Robot Mapping

Grid Maps

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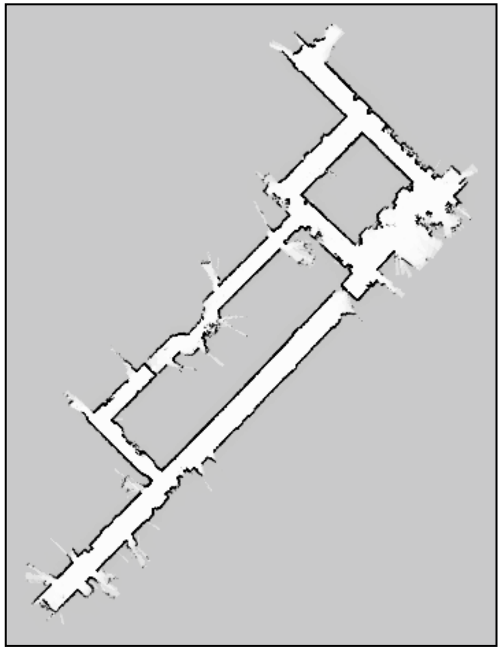
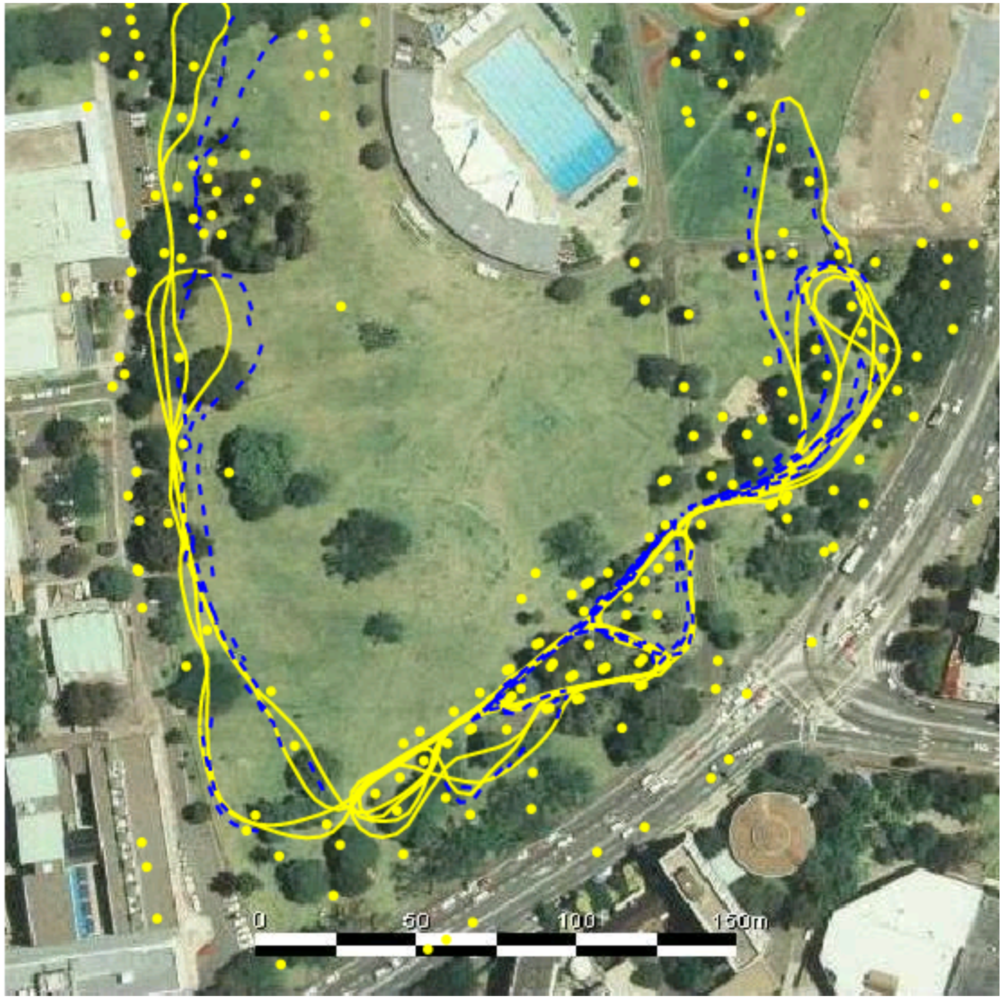
AIS Autonomous Intelligent Systems

Features vs. Volumetric Maps





Courtesy by E. Nebot

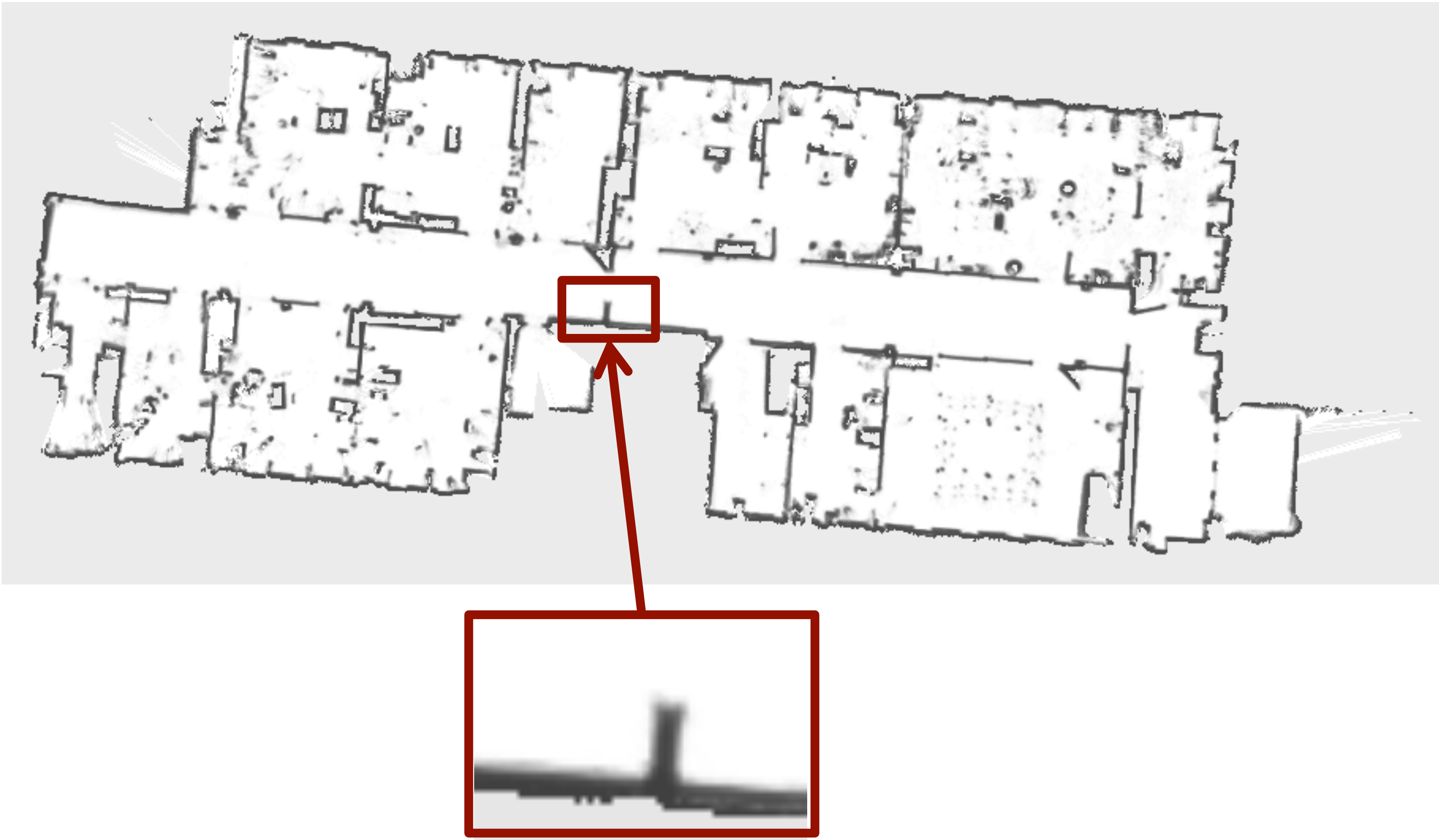
Features

- So far, we only used feature maps
- Natural choice for Kalman filter-based SLAM systems
- Compact representation
- Multiple feature observations improve the landmark position estimate (EKF)

Grid Maps

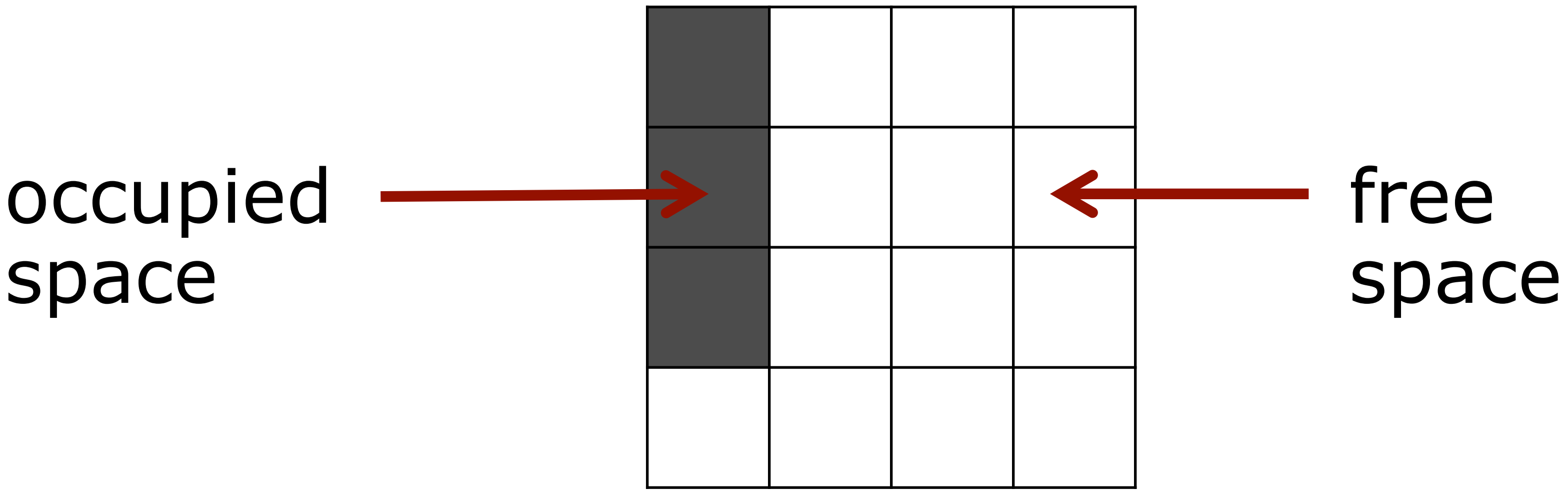
- Discretize the world into cells
- Grid structure is rigid
- Each cell is assumed to be occupied or free space
- Non-parametric model
- Large maps require substantial memory resources
- Do not rely on a feature detector

Example



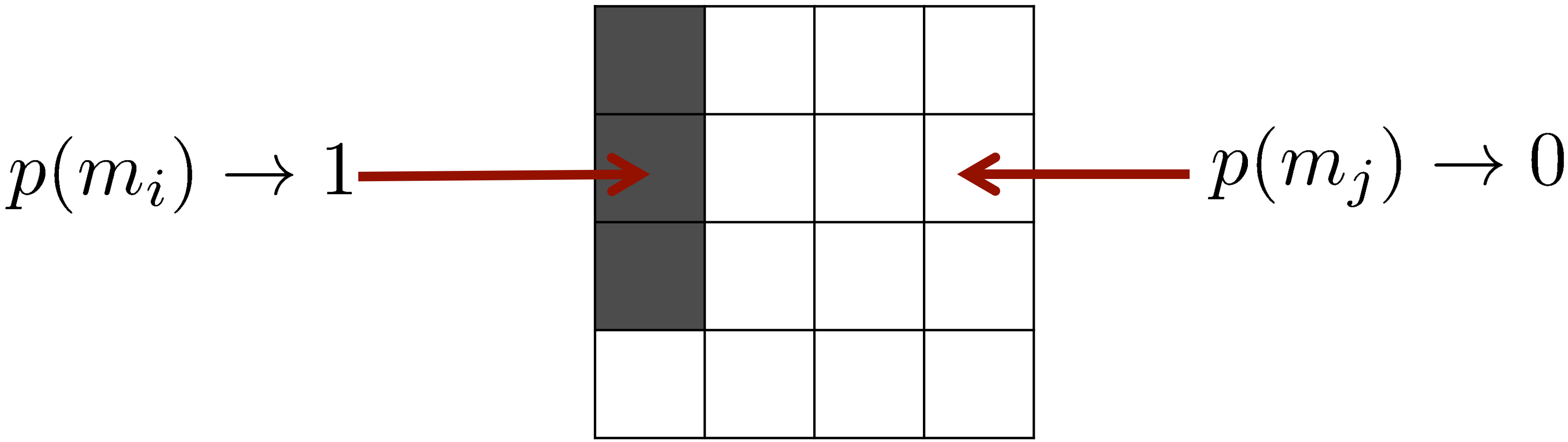
Assumption 1

- The area that corresponds to a cell is either completely free or occupied



Representation

- Each cell is a **binary random variable** that models the occupancy

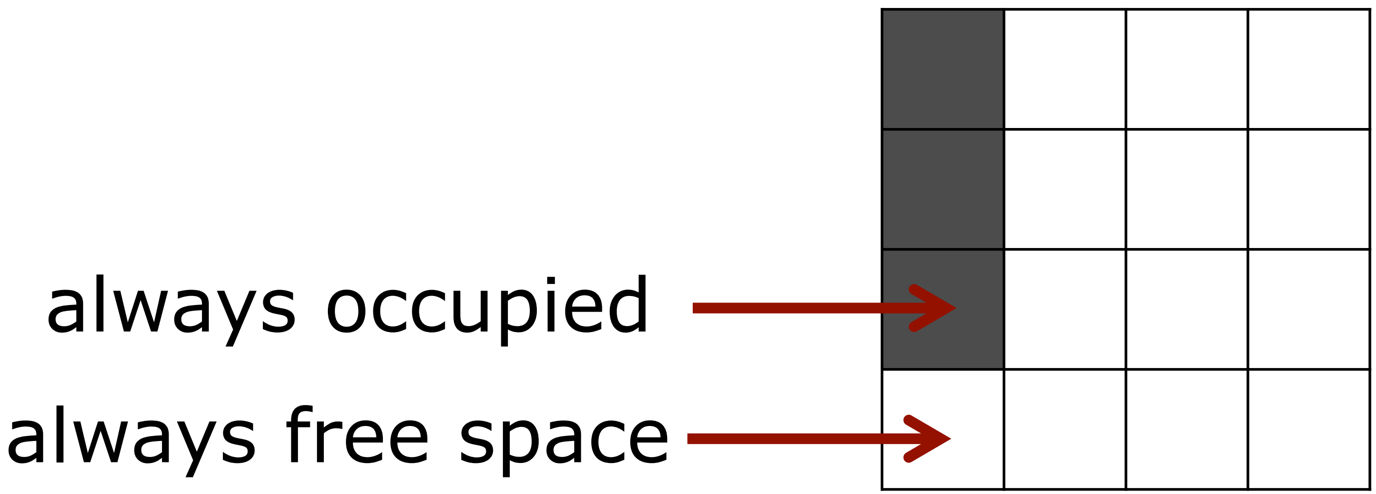


Occupancy Probability

- Each cell is a **binary random variable** that models the occupancy
- Cell is occupied: $p(m_i) = 1$
- Cell is not occupied: $p(m_i) = 0$
- No knowledge: $p(m_i) = 0.5$

Assumption 2

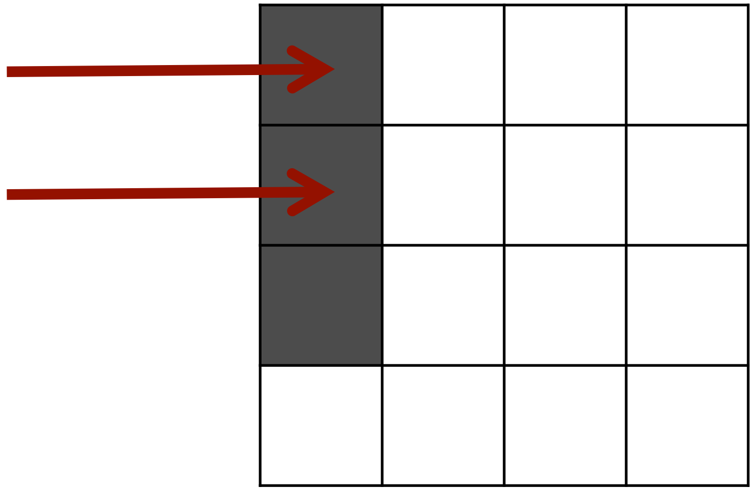
- The world is **static** (most mapping systems make this assumption)



Assumption 3

- The cells (the random variables) are **independent** of each other

no dependency
between the cells



Representation

- The probability distribution of the map is given by the product over the cells

$$p(m) = \prod_i p(m_i)$$

map cell

Representation

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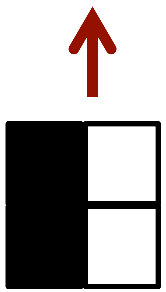
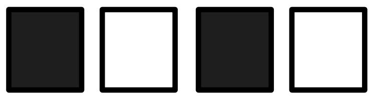


example map
(4-dim state)



4 individual cells

Estimating a Map From Data

- Given sensor data $z_{1:t}$ and the poses $x_{1:t}$ of the sensor, estimate the map

$$p(m \mid z_{1:t}, x_{1:t}) \quad = \quad \prod_i p(m_i \mid z_{1:t}, x_{1:t})$$

binary random variable

➡ Binary Bayes filter
(for a static state)

Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

Static State Binary Bayes Filter

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Static State Binary Bayes Filter

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Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

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$$\overset{\text{Markov}}{=} \frac{p(m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i)\, p(z_t \mid z_{1:t-1}, x_{1:t})}$$

Do exactly the same for the opposite event:

$$p(\neg m_i \mid z_{1:t}, x_{1:t}) \overset{\text{the same}}{=} \frac{p(\neg m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(\neg m_i \mid z_{1:t-1}, x_{1:t-1})}{p(\neg m_i)\, p(z_t \mid z_{1:t-1}, x_{1:t})}$$

Static State Binary Bayes Filter

- By computing the ratio of both probabilities, we obtain:

$$\frac{p(m_i \mid z_{1:t}, x_{1:t})}{p(\neg m_i \mid z_{1:t}, x_{1:t})} = \frac{\dfrac{p(m_i|z_t,x_t)\, p(z_t|x_t)\, p(m_i|z_{1:t-1},x_{1:t-1})}{p(m_i)\, p(z_t|z_{1:t-1},x_{1:t})}}{\dfrac{p(\neg m_i|z_t,x_t)\, p(z_t|x_t)\, p(\neg m_i|z_{1:t-1},x_{1:t-1})}{p(\neg m_i)\, p(z_t|z_{1:t-1},x_{1:t})}}$$

Static State Binary Bayes Filter

- By computing the ratio of both probabilities, we obtain:

$$\frac{p(m_i \mid z_{1:t}, x_{1:t})}{p(\neg m_i \mid z_{1:t}, x_{1:t})}$$

$$= \frac{p(m_i \mid z_t, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})\, p(\neg m_i)}{p(\neg m_i \mid z_t, x_t)\, p(\neg m_i \mid z_{1:t-1}, x_{1:t-1})\, p(m_i)}$$

$$= \frac{p(m_i \mid z_t, x_t)}{1 - p(m_i \mid z_t, x_t)} \frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})} \frac{1 - p(m_i)}{p(m_i)}$$

Static State Binary Bayes Filter

- By computing the ratio of both probabilities, we obtain:

$$\frac{p(m_i \mid z_{1:t}, x_{1:t})}{1 - p(m_i \mid z_{1:t}, x_{1:t})}$$

$$= \frac{p(m_i \mid z_t, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})\, p(\neg m_i)}{p(\neg m_i \mid z_t, x_t)\, p(\neg m_i \mid z_{1:t-1}, x_{1:t-1})\, p(m_i)}$$

$$= \underbrace{\frac{p(m_i \mid z_t, x_t)}{1 - p(m_i \mid z_t, x_t)}}_{\text{uses } z_t} \underbrace{\frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})}}_{\text{recursive term}} \underbrace{\frac{1 - p(m_i)}{p(m_i)}}_{\text{prior}}$$

From Ratio to Probability

- We can easily turn the ration into the probability

$$
\begin{aligned}
\frac{p(x)}{1-p(x)} &= Y \\
p(x) &= Y - Y\,p(x) \\
p(x)\,(1+Y) &= Y \\
p(x) &= \frac{Y}{1+Y} \\
p(x) &= \frac{1}{1+\frac{1}{Y}}
\end{aligned}
$$

From Ratio to Probability

- Using $p(x) = \left[1 + Y^{-1}\right]^{-1}$ directly leads to

$$p(m_i \mid z_{1:t}, x_{1:t})$$
$$= \left[1 + \frac{1 - p(m_i \mid z_t, x_t)}{p(m_i \mid z_t, x_t)} \frac{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i \mid z_{1:t-1}, x_{1:t-1})} \frac{p(m_i)}{1 - p(m_i)}\right]^{-1}$$

For reasons of efficiency, one performs the calculations in the log odds notation

Log Odds Notation

- The log odds notation computes the logarithm of the ratio of probabilities

$$\frac{p(m_i \mid z_{1:t}, x_{1:t})}{1 - p(m_i \mid z_{1:t}, x_{1:t})}$$

$$= \underbrace{\frac{p(m_i \mid z_t, x_t)}{1 - p(m_i \mid z_t, x_t)}}_{\text{uses } z_t} \underbrace{\frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})}}_{\text{recursive term}} \underbrace{\frac{1 - p(m_i)}{p(m_i)}}_{\text{prior}}$$

$$\Rightarrow \quad l(m_i \mid z_{1:t}, x_{1:t}) = \log\left(\frac{p(m_i \mid z_{1:t}, x_{1:t})}{1 - p(m_i \mid z_{1:t}, x_{1:t})}\right)$$

Log Odds Notation

- Log odds ratio is defined as

$$l(x) = \log \frac{p(x)}{1 - p(x)}$$

- and with the ability to retrieve $p(x)$

$$p(x) = 1 - \frac{1}{1 + \exp l(x)}$$

Occupancy Mapping in Log Odds Form

- The product turns into a sum

$$l(m_i \mid z_{1:t}, x_{1:t}) = \underbrace{l(m_i \mid z_t, x_t)}_{\text{inverse sensor model}} + \underbrace{l(m_i \mid z_{1:t-1}, x_{1:t-1})}_{\text{recursive term}} - \underbrace{l(m_i)}_{\text{prior}}$$

- or in short

$$l_{t,i} = \text{inv\_sensor\_model}(m_i, x_t, z_t) + l_{t-1,i} - l_0$$

Occupancy Mapping Algorithm

occupancy_grid_mapping($\{l_{t-1,i}\}, x_t, z_t$):

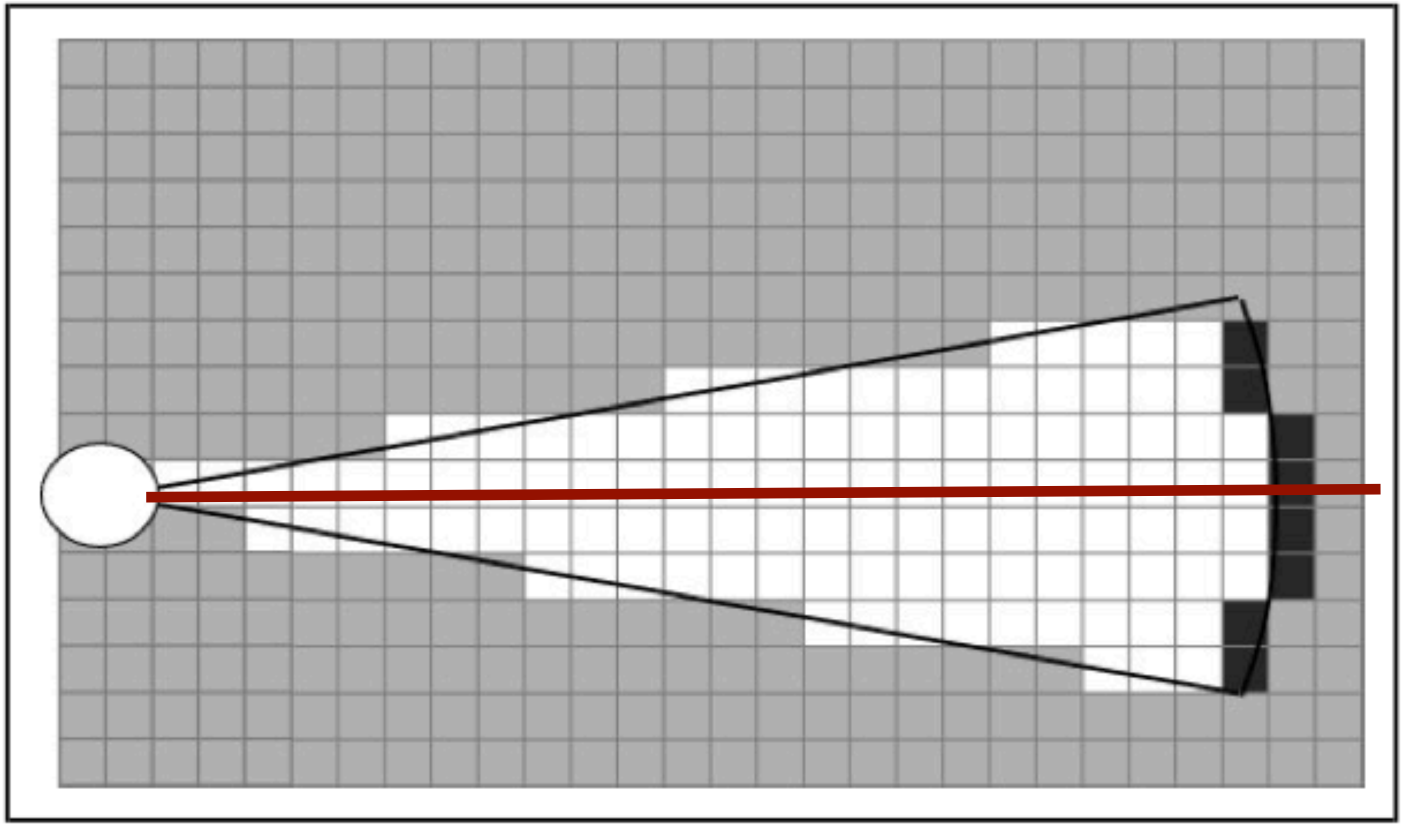
for all cells m_i *do*
    if m_i *in perceptual field of* z_t *then*
        $l_{t,i} = l_{t-1,i} + \text{inv\_sensor\_model}(m_i, x_t, z_t) - l_0$
    else
        $l_{t,i} = l_{t-1,i}$
    endif
endfor
return $\{l_{t,i}\}$

highly efficient, we only have to compute sums

Occupancy Grid Mapping

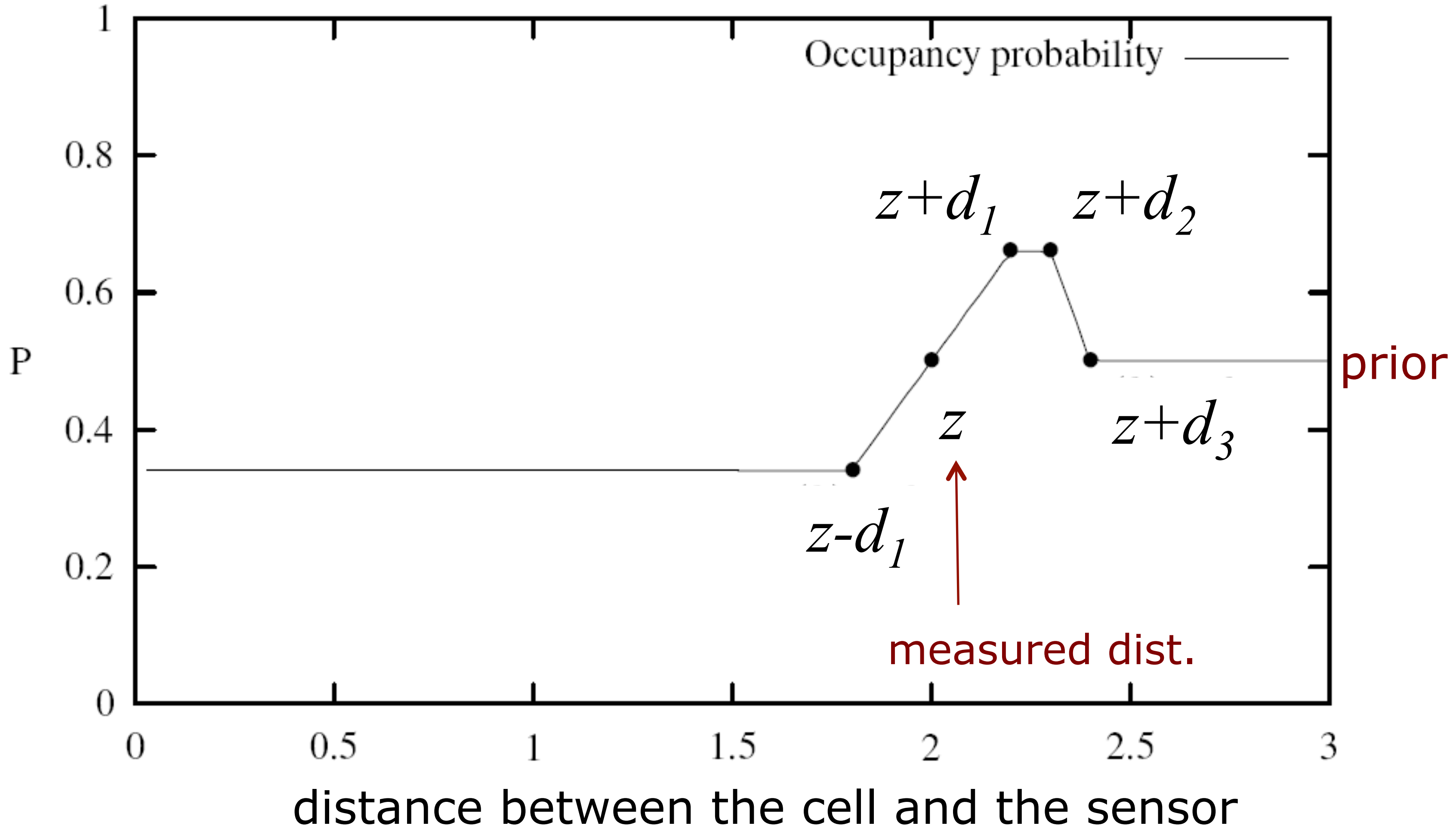
- Moravec and Elfes proposed occupancy grid mapping in the mid 80’ies
- Developed for noisy sonar sensors
- Also called “mapping with know poses”

Inverse Sensor Model for Sonar Range Sensors



In the following, consider the cells along the optical axis (red line)

Occupancy Value Depending on the Measured Distance



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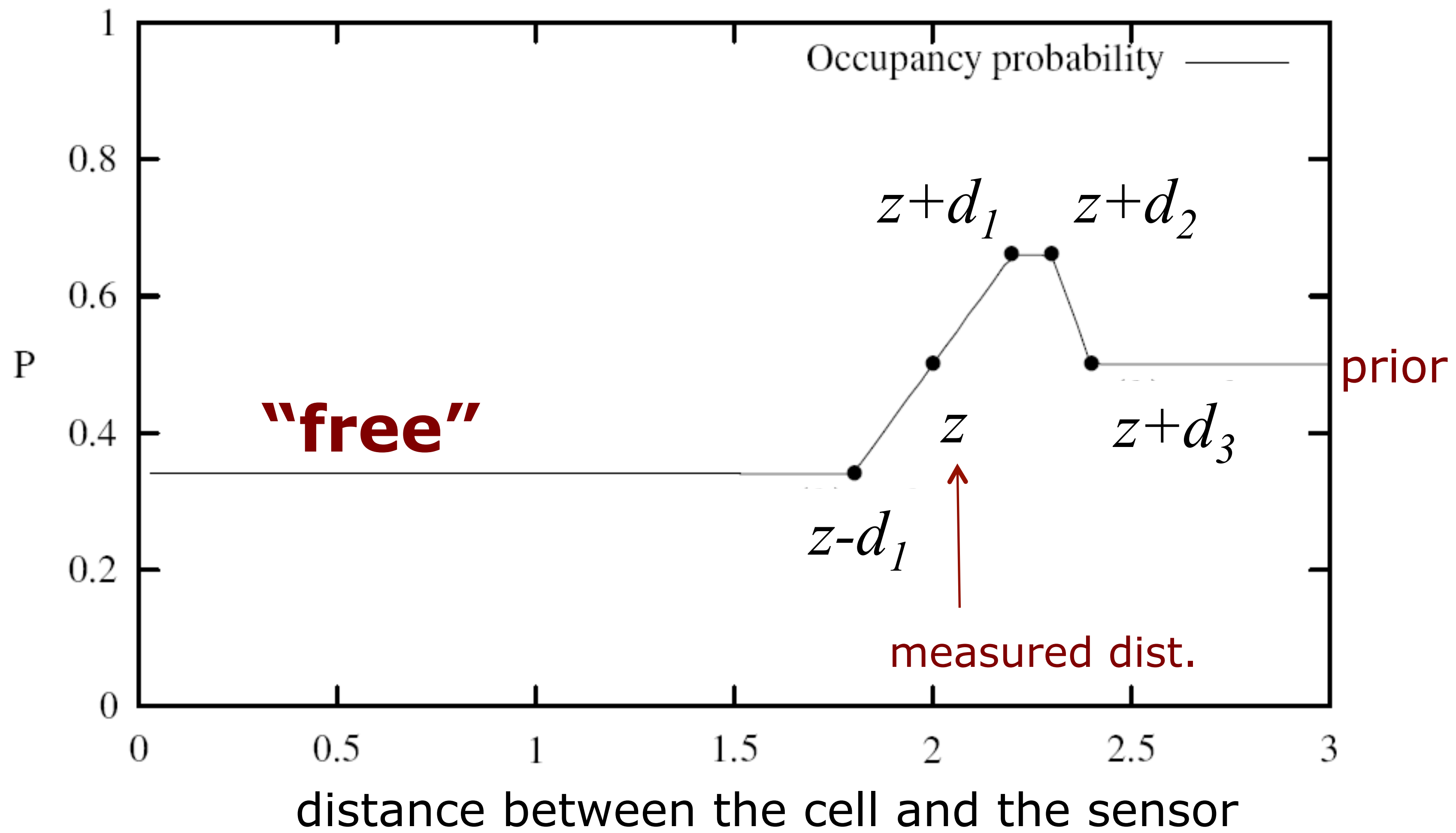

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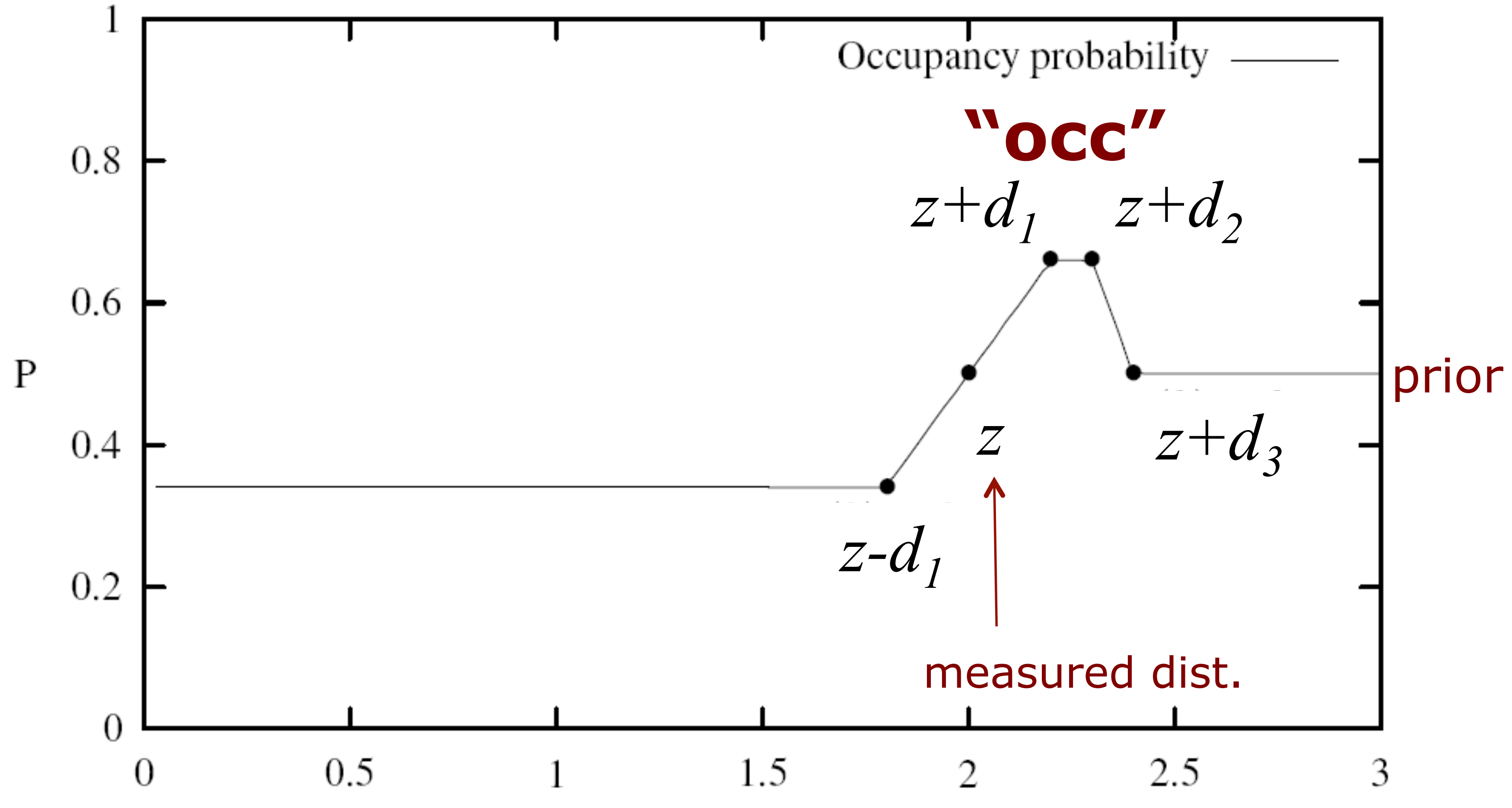
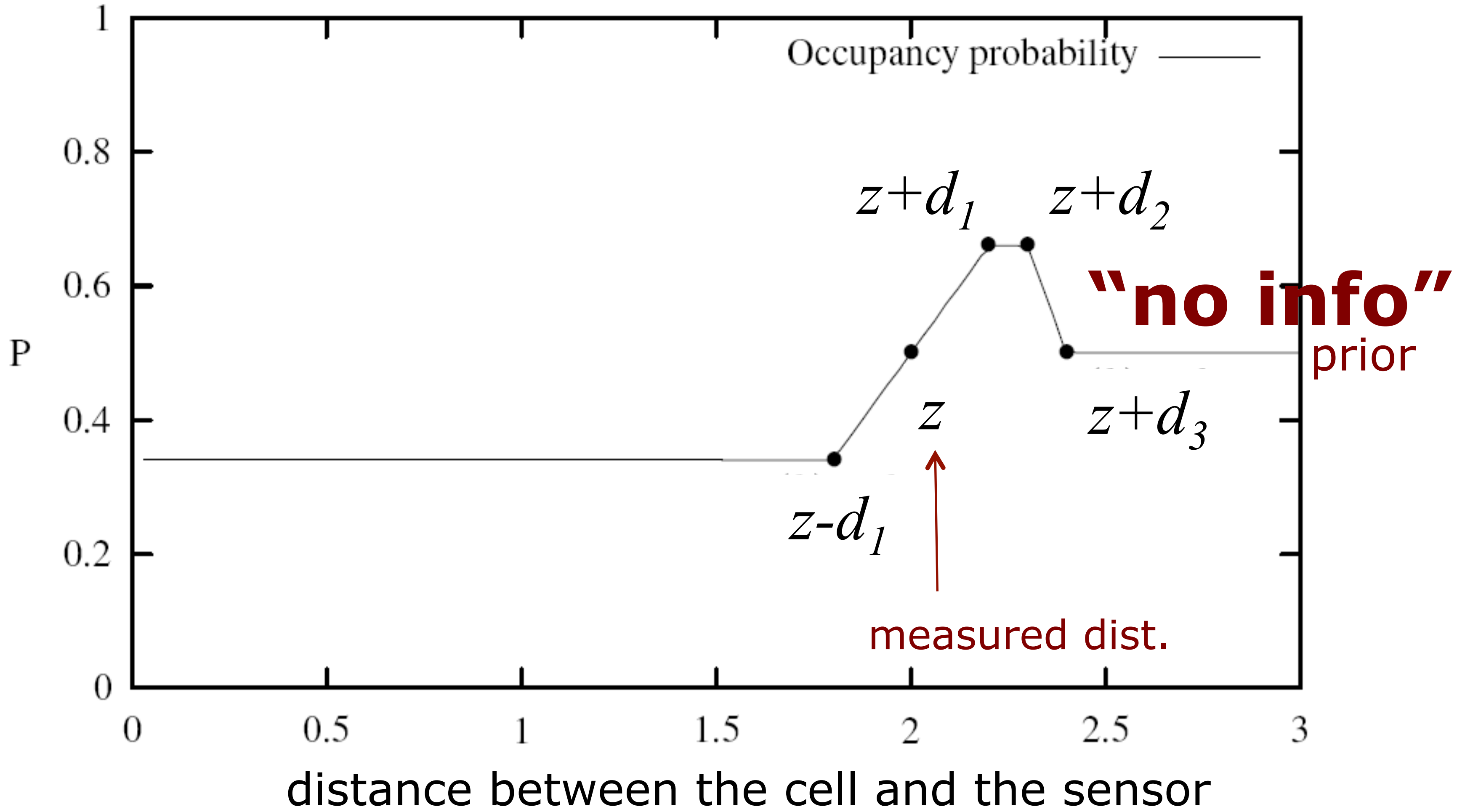
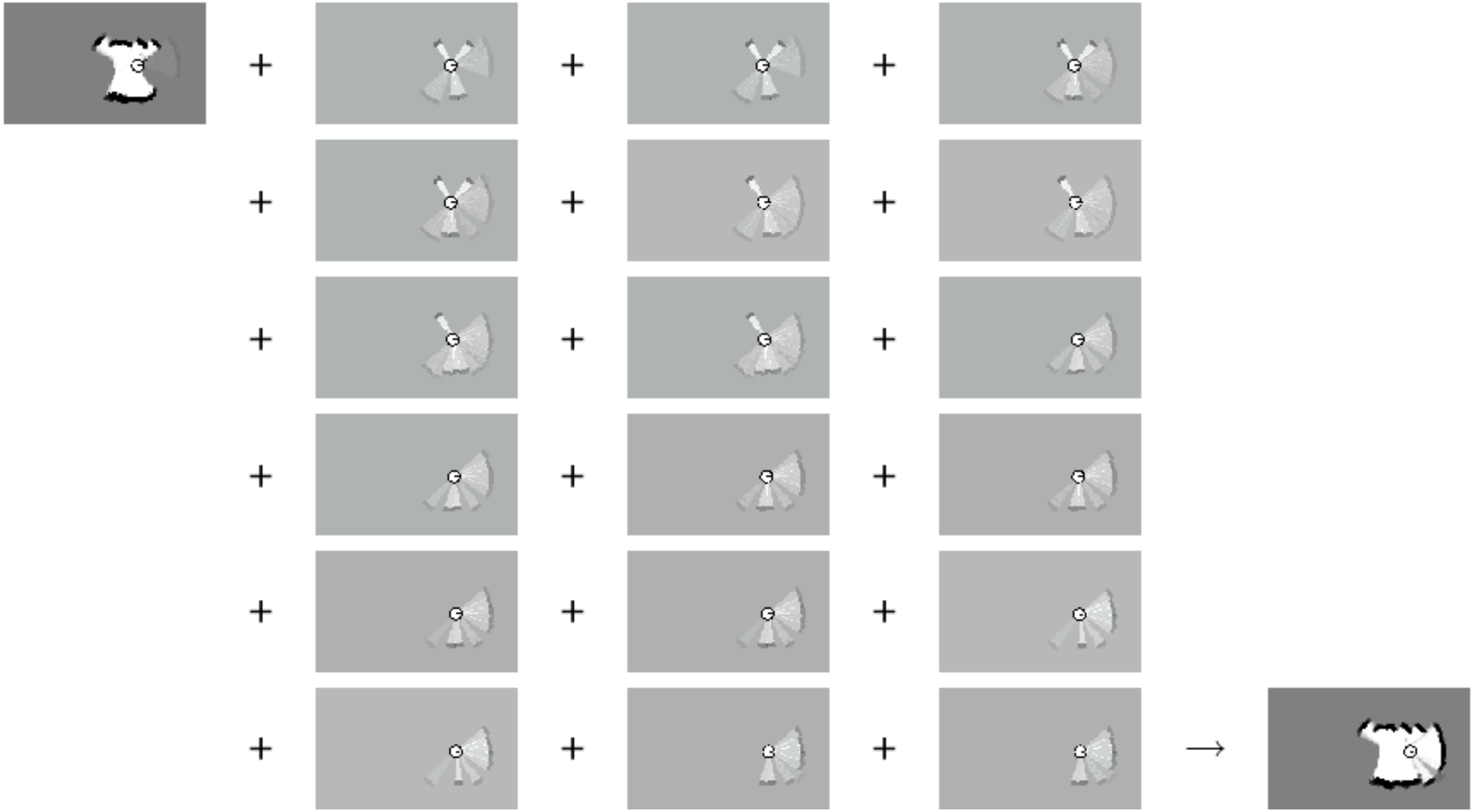


distance between the cell and the sensor

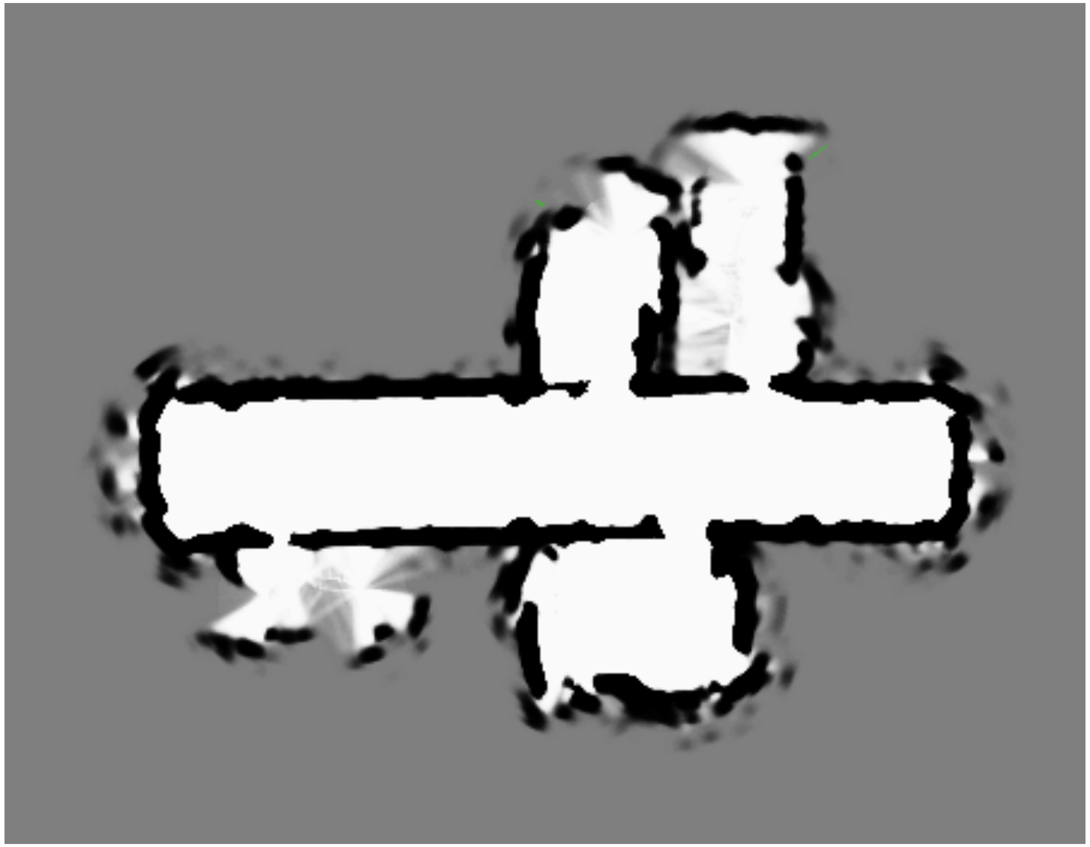
Occupancy Value Depending on the Measured Distance



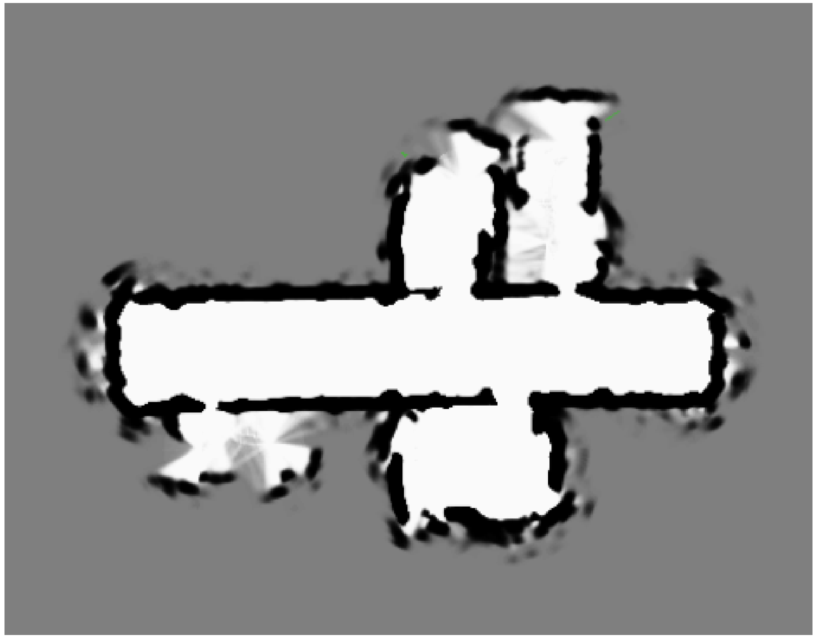
Example: Incremental Updating of Occupancy Grids

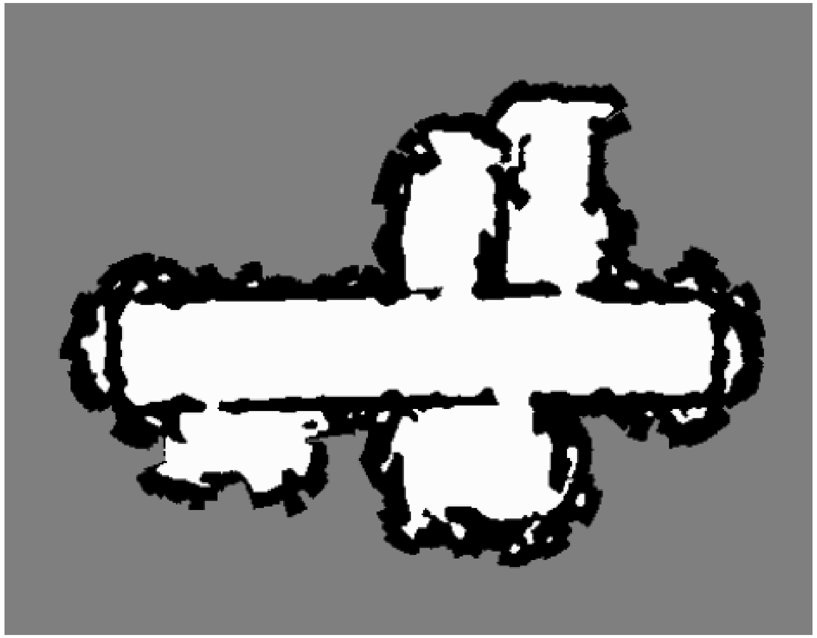


Resulting Map Obtained with 24 Sonar Range Sensors



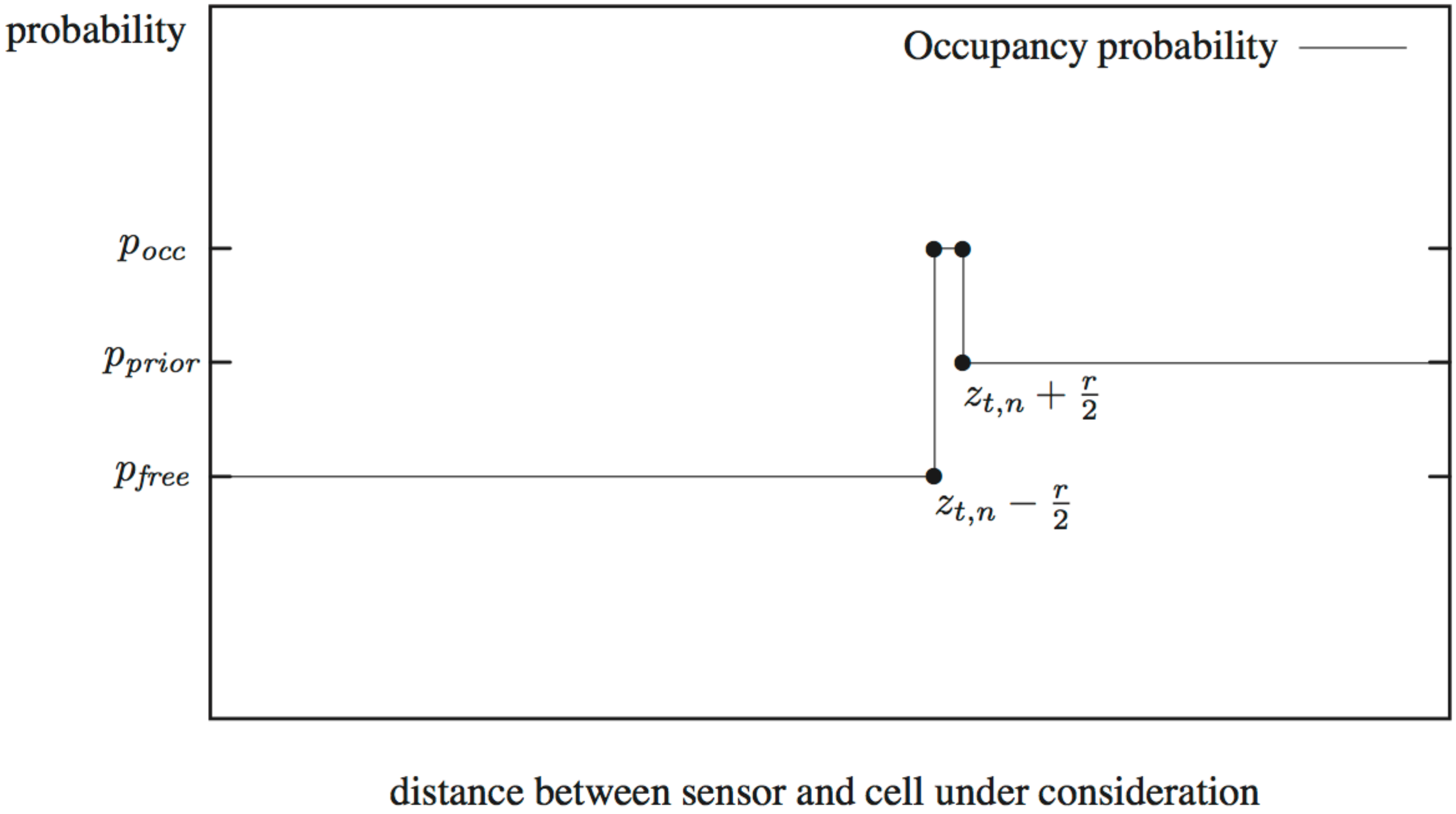
Resulting Occupancy and Maximum Likelihood Map



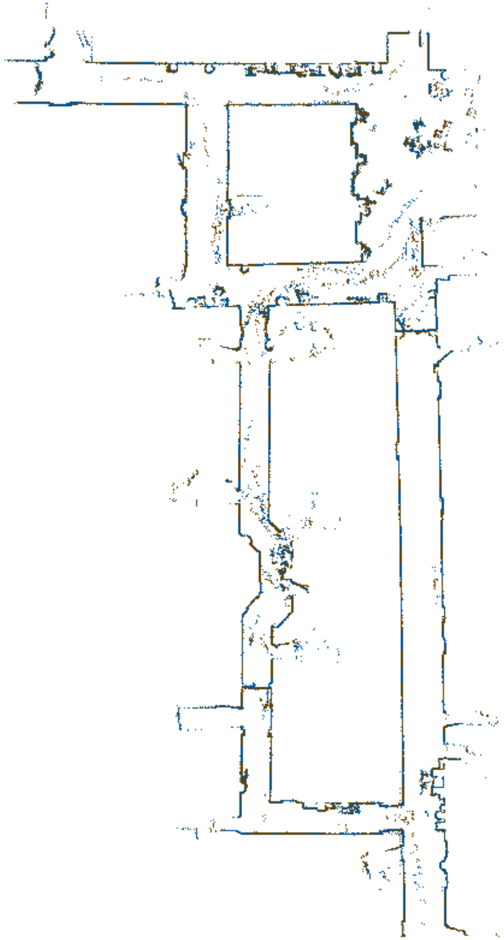


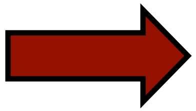
The maximum likelihood map is obtained by rounding the probability for each cell to 0 or 1.

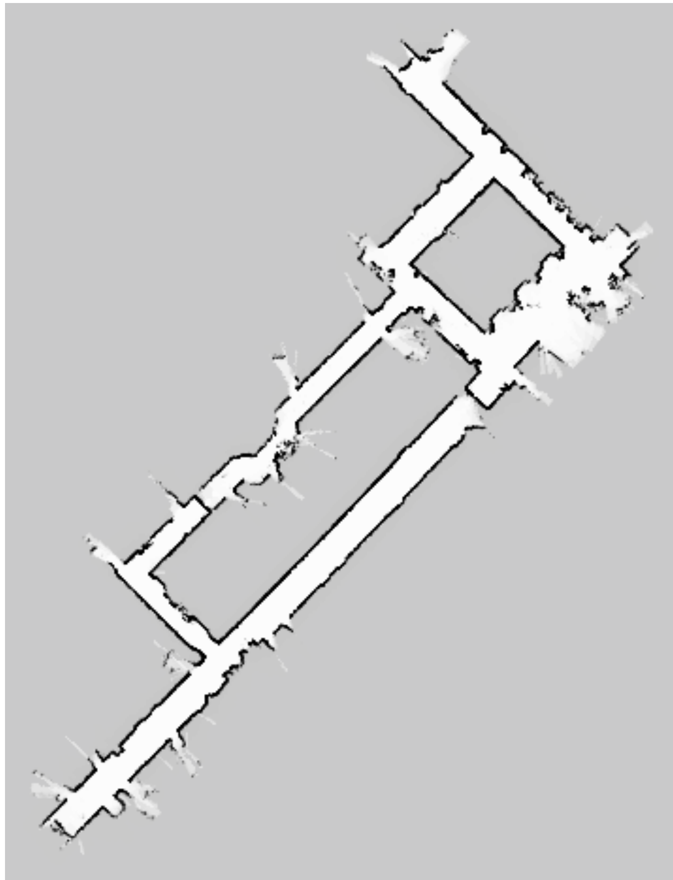
Inverse Sensor Model for Laser Range Finders



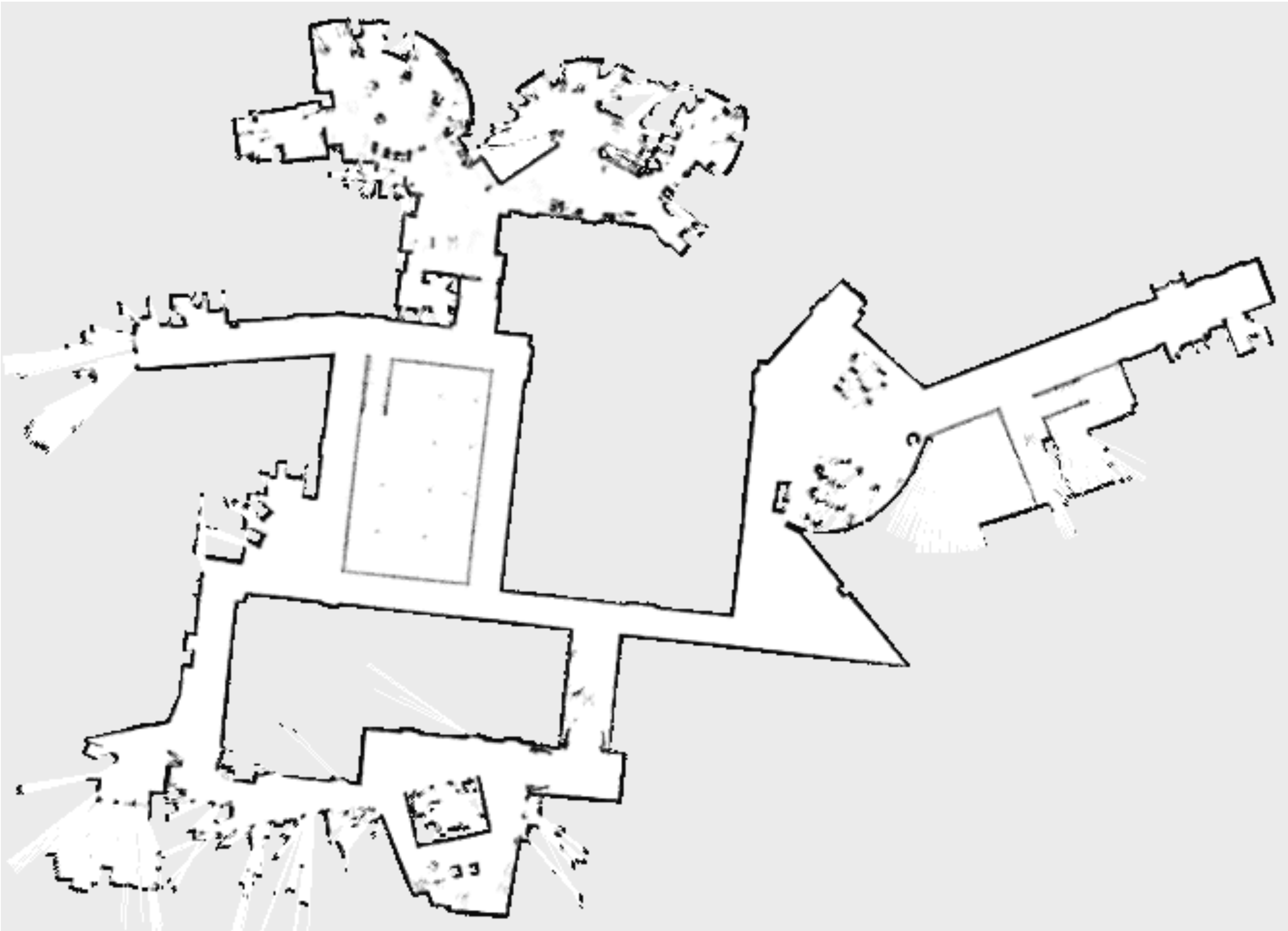
Occupancy Grids
From Laser Scans to Maps



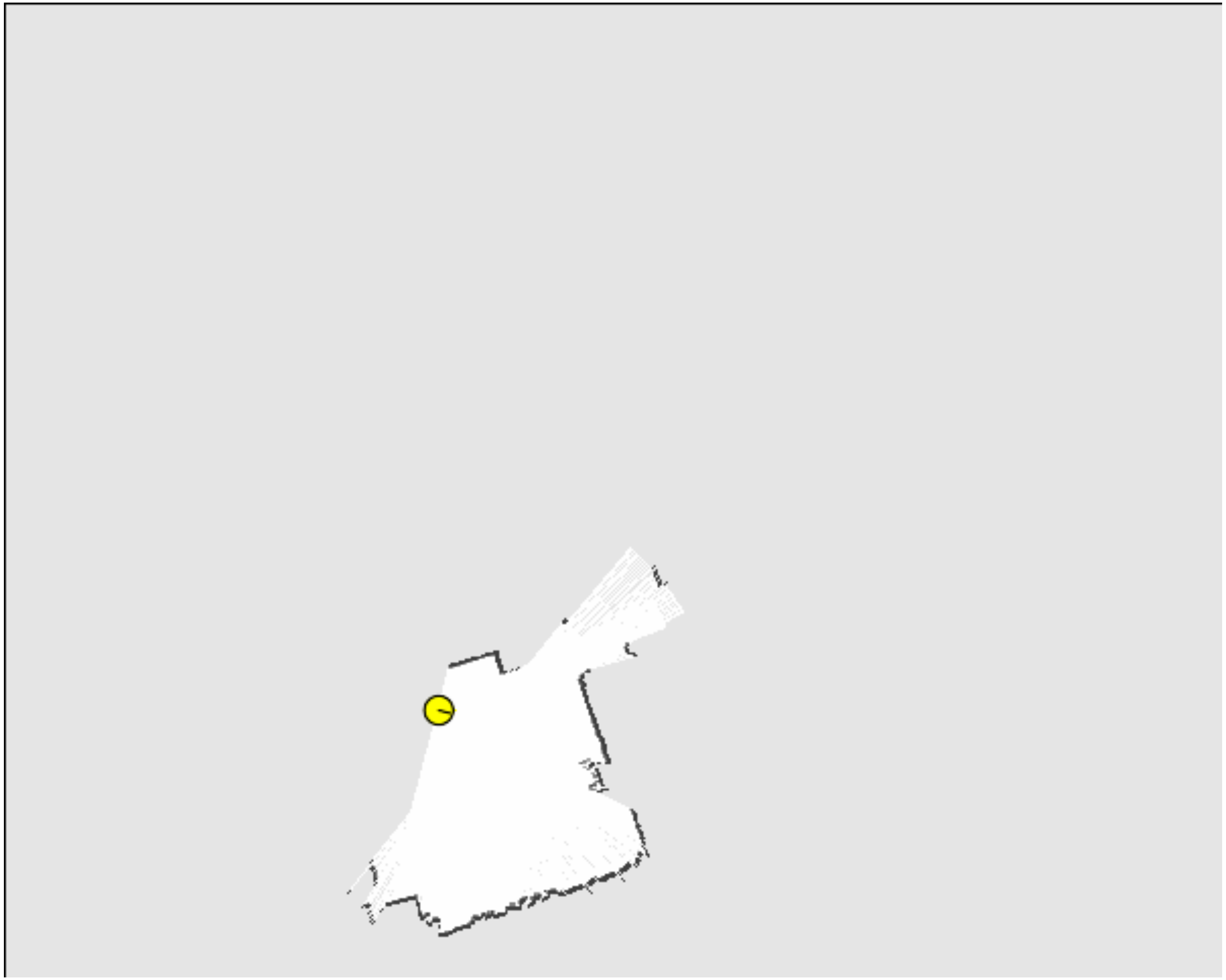




Example: MIT CSAIL 3rd Floor



Uni Freiburg Building 106

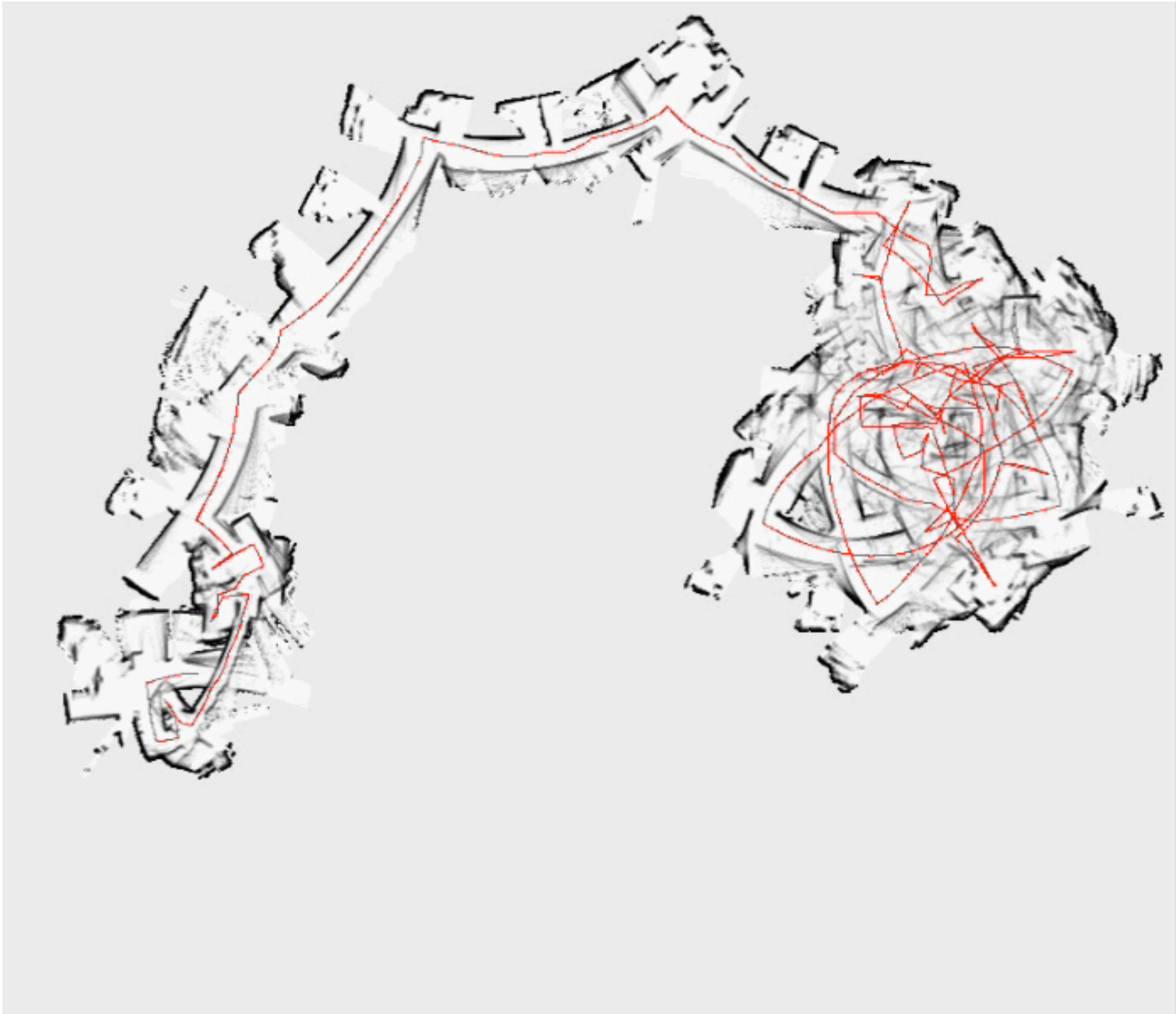


Occupancy Grid Map Summary

- Occupancy grid maps discretize the space into independent cells
- Each cell is a binary random variable estimating if the cell is occupied
- Static state binary Bayes filter per cell
- Mapping with known poses is easy
- Log odds model is fast to compute
- No need for predefined features

Grid Mapping Meets Reality…

Mapping With Raw Odometry



Courtesy by D. Hähnel

Incremental Scan Alignment

- Motion is noisy, we cannot ignore it
- Assuming known poses fails!
- Often, the sensor is rather precise

- Scan-matching tries to incrementally align two scans or a map to a scan, without revising the past/map

Pose Correction Using Scan-Matching

Maximize the likelihood of the **current** pose relative to the **previous** pose and map

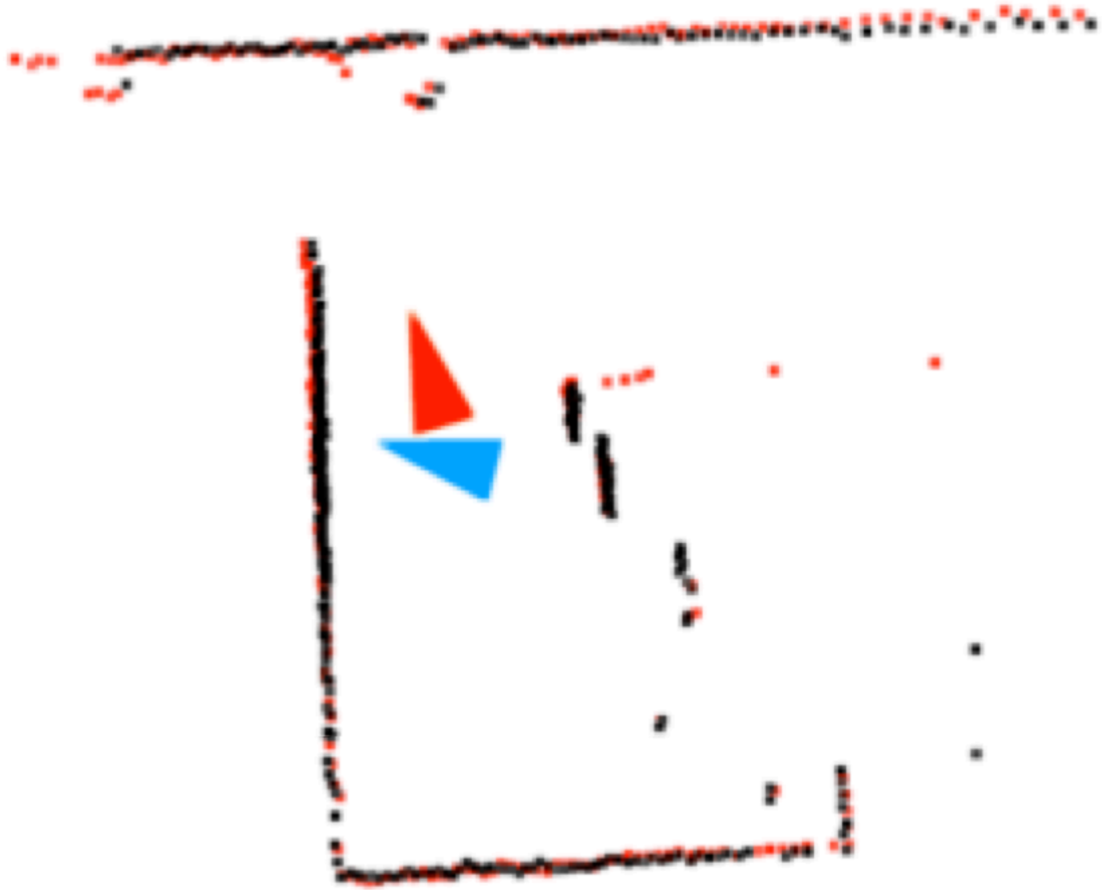
$$x_t^* = \underset{x_t}{\operatorname{argmax}} \left\{ p(z_t \mid x_t, m_{t-1}) \, p(x_t \mid u_{t-1}, x_{t-1}^*) \right\}$$

current measurement

map constructed so far

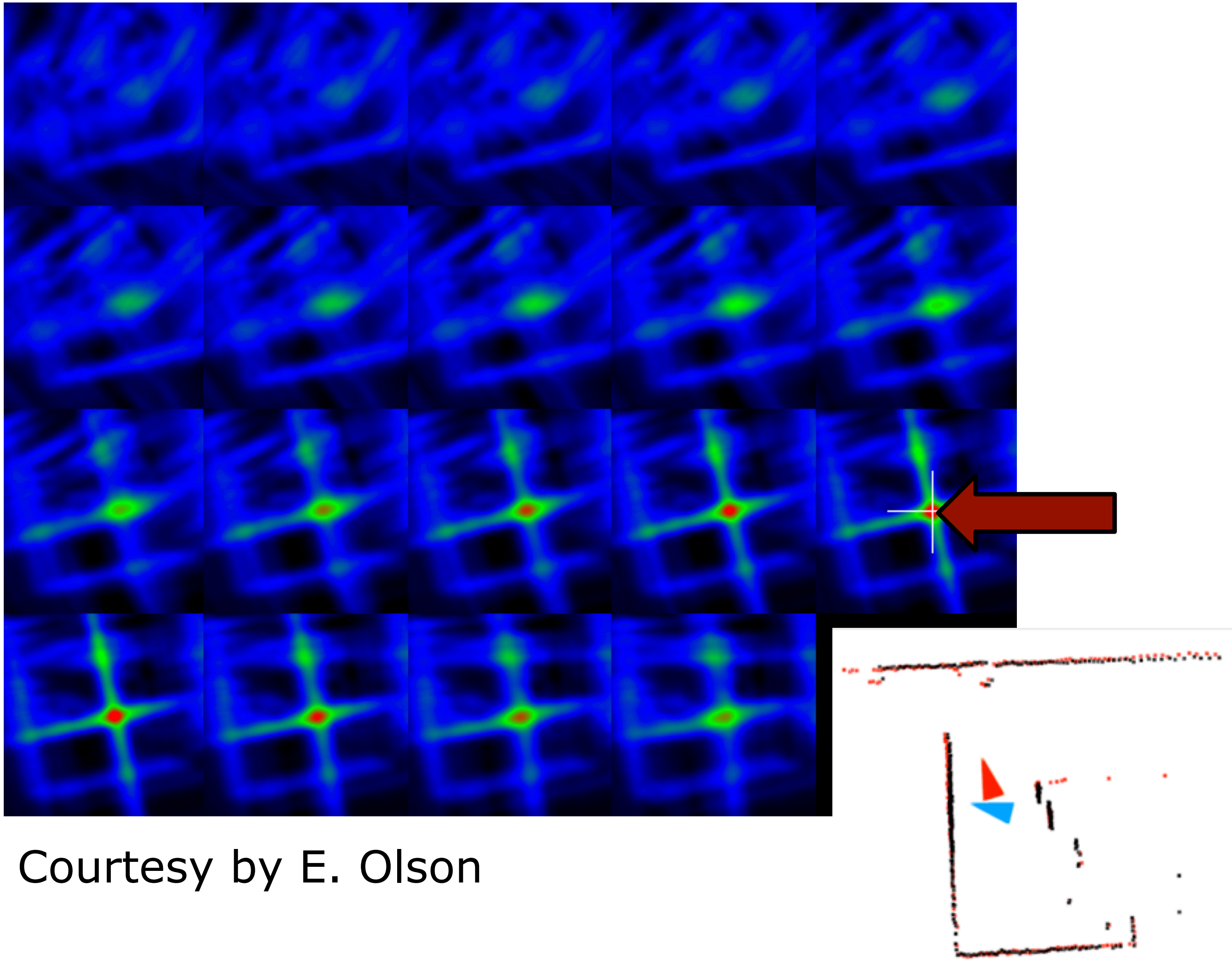
robot motion

Incremental Alignment



Courtesy by E. Olson

Incremental Alignment

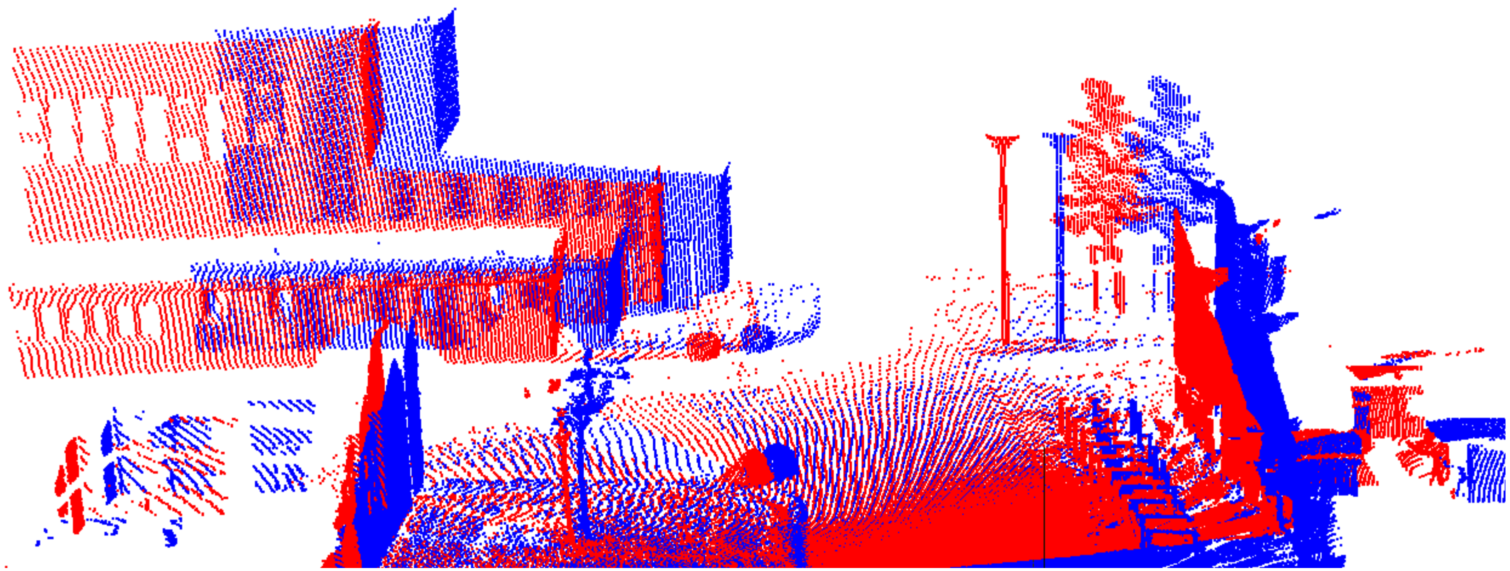


Courtesy by E. Olson

Various Different Ways to Realize Scan-Matching

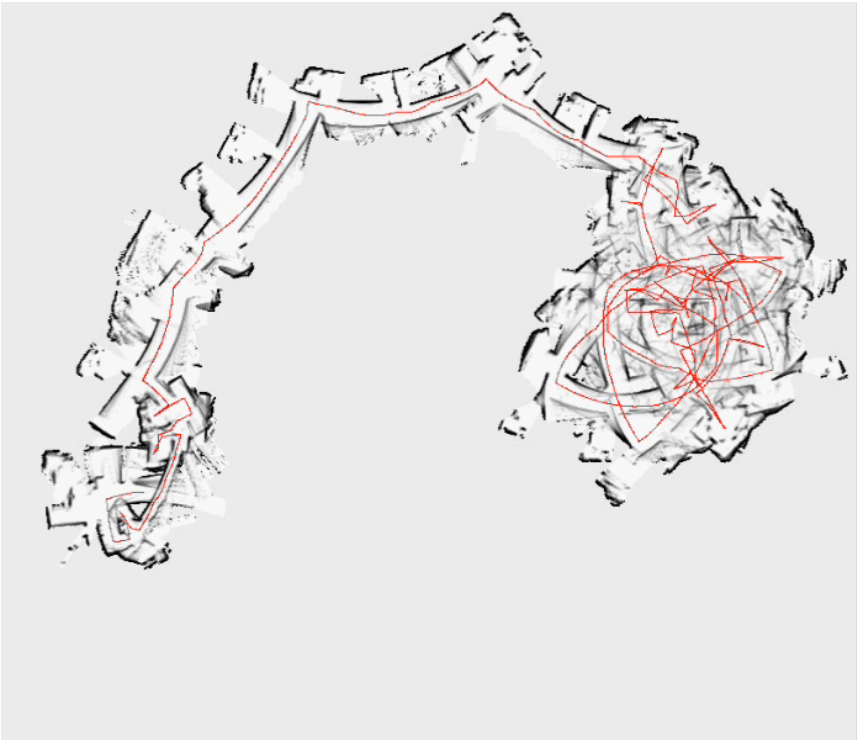
- Iterative closest point (ICP)
- Scan-to-scan
- Scan-to-map
- Map-to-map
- Feature-based
- RANSAC for outlier rejection
- Correlative matching
- …

Example: Aligning Two 3D Maps



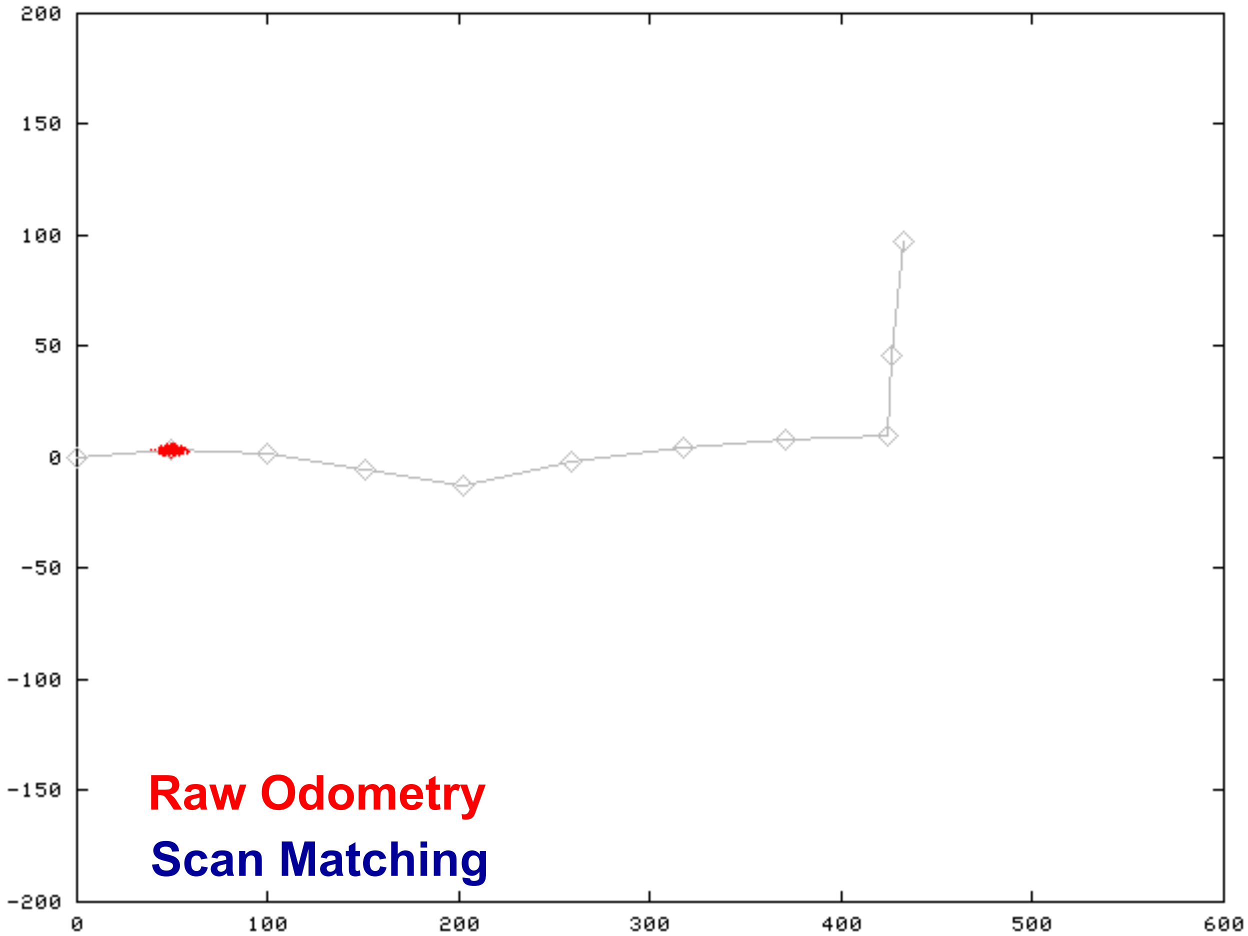
Courtesy by P. Pfaff

With and Without Scan-Matching



Courtesy by D. Hähnel

Motion Model for Scan Matching



Courtesy by D. Hähnel

Scan Matching Summary

- Scan-matching improves the pose estimate (and thus mapping) substantially
- Locally consistent estimates
- Often scan-matching is not sufficient to build a (large) consistent map

Literature

Static state binary Bayes filter

- Thrun et al.: "Probabilistic Robotics", Chapter 4.2

Occupancy Grid Mapping

- Thrun et al.: "Probabilistic Robotics", Chapter 9.1+9.2

Scan-Matching

- Besl and McKay. A method for Registration of 3-D Shapes, 1992
- Olson. Real-Time Correlative Scan Matching, 2009