

Robot Mapping

Max-Mixture and Robust Least Squares for SLAM

Cyrill Stachniss



Courtesy for most images: Pratik Agarwal

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Least Squares in General

- Minimizes the **sum of the squared errors**
- Strong relation to ML estimation in the Gaussian case

Problems:

- **Sensitive to outliers**
- **Only Gaussians** (single modes)

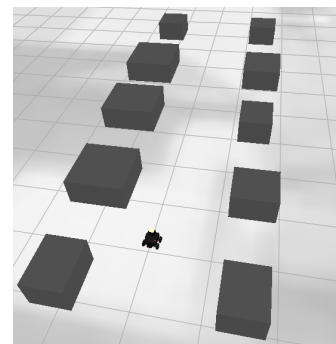
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Data Association Is Ambiguous And Not Always Perfect

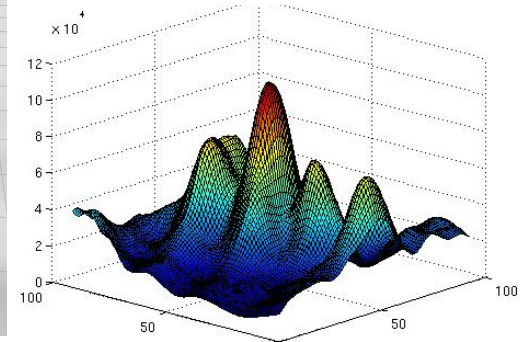
- Places that look identical
- Similar rooms in the same building
- Cluttered scenes
- GPS multi pass (signal reflections)
- ...

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Example



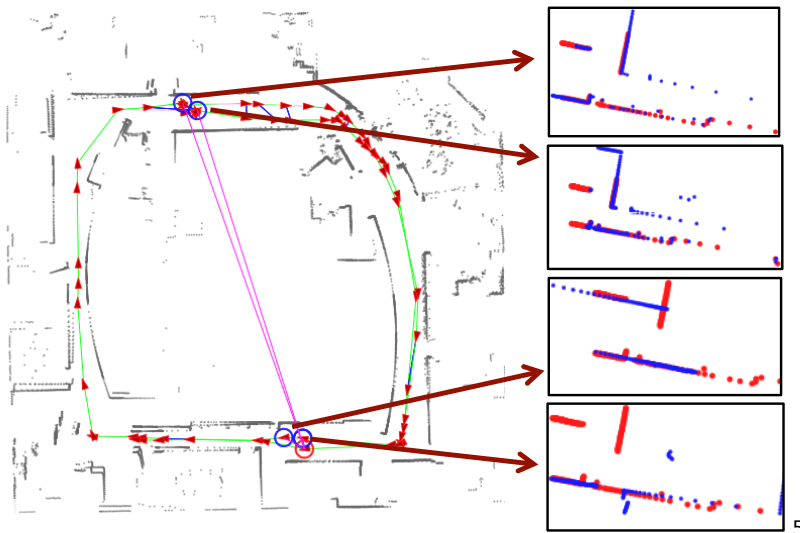
3D world



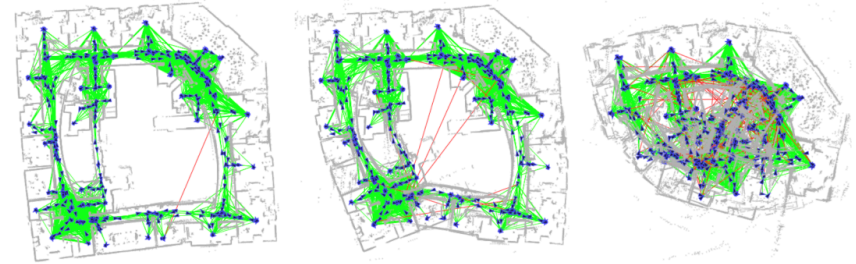
belief about the robot's pose

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Such Situations Occur In Reality



Committing To The Wrong Mode Can Lead to Mapping Failures



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How to incorporate that into graph-based SLAM?

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Mathematical Model

- We can express a multi-modal belief by a sum of Gaussians

$$p(\mathbf{z} | \mathbf{x}) = \eta \exp\left(-\frac{1}{2} \mathbf{e}_{ij}^T \boldsymbol{\Omega}_{ij} \mathbf{e}_{ij}\right)$$



$$p(\mathbf{z} | \mathbf{x}) = \sum_k w_k \eta_k \exp\left(-\frac{1}{2} \mathbf{e}_{ijk}^T \boldsymbol{\Omega}_{ijk} \mathbf{e}_{ijk}\right)$$

Sum of Gaussians with k modes

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Problem

- During error minimization, we consider the negative log likelihood

$$-\log p(\mathbf{z} | \mathbf{x}) = \frac{1}{2} \mathbf{e}_{ij}^T \boldsymbol{\Omega}_{ij} \mathbf{e}_{ij} - \log \eta$$



$$-\log p(\mathbf{z} | \mathbf{x}) = -\log \sum_k w_k \eta_k \exp\left(-\frac{1}{2} \mathbf{e}_{ijk}^T \boldsymbol{\Omega}_{ijk} \mathbf{e}_{ijk}\right)$$

The log cannot be moved inside the sum!

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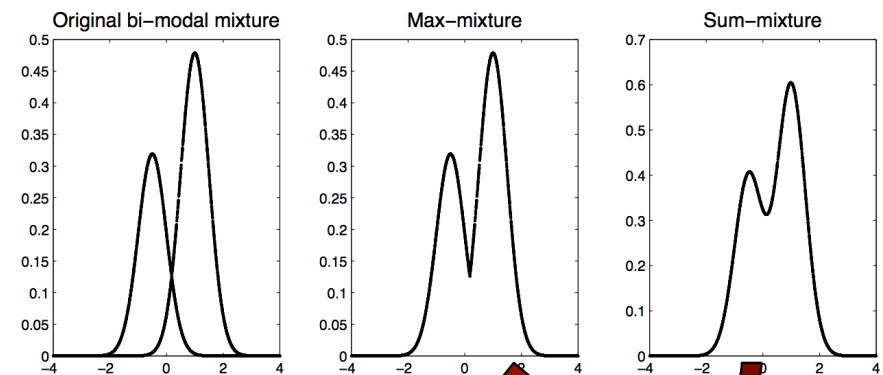
Max-Mixture Approximation

- Instead of computing the sum of Gaussians at \mathbf{X} , compute the maximum of the Gaussians

$$\begin{aligned} p(\mathbf{z} | \mathbf{x}) &= \sum_k w_k \eta_k \exp\left(-\frac{1}{2} \mathbf{e}_{ijk}^T \boldsymbol{\Omega}_{ijk} \mathbf{e}_{ijk}\right) \\ &\simeq \max_k w_k \eta_k \exp\left(-\frac{1}{2} \mathbf{e}_{ijk}^T \boldsymbol{\Omega}_{ijk} \mathbf{e}_{ijk}\right) \end{aligned}$$

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Max-Mixture Approximation



approximation error

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Log Likelihood Of The Max-Mixture Formulation

- The log can be moved inside the max operator

$$p(\mathbf{z} | \mathbf{x}) \simeq \max_k w_k \eta_k \exp\left(-\frac{1}{2} \mathbf{e}_{ijk}^T \boldsymbol{\Omega}_{ijk} \mathbf{e}_{ijk}\right)$$



$$\log p(\mathbf{z} | \mathbf{x}) \simeq \max_k -\frac{1}{2} \mathbf{e}_{ijk}^T \boldsymbol{\Omega}_{ijk} \mathbf{e}_{ijk} + \log(w_k \eta_k)$$

$$\text{or: } -\log p(\mathbf{z} | \mathbf{x}) \simeq \min_k \frac{1}{2} \mathbf{e}_{ijk}^T \boldsymbol{\Omega}_{ijk} \mathbf{e}_{ijk} - \log(w_k \eta_k)$$

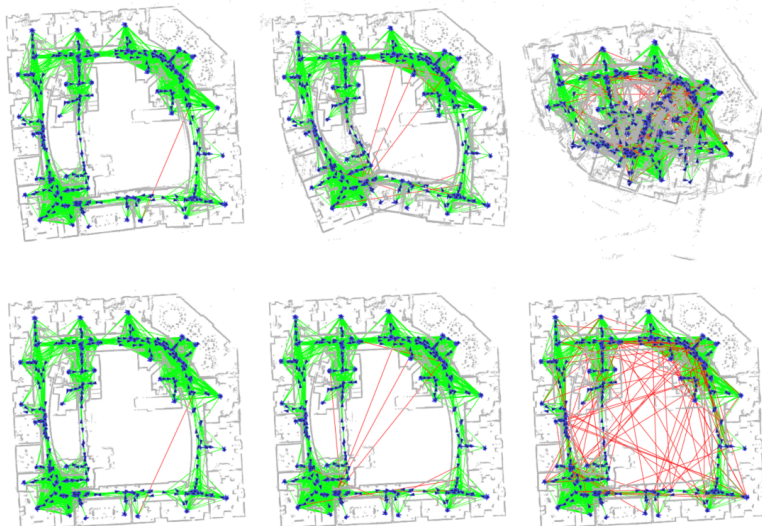
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Integration

- With the max-mixture formulation, the log likelihood again results in local quadratic forms
- Easy to integrate in the optimizer:
 1. Evaluate all k components
 2. Select the component with the maximum log likelihood
 3. Perform the optimization as before using only the max components (as a single Gaussian)

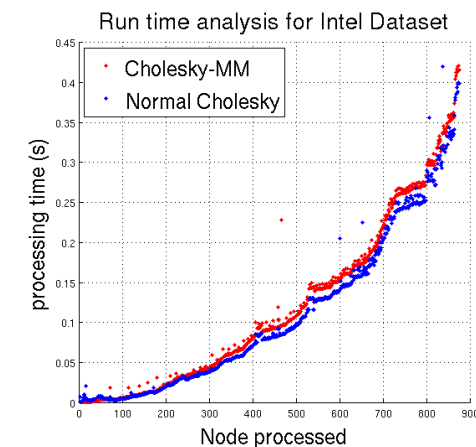
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Performance (Gauss vs. MM)



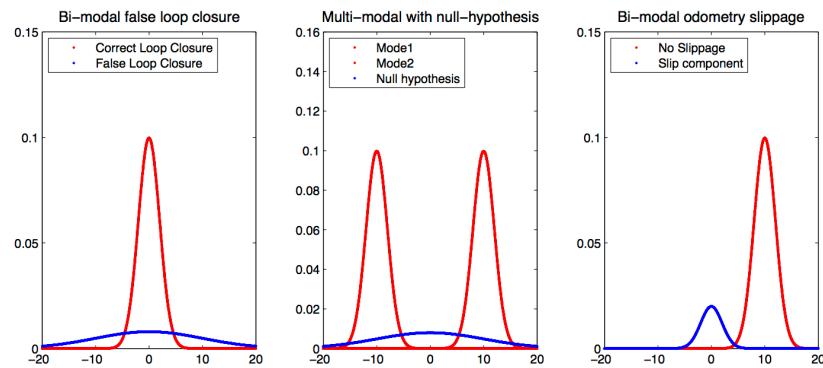
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Runtime



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MM For Outlier Rejection



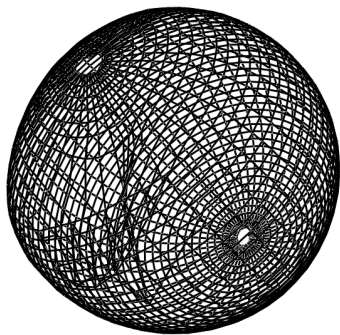
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Max-Mixture and Outliers

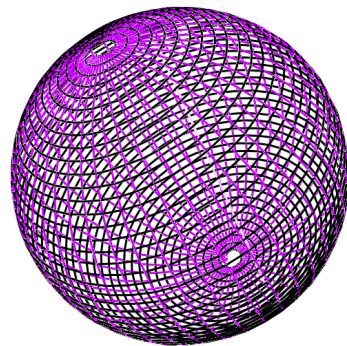
- MM formulation is useful for multi-model constraints (D.A. ambiguities)
- MM is also a handy tool outliers (D.A. failures)
- Here, one mode represents the edge and a second model uses a flat Gaussian for the outlier hypothesis

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Performance (1 outlier)



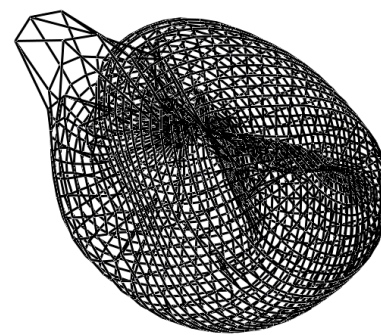
Gauss-Newton



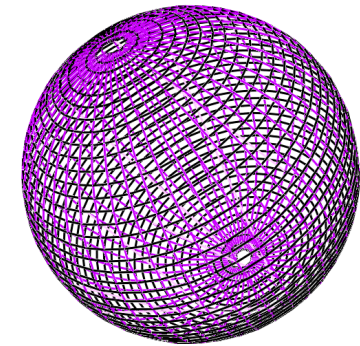
MM Gauss-Newton

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Performance (10 outliers)



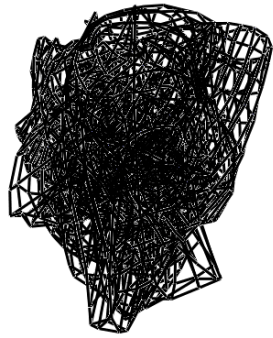
Gauss-Newton



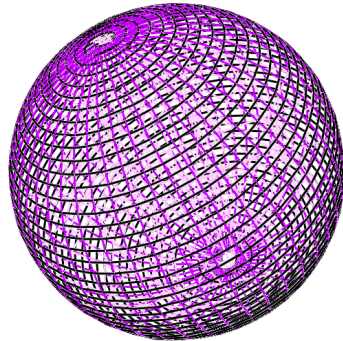
MM Gauss-Newton

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Performance (100 outliers)



Gauss-Newton

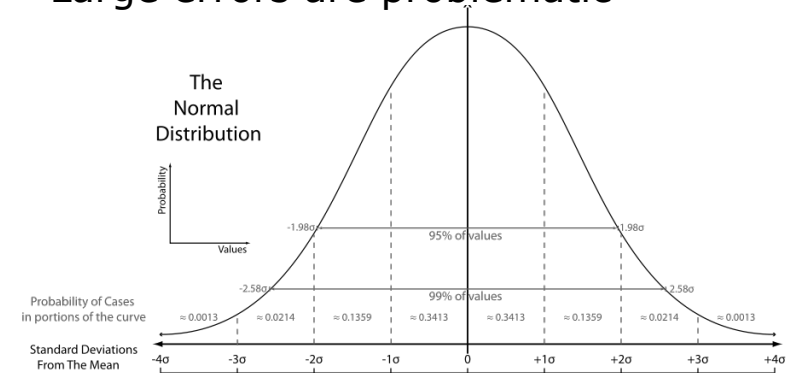


MM Gauss-Newton

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Optimizing With Outliers

- Assuming a Gaussian error in the constraints is not always realistic
- Large errors are problematic



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Robust M-Estimators

- Assume non-normally-distributed noise
- Intuitively: PDF with "heavy tails"
- $\rho(e)$ function used to define the PDF

$$p(e) = \exp(-\rho(e))$$

- Minimizing the neg. log likelihood

$$\mathbf{x}^* = \operatorname{argmin}_{\mathbf{x}} \sum_i \rho(e_i(\mathbf{x}))$$

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Different Rho Functions

- Gaussian: $\rho(e) = e^2$
- Absolute values (L1 norm): $\rho(e) = |e|$
- Huber M-estimator

$$\rho(e) = \begin{cases} \frac{e^2}{2} & \text{if } |e| < c \\ c(|e| - \frac{c}{2}) & \text{otherwise} \end{cases}$$

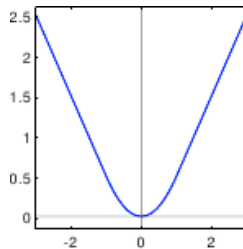
- Several others (Tukey, Cauchy, Blake-Zisserman, Corrupted Gaussian, ...)

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Huber

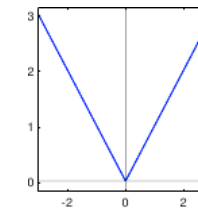
- Mixture of a quadratic and a linear function

$$\rho(e) = \begin{cases} \frac{e^2}{2} & \text{if } |e| < c \\ c(|e| - \frac{c}{2}) & \text{otherwise} \end{cases}$$

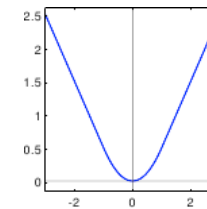


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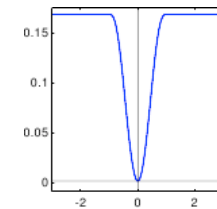
Different Rho Functions



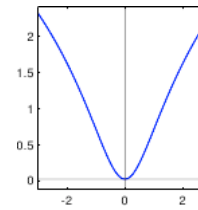
L1 norm



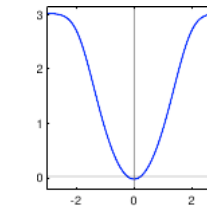
Huber



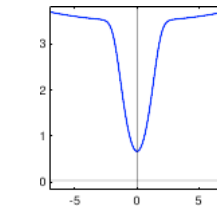
Tukey



Cauchy



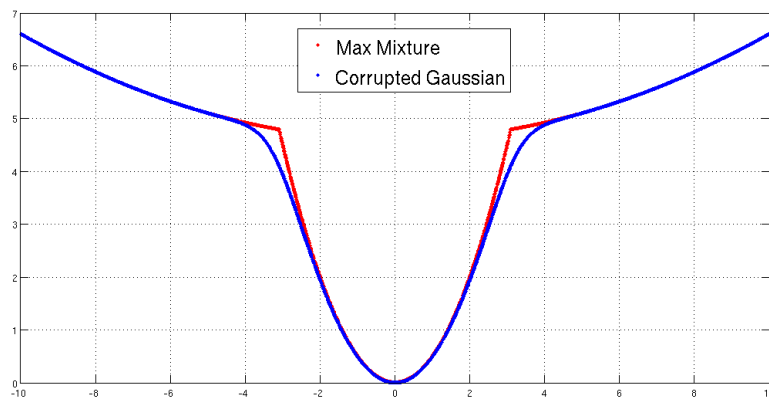
Blake-Zisserman



Corrupted G.

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MM Cost Function For Outliers



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Robust Estimation

- Choice of the rho function depends on the problem at hand
- Huber function is often used
- MM for outlier handling is similar to a corrupted Gaussian
- MM additionally supports multi-model constraints
- Combinations of MM for multi-modalities and Huber possible

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Conclusions

- Sum of Gaussians cannot be used easily in the optimization framework
- Max-Mixture formulation approximates the sum by the max operator
- This allows for handling data association ambiguities and failures
- Minimal performance overhead
- Minimal code changes for integration

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Literature

Max-Mixture Approach:

- Olson, Agarwal: "Inference on Networks of Mixtures for Robust Robot Mapping"

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