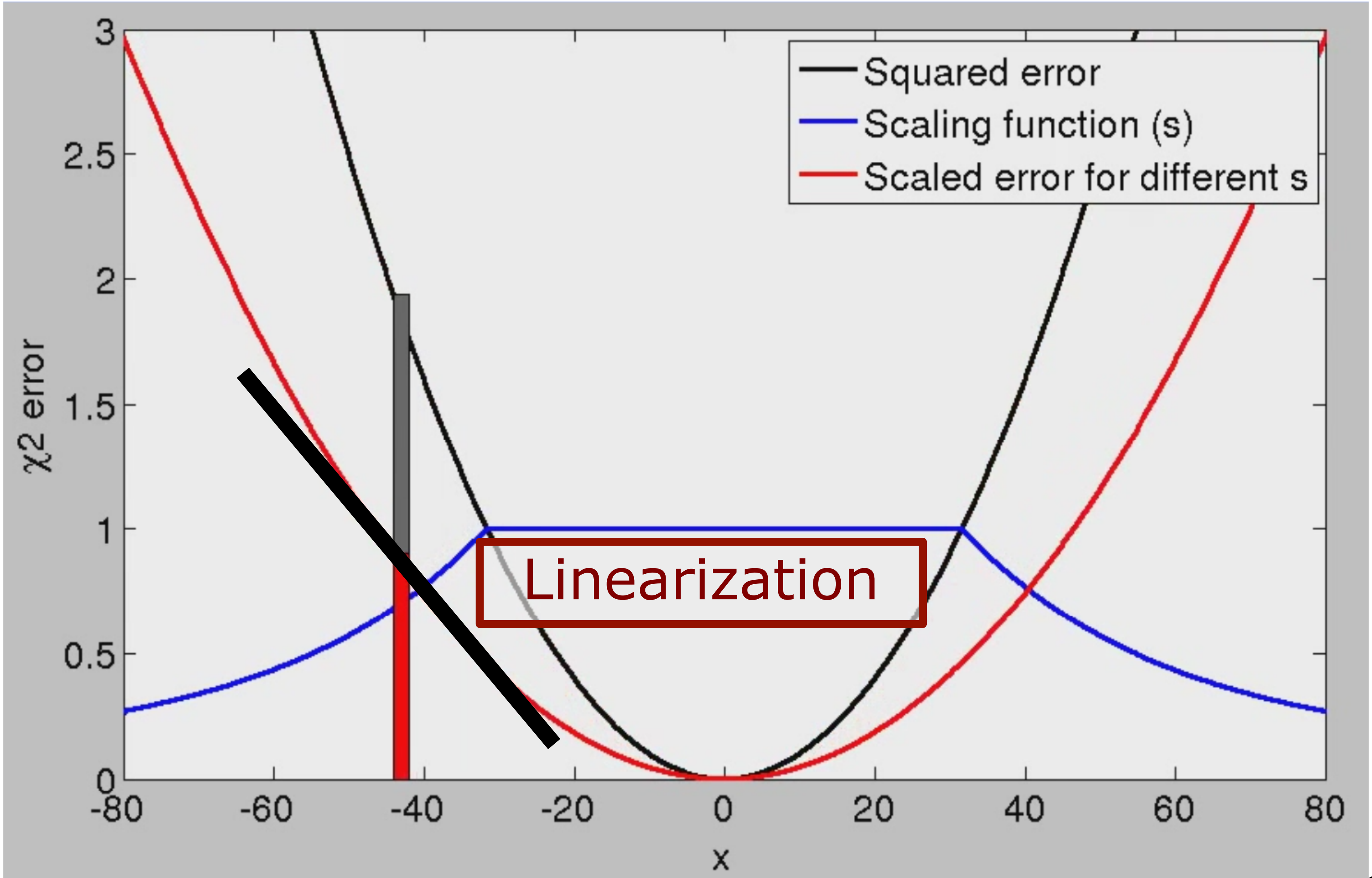
# Dynamic Covariance Scaling



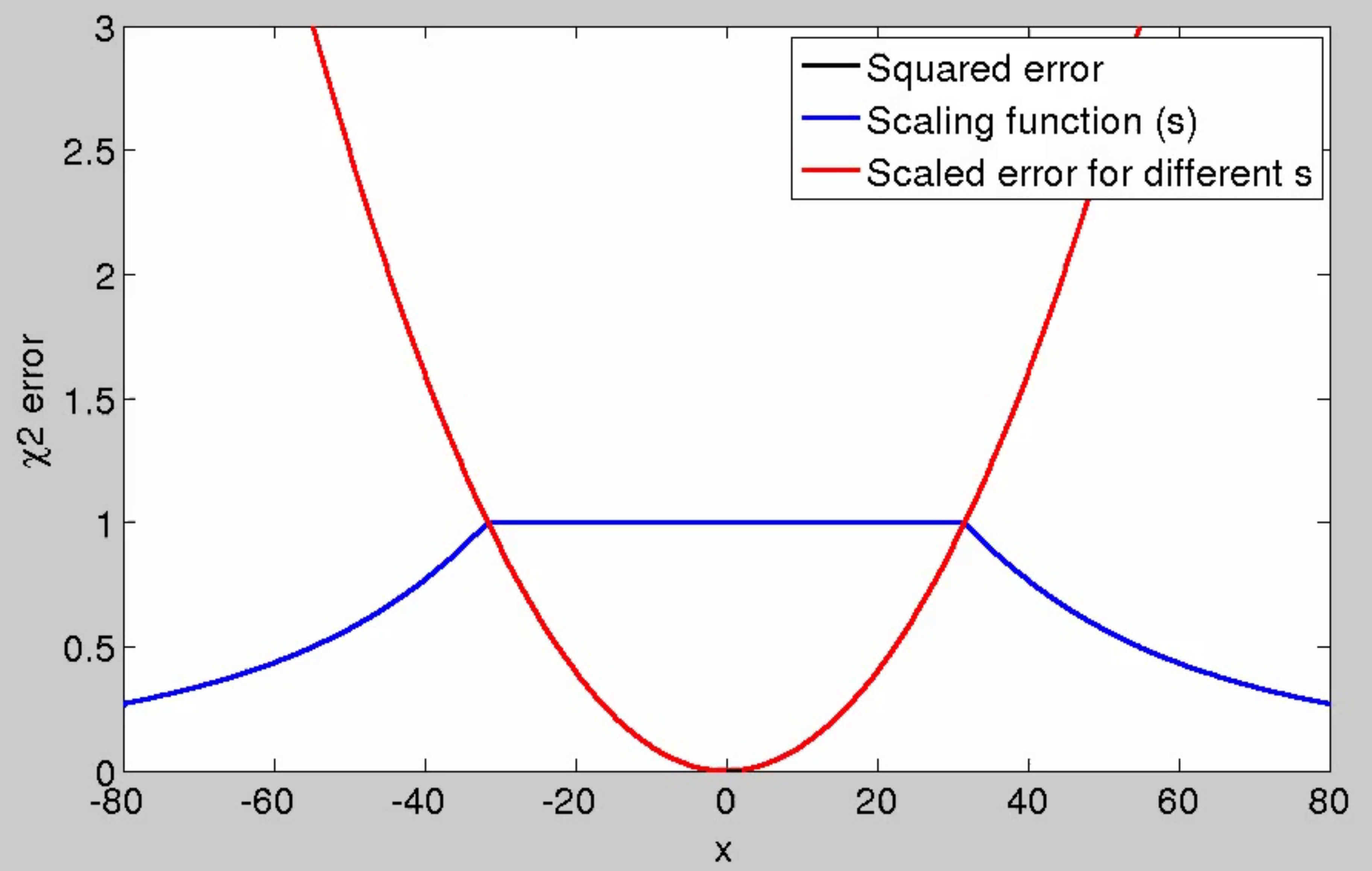
# Dynamic Covariance Scaling



# Dynamic Covariance Scaling

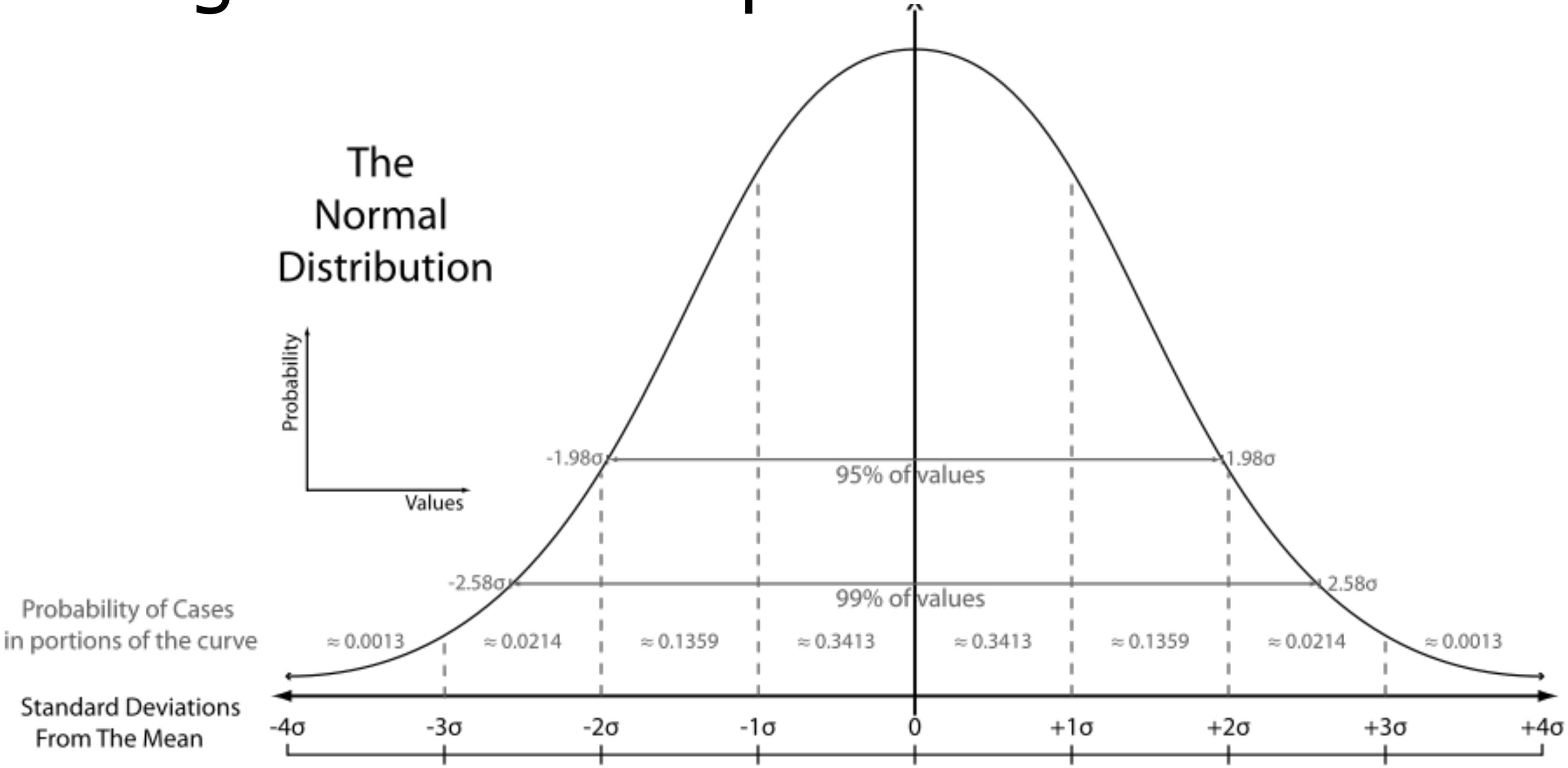


# Dynamic Covariance Scaling



# Optimizing With Outliers

- Assuming a Gaussian error in the constraints is not always realistic
- Large errors are problematic



# Robust M-Estimators

- Assume non-normally-distributed noise
- Intuitively: PDF with “heavy tails”
- $\rho(e)$ function used to define the PDF

$$p(e) \;=\; \exp(-\rho(e))$$

- Minimizing the neg. log likelihood

$$\mathbf{x}^* \;=\; \operatorname*{argmin}_{\mathbf{x}} \sum_i \rho(e_i(\mathbf{x}))$$

# Different Rho Functions

- Gaussian: $\rho(e) \;=\; e^2$
- Absolute values (L1 norm): $\rho(e) \;=\; |e|$
- Huber M-estimator

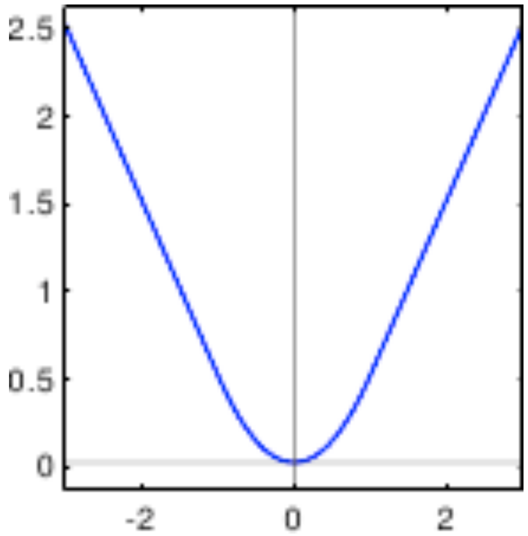
$$\rho(e) \;=\; \begin{cases} \frac{e^2}{2} & \text{if } |e| < c \\ c(|e| - \frac{c}{2}) & \text{otherwise} \end{cases}$$

- Several others (Tukey, Cauchy, Blake-Zisserman, Corrupted Gaussian, …)

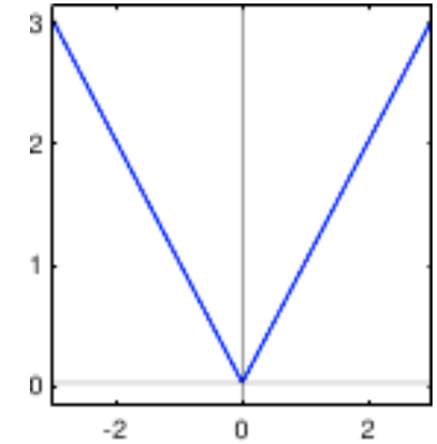
# Huber

- Mixture of a quadratic and a linear function

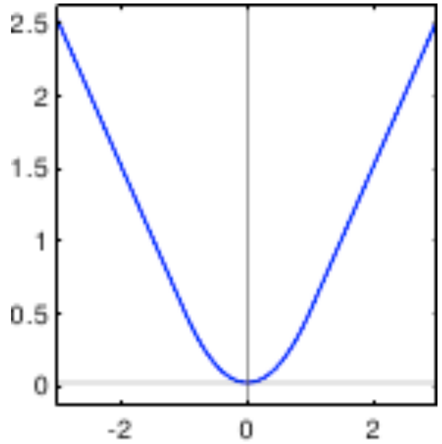
$$\rho(e) = \begin{cases} \frac{e^2}{2} & \text{if } |e| < c \\ c(|e| - \frac{c}{2}) & \text{otherwise} \end{cases}$$



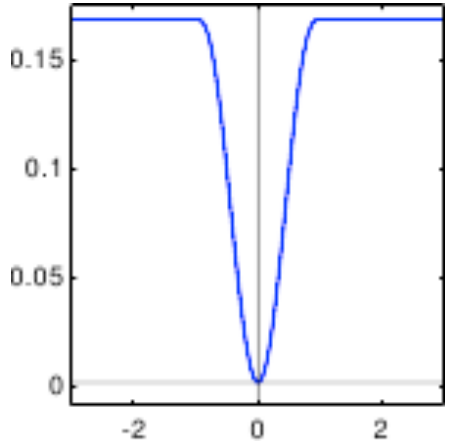
# Different Rho Functions



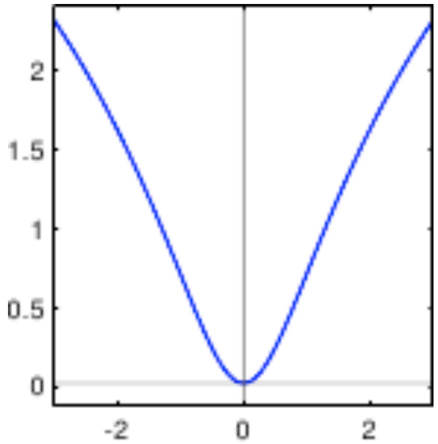

L1 norm



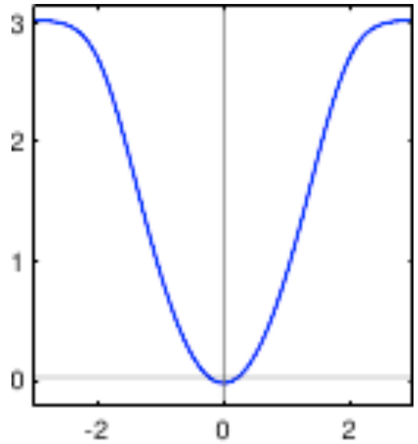

Huber



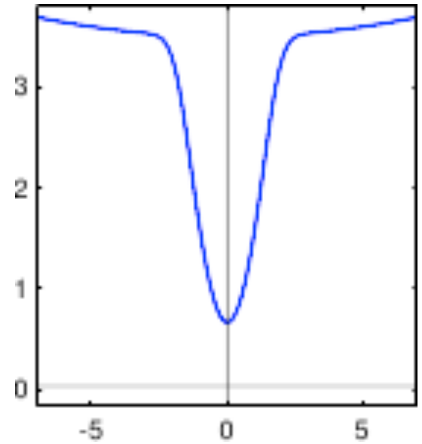

Tukey




Cauchy

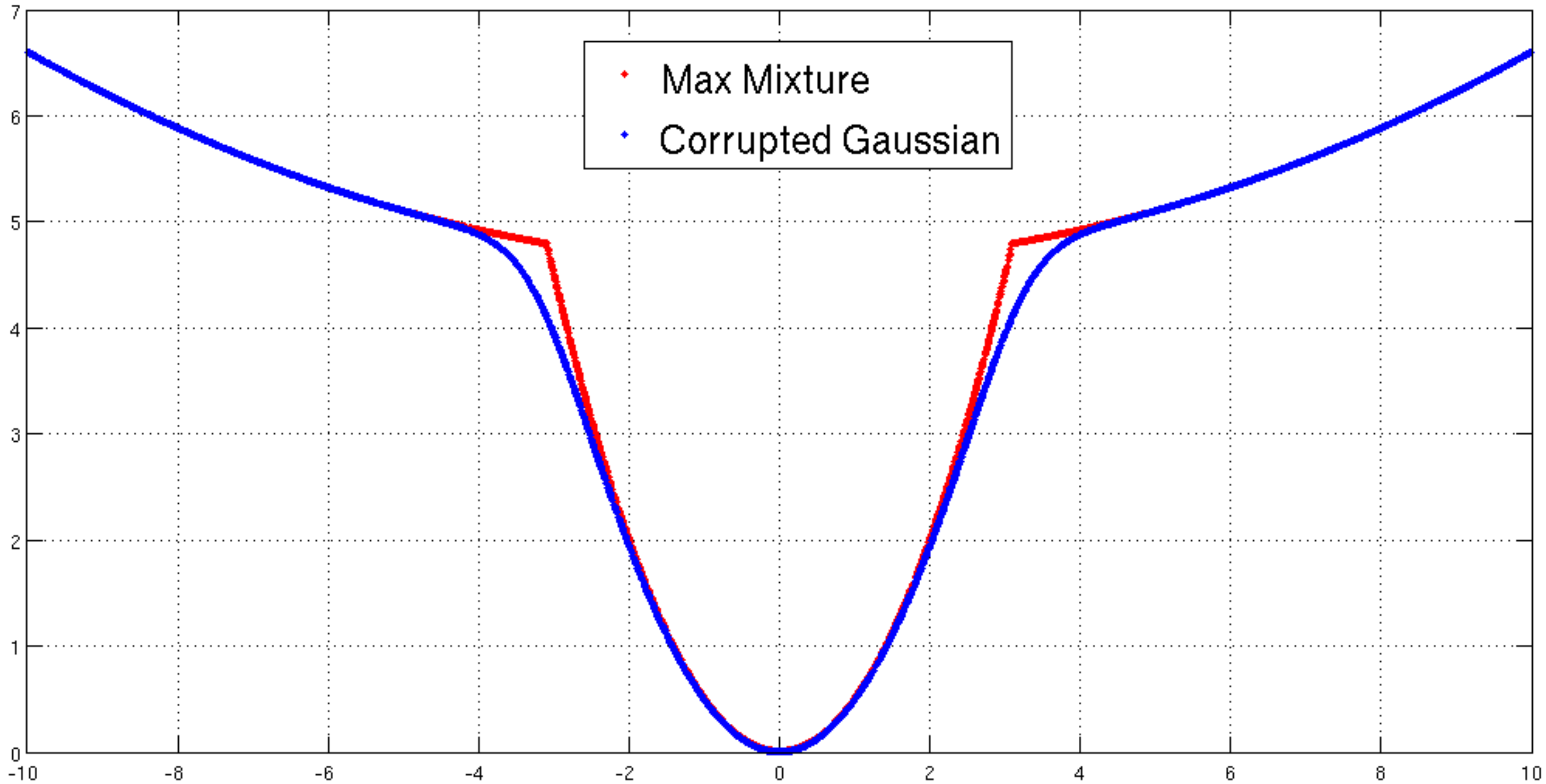



Blake-Zisserman




Corrupted G.

# MM Cost Function For Outliers



# Robust Estimation

- Choice of the rho function depends on the problem at hand
- Huber function is often used
- MM for outlier handling is similar to a corrupted Gaussian
- MM additionally supports multi-model constraints
- Dynamic Covariance Scaling is a robust M-estimator

# Conclusions

- Sum of Gaussians cannot be used easily in the optimization framework
- Max-Mixture formulation approximates the sum by the max operator
- This allows for handling data association ambiguities and failures
- Minimal performance overhead
- Minimal code changes for integration

# Literature

**Max-Mixture Approach:**

- Olson, Agarwal: "Inference on Networks of Mixtures for Robust Robot Mapping"

**Dynamic Covariance Scaling:**

- Agarwal, Tipaldi, Spinello, Stachniss, Burgard: "Robust Map Optimization Using Dynamic Covariance Scaling"