Robot Mapping

Summary on the Kalman Filter & Friends: KF, EKF, UKF, EIF, SEIF

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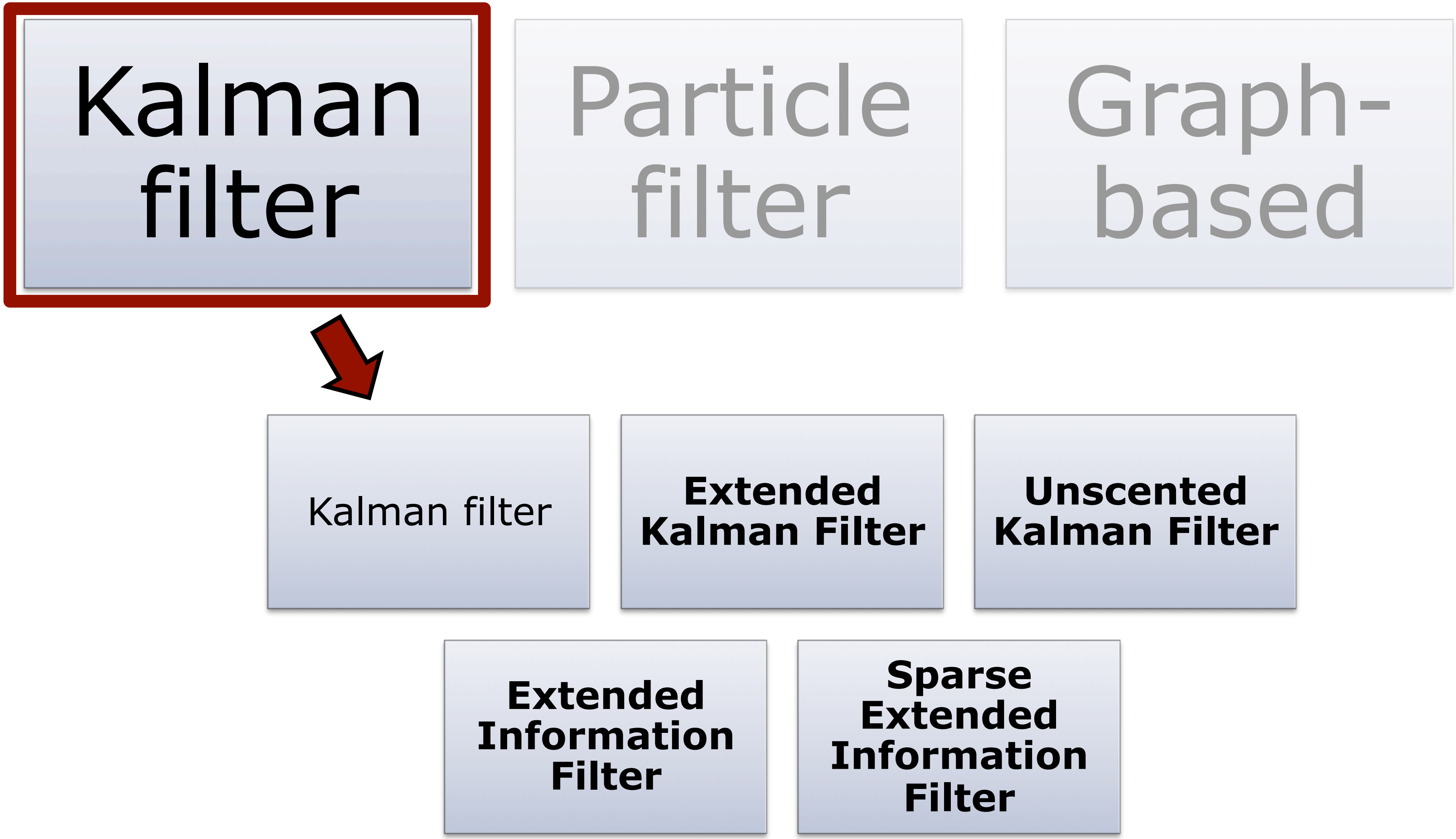
Three Main SLAM Paradigms

- **Kalman filter**
- Particle filter
- Graph-based

Kalman Filter & Its Friends



Kalman Filter Algorithm

1: $\textbf{Kalman\_filter}(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2: $\bar{\mu}_t = A_t \, \mu_{t-1} + B_t \, u_t$

3: $\bar{\Sigma}_t = A_t \, \Sigma_{t-1} \, A_t^T + R_t$

prediction

4: $K_t = \bar{\Sigma}_t \, C_t^T (C_t \, \bar{\Sigma}_t \, C_t^T + Q_t)^{-1}$

5: $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \, \bar{\mu}_t)$

6: $\Sigma_t = (I - K_t \, C_t) \, \bar{\Sigma}_t$

7: $return \; \mu_t, \Sigma_t$

correction

Non-linear Dynamic Systems

- Most realistic problems in robotics involve nonlinear functions

$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t \qquad z_t = C_t x_t + \delta_t$$

$$x_t = g(u_t, x_{t-1}) + \epsilon_t \qquad z_t = h(x_t) + \delta_t$$

requires linearization

➡ **EKF**

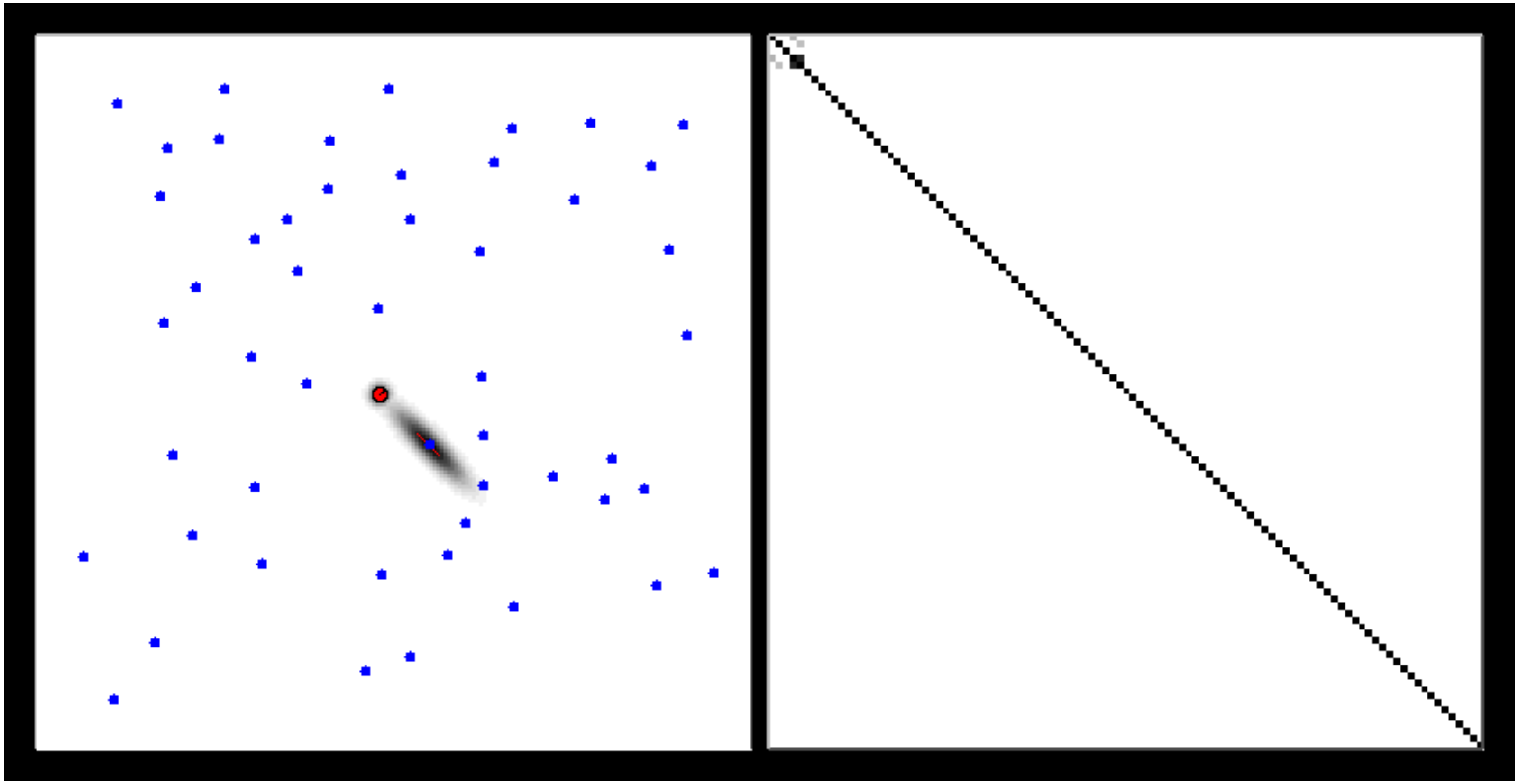
KF vs. EKF

- EKF is an extension of the KF
- Approach to handle the non-linearities
- Performs local linearizations
- Works well in practice for moderate non-linearities and uncertainty

EKF for SLAM

$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

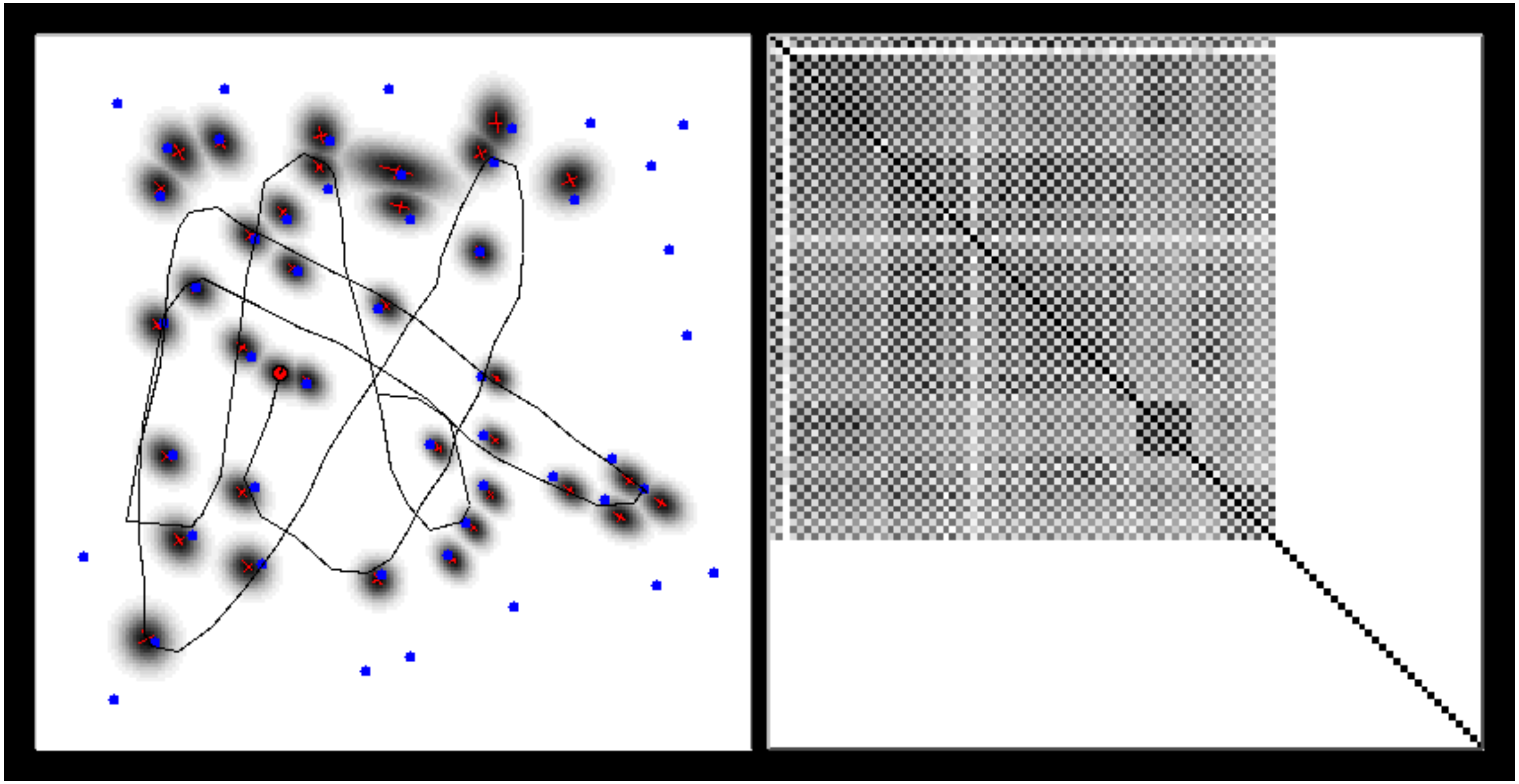
EKF SLAM



Map Correlation matrix

Courtesy of M. Montemerlo

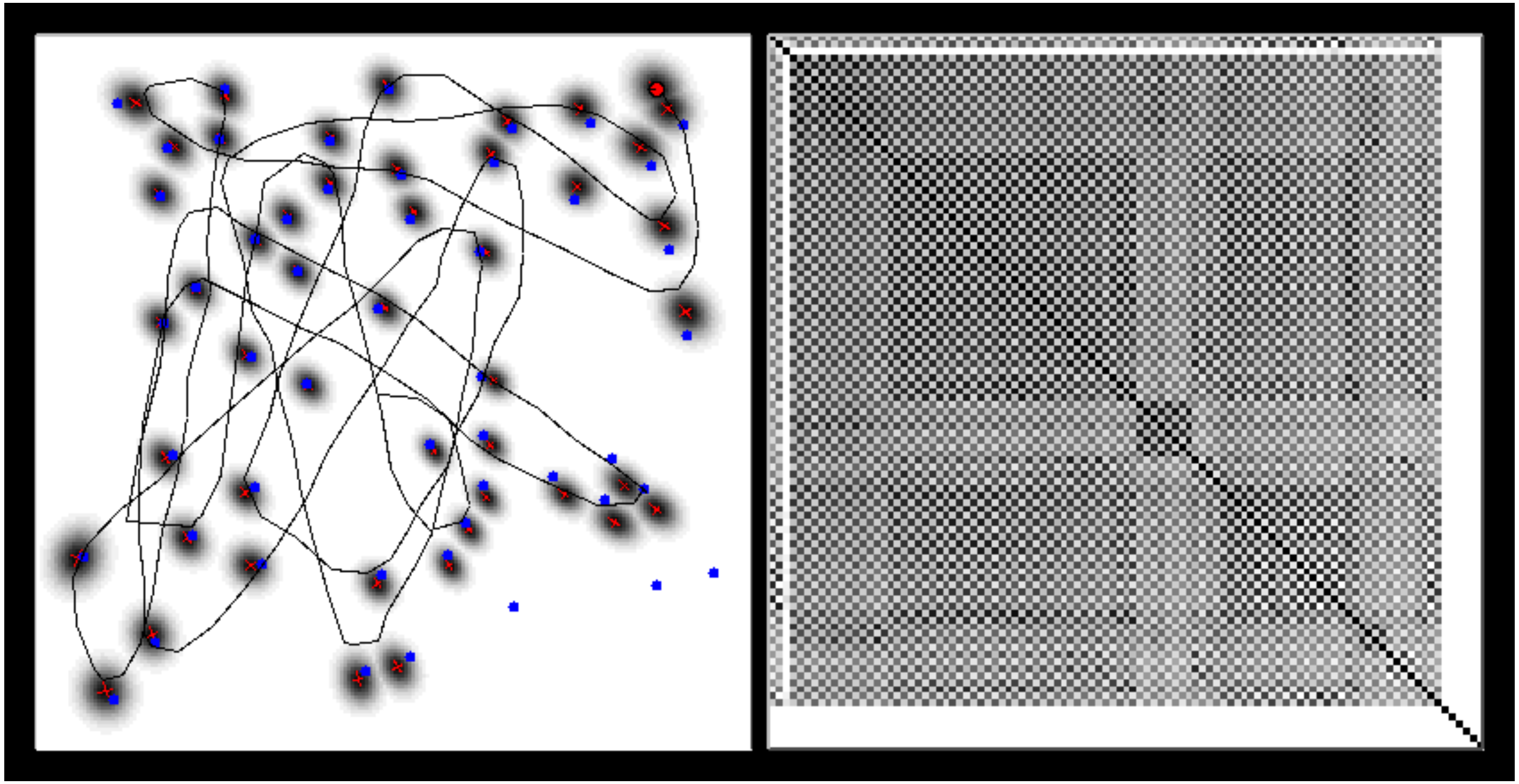
EKF SLAM



Map Correlation matrix

Courtesy of M. Montemerlo

EKF SLAM

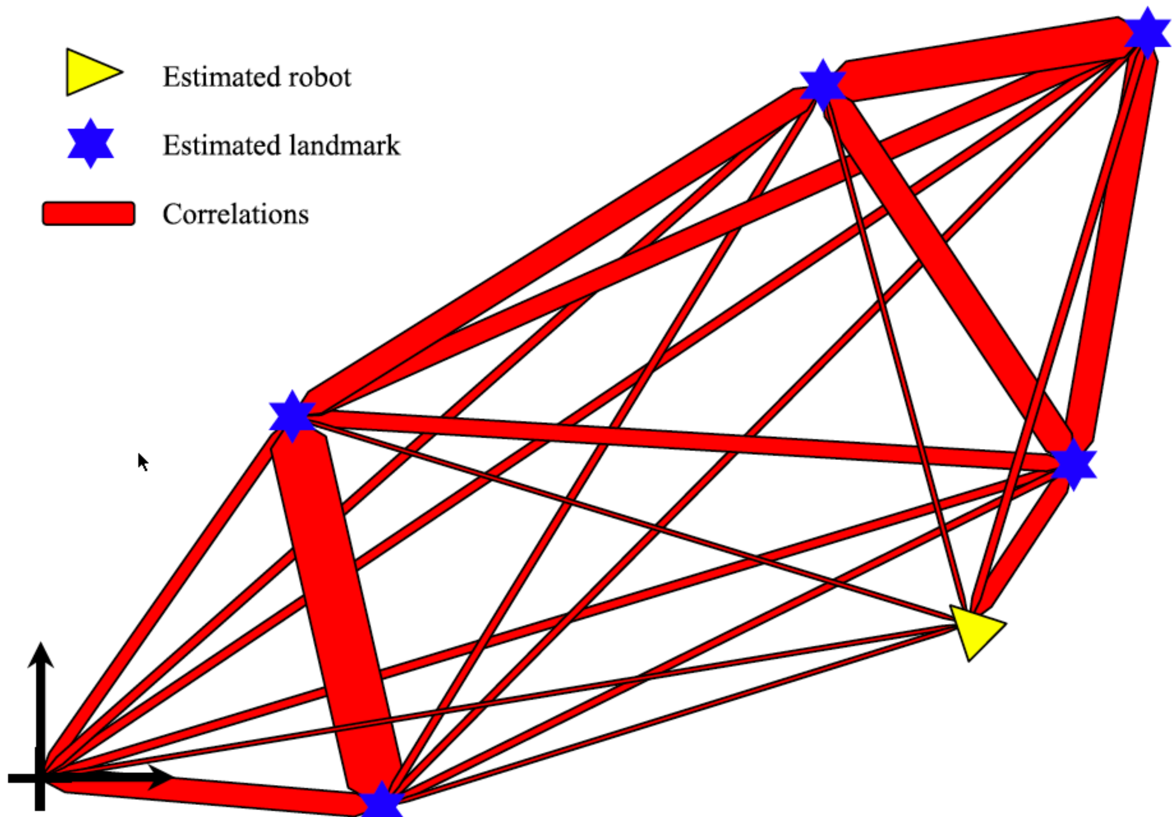


Map Correlation matrix

Courtesy of M. Montemerlo

EKF-SLAM Properties

- In the limit, the landmark estimates become **fully correlated**



[Dissanayake et al., 2001]

EKF-SLAM Complexity

- Cubic complexity only on the measurement dimensionality
- Cost per step: dominated by the number of landmarks: $O(n^2)$
- Memory consumption: $O(n^2)$
- The EKF becomes computationally intractable for large maps!

Unscented Kalman Filter (UKF)

UKF Motivation

- Kalman filter requires linear models
- EKF linearizes via Taylor expansion

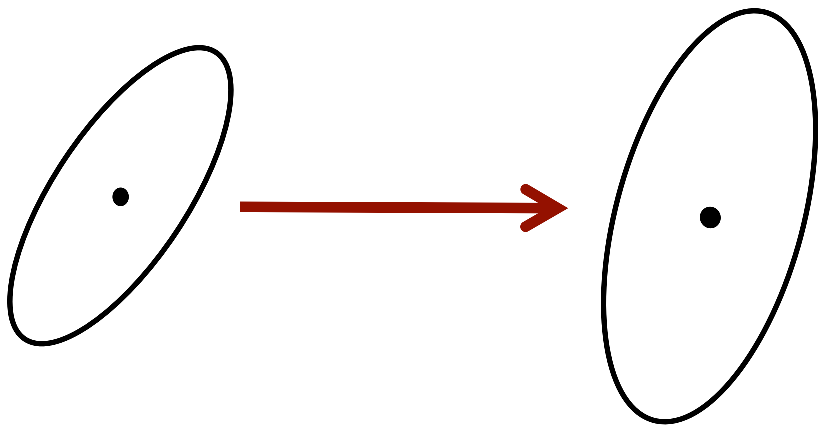
Is there a better way to linearize?

Unscented Transform

⬇

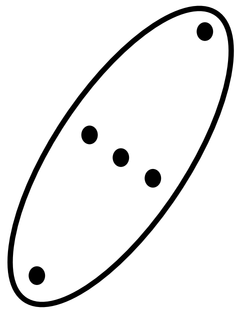
Unscented Kalman Filter (UKF)

Taylor Approximation (EKF)



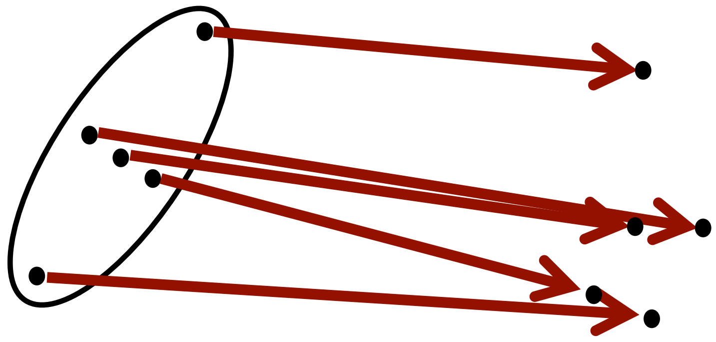
Linearization of the non-linear function through Taylor expansion

Unscented Transform



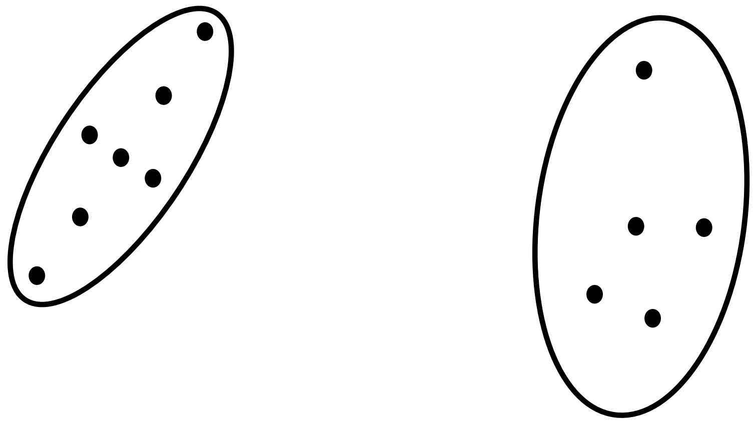
Compute a set of (so-called) sigma points

Unscented Transform



Transform each sigma point through the non-linear motion and measurement functions

Unscented Transform



Reconstruct a Gaussian from the transformed and weighted points

UKF vs. EKF

- Same results as EKF for linear models
- Better approximation than EKF for non-linear models
- Differences often “somewhat small”
- No Jacobians needed for the UKF
- Same complexity class
- Slightly slower than the EKF

EIF: Two Parameterizations for a Gaussian Distribution

moments	**canonical**
$\Sigma = \Omega^{-1}$	$\Omega = \Sigma^{-1}$
$\mu = \Omega^{-1}\xi$	$\xi = \Sigma^{-1}\mu$
covariance matrix mean vector	information matrix information vector

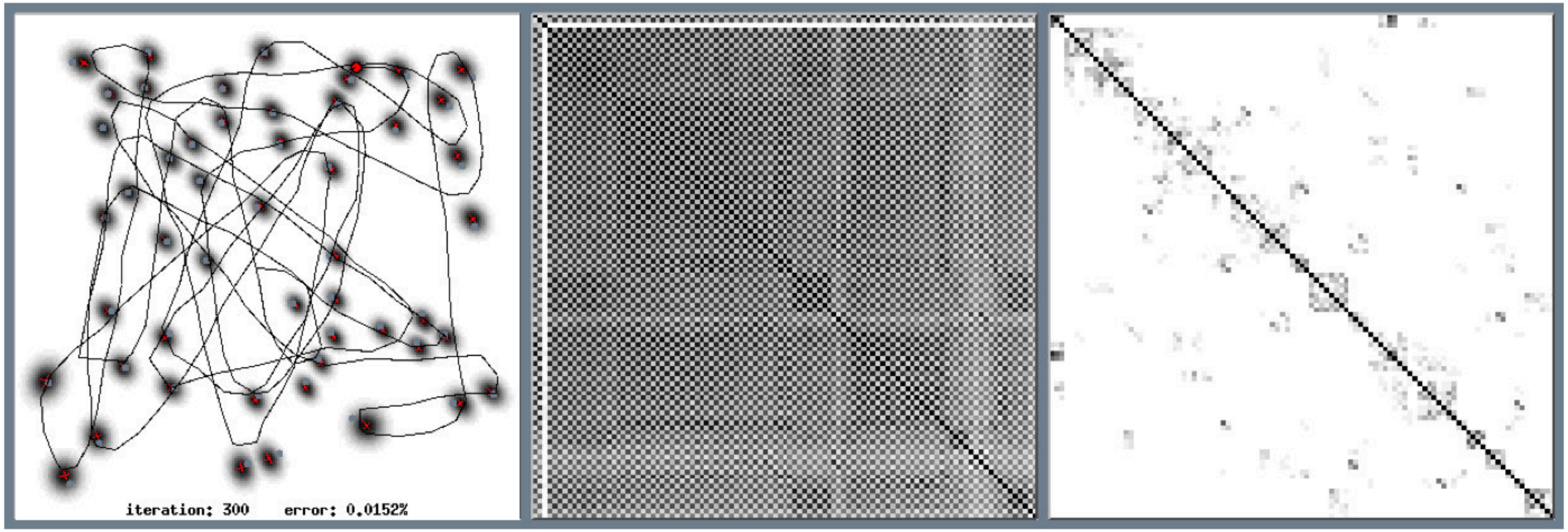
Extended Information Filter

- The EIF is the EKF in information form
- Instead of the moments Σ, μ the canonical form is maintained using Ω, ξ
- Conversion between information for and canonical form is expensive
- EIF has the same expressiveness than the EKF

EIF vs. EKF

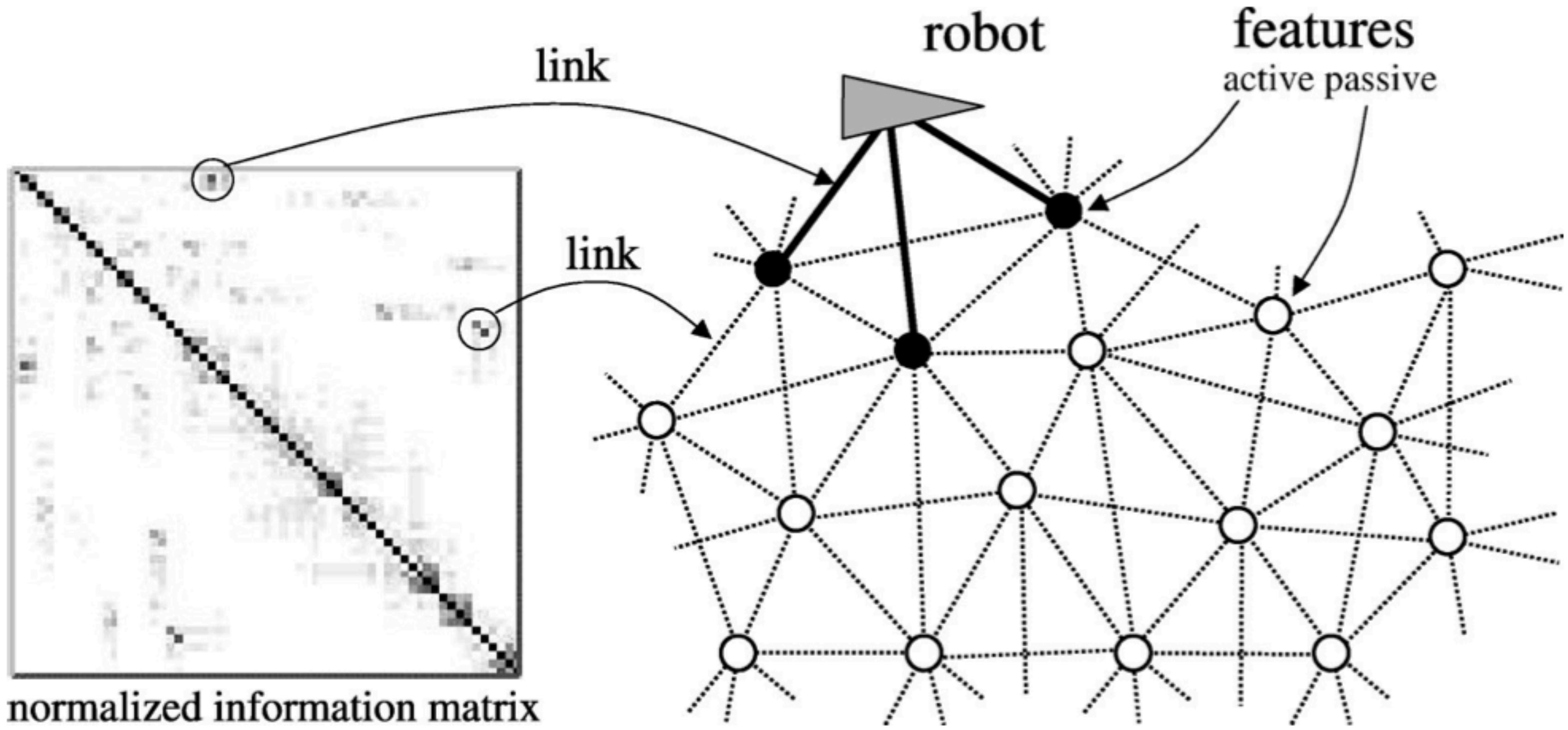
- Complexity of the prediction and corrections steps differs
- KF: efficient prediction, slow correction
- IF: slow prediction, efficient correction
- "The application determines the filter"
- In practice, the EKF is more popular than the EIF

Motivation for SEIF SLAM



Gaussian estimate (map & pose)	normalized covariance matrix	normalized information matrix

Keep the Links Between in the Information Matrix Bounded



Four Steps of SEIF SLAM

1. Motion update
2. Measurement update
3. Update of the state estimate
4. Sparsification

Efficiency of SEIF SLAM

- Maintains the robot-landmark links only for a small set of landmarks at a time
- Removes robot-landmark links by sparsification (equal to assuming conditional independence)
- This also bounds the number of landmark-landmark links
- Exploits the sparsity of the information matrix in all computations

SEIF SLAM vs. EKF SLAM

- SEIFs are an efficient **approximation** of the EIF for the SLAM problem
- Neglects links by sparsification
- **Constant time** updates of the filter (for known correspondences)
- **Linear memory** complexity
- **Inferior quality** compared to EKF SLAM

Summary

- KFs deal differently with non-linear motion and measurement functions
- KF, EKF, UKF, EIF suffer from complexity issues for large maps
- SEIF approximations lead to sub-quadratic memory and runtime complexity
- All filters presented so far, **require Gaussian distributions**