Robot Mapping

FastSLAM – Feature-based SLAM with Particle Filters

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Particle Filter in Brief

- Non-parametric, recursive Bayes filter
- Posterior is represented by a set of weighted samples
- Not limited to Gaussians
- Proposal to draw new samples
- Weight to account for the differences between the proposal and the target
- Works well in low-dimensional spaces

Particle Filter Algorithm

1. Sample the next particle set using the proposal distribution

$$x_t^{[i]} \sim proposal(x_t \mid \ldots)$$

2. Compute the importance weights

$$w_t^{[i]} = \frac{target(x_t^{[i]})}{proposal(x_t^{[i]})}$$

3. Resampling: "Replace unlikely samples by more likely ones"

Particle Representation

A set of weighted samples

$$\mathcal{X} = \left\{ \left\langle x^{[i]}, w^{[i]} \right\rangle \right\}_{i=1,\dots,N}$$

- Think of a sample as one hypothesis about the state
- For feature-based SLAM:

$$x = (x_{1:t}, l_{1,x}, l_{1,y}, \dots, l_{M,x}, l_{M,y})^T$$

pose landmarks

Dimensionality Problem

Particle filters are effective in low dimensional spaces as the likely volumes of the state space need to be covered with samples.

$$x = (x_{1:t}, l_{1,x}, l_{1,y}, \dots, l_{M,x}, l_{M,y})^T$$

high-dimensional

Can We Exploit Dependencies Between the Different Dimensions of the State Space?

$x_{1:t}, l_1, \ldots, l_M$

If We Know the Poses of the Robot, Mapping is Easy!

 $x_{1:t}, l_1, \ldots, l_M$

Key Idea $x_{1:t}, l_1, \ldots, l_M$

If we use the particle set only to model the robot's path, each sample is a path hypothesis. For each sample, we can then compute an individual map of landmarks

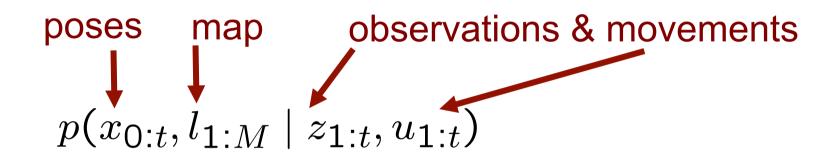
Rao-Blackwellization

 Factorization to exploit dependencies between variables:

$$p(a,b) = p(b \mid a) p(a)$$

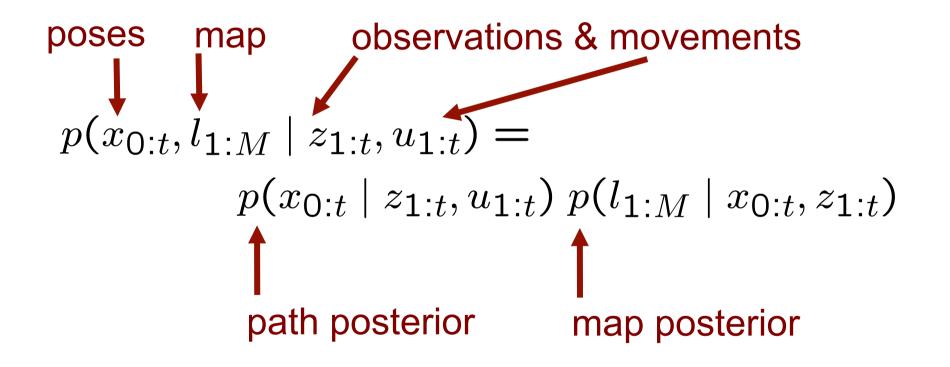
 If p(b | a) can be computed in closed form, represent only p(a) with samples and compute p(b | a) for every sample

Factorization of the SLAM posterior



First introduced for SLAM by Murphy in 1999

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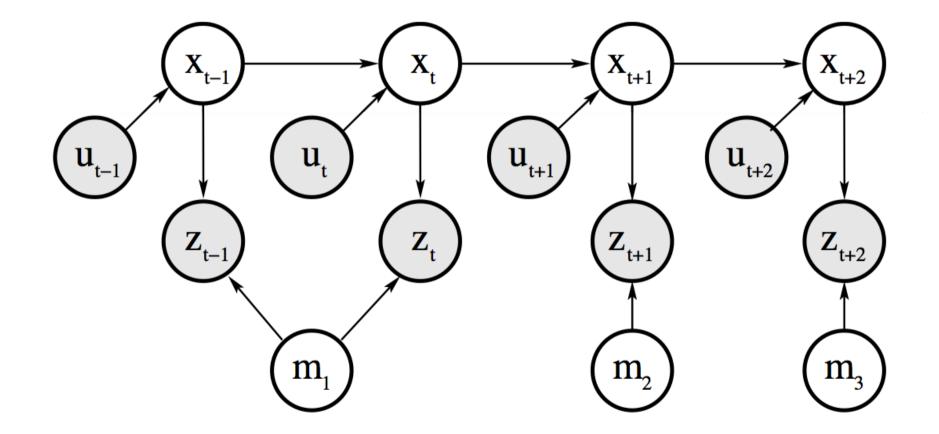
Factorization of the SLAM posterior

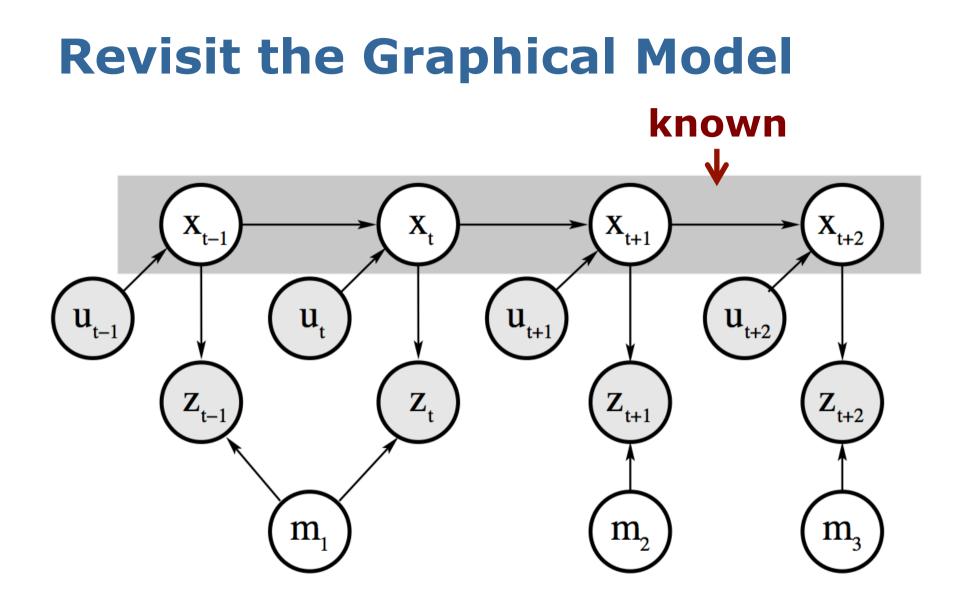
 $p(x_{0:t}, l_{1:M} \mid z_{1:t}, u_{1:t}) = p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(l_{1:M} \mid x_{0:t}, z_{1:t})$

How to compute this term efficiently?

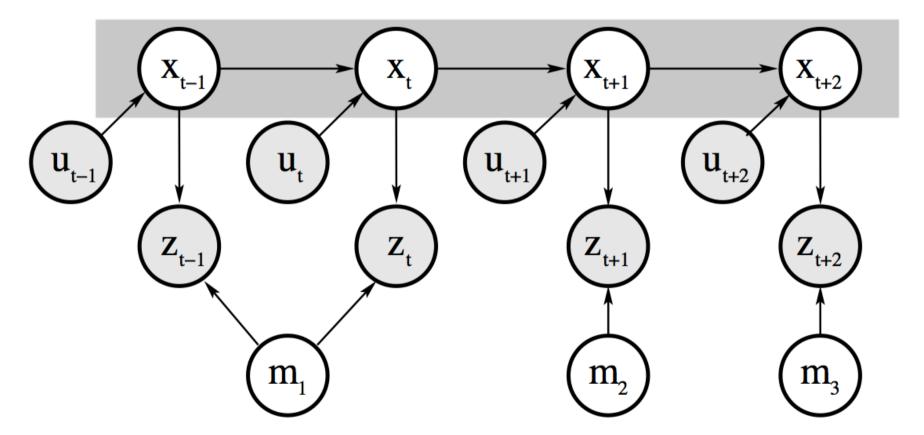
First introduced for SLAM by Murphy in 1999

Revisit the Graphical Model





Landmarks are Conditionally Independent Given the Poses



Landmark variables are all disconnected (i.e. independent) given the robot's path

Factorization of the SLAM posterior

 $p(x_{0:t}, l_{1:M} \mid z_{1:t}, u_{1:t}) = p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(l_{1:M} \mid x_{0:t}, z_{1:t})$

Landmarks are conditionally independent given the poses

Factorization of the SLAM posterior

 $p(x_{0:t}, l_{1:M} | z_{1:t}, u_{1:t}) = p(x_{0:t} | z_{1:t}, u_{1:t}) p(l_{1:M} | x_{0:t}, z_{1:t}) \\= p(x_{0:t} | z_{1:t}, u_{1:t}) \prod_{i=1}^{M} p(l_i | x_{0:t}, z_{1:t})$

First exploited in FastSLAM by Montemerlo et al., 2002

Factorization of the SLAM posterior

 $p(x_0 \cdot t, l_1 \cdot M \mid z_1 \cdot t, u_1 \cdot t)$ $= p(x_{0:t} | z_{1:t}, u_{1:t}) p(l_{1:M} | x_{0:t}, z_{1:t})$ $= p(x_{0:t} | z_{1:t}, u_{1:t}) \prod p(l_i | x_{0:t}, z_{1:t})$ $i \equiv 1$ **2-dimensional EKFs!**

First exploited in FastSLAM by Montemerlo et al., 2002

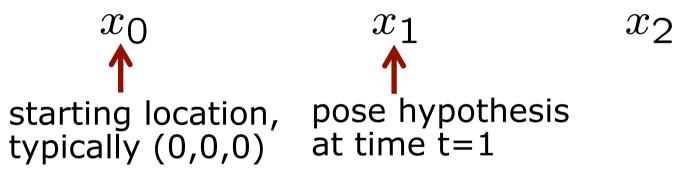
Factorization of the SLAM posterior

 $p(x_0 \cdot t, l_1 \cdot M \mid z_1 \cdot t, u_1 \cdot t)$ $= p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(l_{1:M} \mid x_{0:t}, z_{1:t})$ $= p(x_{0:t} \mid z_{1:t}, u_{1:t}) \prod p(l_i \mid x_{0:t}, z_{1:t})$ $\int_{0}^{-1} i=1$ particle filter similar to MCL 2-dimensional EKFs!

First exploited in FastSLAM by Montemerlo et al., 2002

Modeling the Robot's Path

- Sample-based representation for $p(x_{0:t} \mid z_{1:t}, u_{1:t})$
- Each sample is a path hypothesis



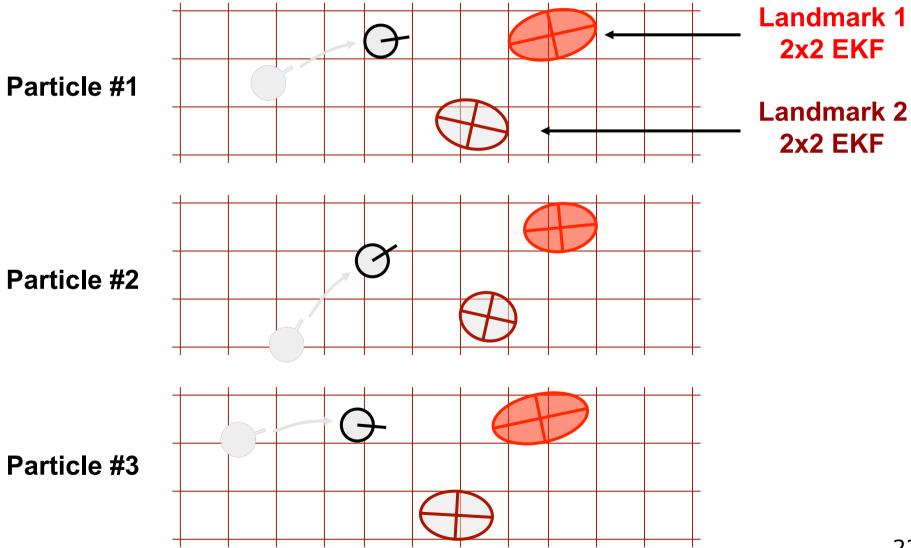
- Past poses of a sample are not revised
- No need to maintain past poses in the sample set

FastSLAM

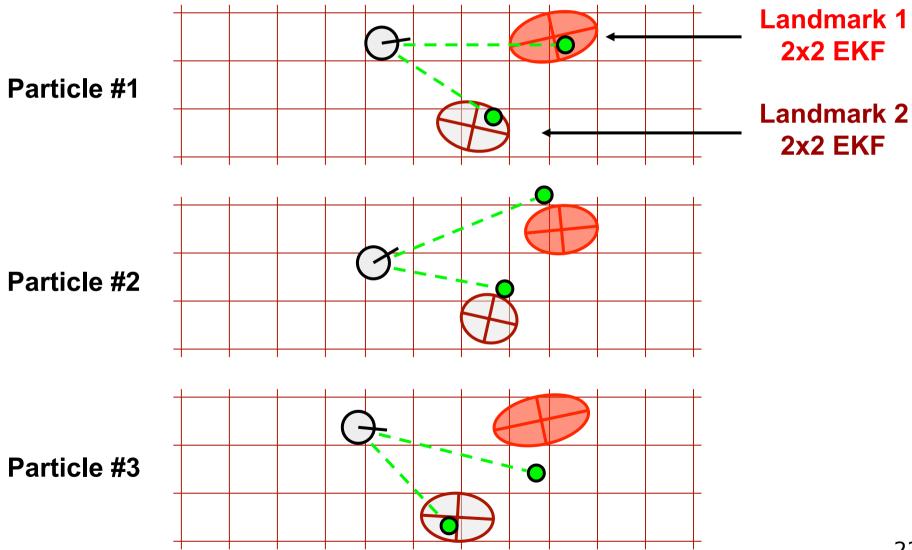
- Proposed by Montemerlo et al. in 2002
- Each landmark is represented by a 2x2 EKF
- Each particle therefore has to maintain M individual EKFs



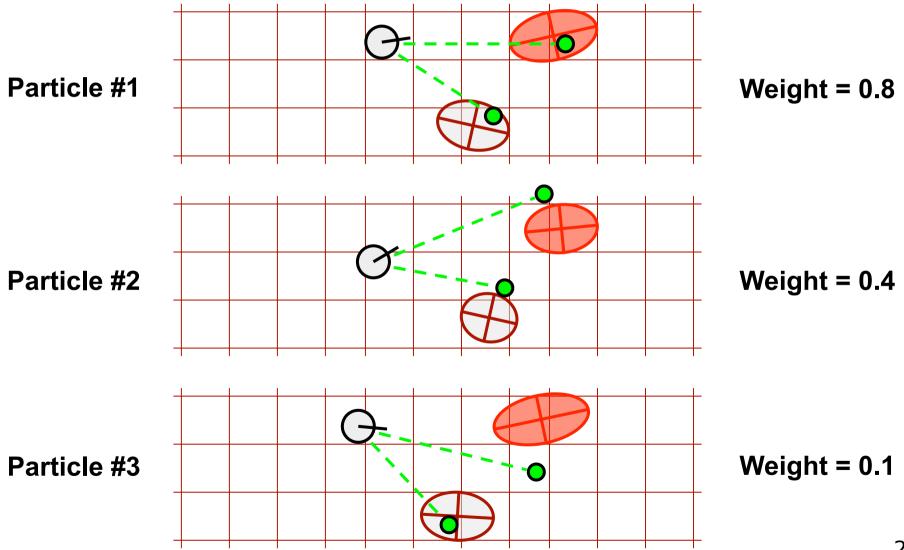
FastSLAM – Action Update



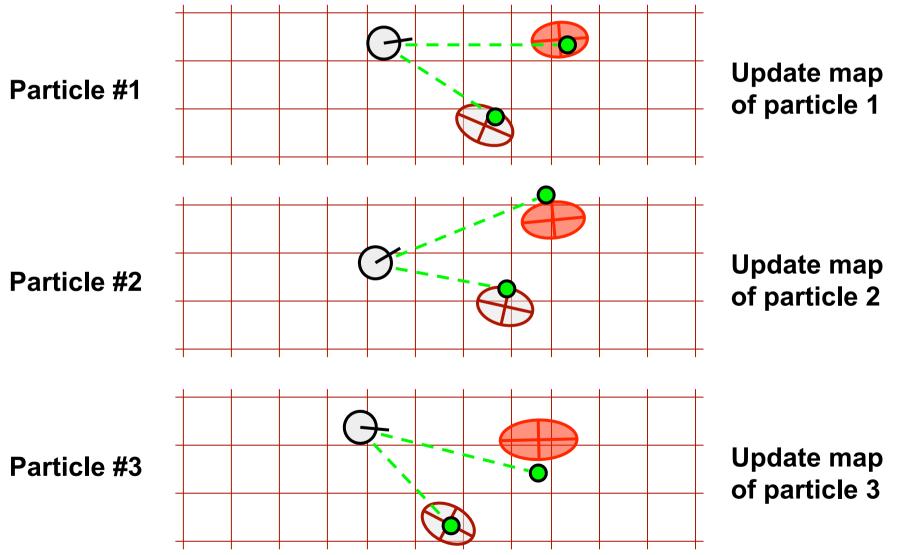
FastSLAM – Sensor Update



FastSLAM – Sensor Update



FastSLAM – Sensor Update



Key Steps of FastSLAM 1.0

 Extend the path posterior by sampling a new pose for each sample

 $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$

• Compute particle weight $w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$

measurement covariance

- Update belief of observed landmarks (EKF update rule)
- Resample

FastSLAM 1.0 – Part 1

- 1: FastSLAM1.0_known_correspondence($z_t, c_t, u_t, \mathcal{X}_{t-1}$):
- 2: for k = 1 to N do 3: Let $\left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle, \ldots \right\rangle$ be particle k in \mathcal{X}_{t-1} 4: $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$ // sample pose

FastSLAM 1.0 – Part 1

1: FastSLAM1.0_known_correspondence($z_t, c_t, u_t, \mathcal{X}_{t-1}$):

2: for k = 1 to N do // loop over all particles 3: Let $\left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle, \ldots \right\rangle$ be particle k in \mathcal{X}_{t-1}

4:
$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

// observed feature

// sample pose

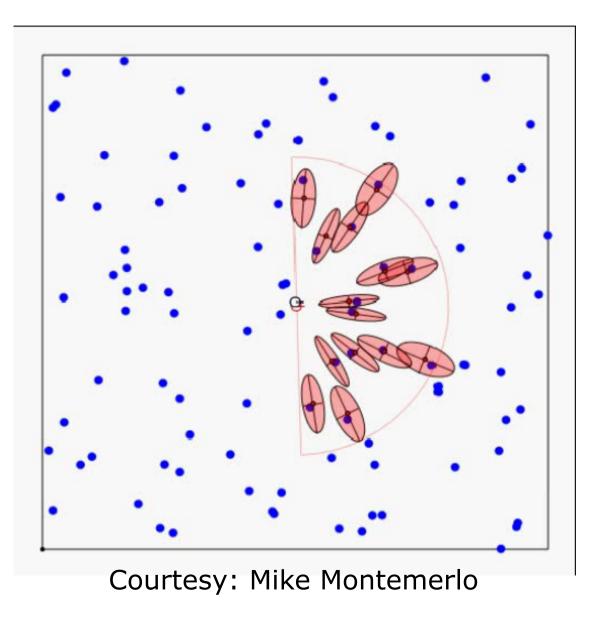
5:
$$j = c_t$$
 // observed feature
6: if feature j never seen before
7: $\mu_{j,t}^{[k]} = h^{-1}(z_t, x_t^{[k]})$ // initialize mean
8: $H = h'(\mu_{j,t}^{[k]}, x_t^{[k]})$ // calculate Jacobian
9: $\Sigma_{j,t}^{[k]} = H^{-1} Q_t (H^{-1})^T$ // initialize covariance
10: $w^{[k]} = p_0$ // default importance weight
11: else

FastSLAM 1.0 – Part 2

11:	else
12:	$ \langle \mu_{j,t}^{[k]}, \Sigma_{j,t}^{[k]} \rangle = EKF\text{-}Update(\dots) // \text{ update landmark} \\ w^{[k]} = 2\pi Q ^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\} $
13:	$w^{[k]} = 2\pi Q ^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$
measurement cov. $Q = H \Sigma_{j,t-1}^{[k]} H^T + Q_t$ exp. observation	
14:	endif
15:	for all unobserved features j' do
16:	$\langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} \rangle // \text{ leave unchanged }$
17:	endfor
18:	endfor
19:	$\mathcal{X}_{t} = \text{resample}\left(\left\langle x_{t}^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)$
20:	return \mathcal{X}_t

FastSLAM 1.0 – Part 2 (long)

FastSLAM in Action



- Derivation of the importance weight
- Based on the importance sampling principle

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

The target distribution is

 $p(x_{1:t} \mid z_{1:t}, u_{1:t})$

The proposal distribution is

$$p(x_{1:t} \mid z_{1:t-1}, u_{1:t})$$

Proposal is used step-by-step

$$p(x_{1:t} \mid z_{1:t-1}, u_{1:t}) = \underbrace{p(x_t \mid x_{t-1}, u_t)}_{\text{from } \mathcal{X}_{t-1} \text{ to } \bar{\mathcal{X}}_t} \underbrace{p(x_{1:t-1} \mid z_{1:t-1}, u_{1:t-1})}_{\mathcal{X}_{t-1}}$$
33

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_{t}^{[k]} \mid x_{t-1}, u_{t}) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$
$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) \ p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

Bayes rule + factorization

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$\frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$\frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

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$$= \frac{\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$= \frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} = \eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

= $\eta \int_{l_j} p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, l_j) p(l_j \mid x_{1:t}^{[k]}, z_{1:t-1}) dl_j$

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} = \eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

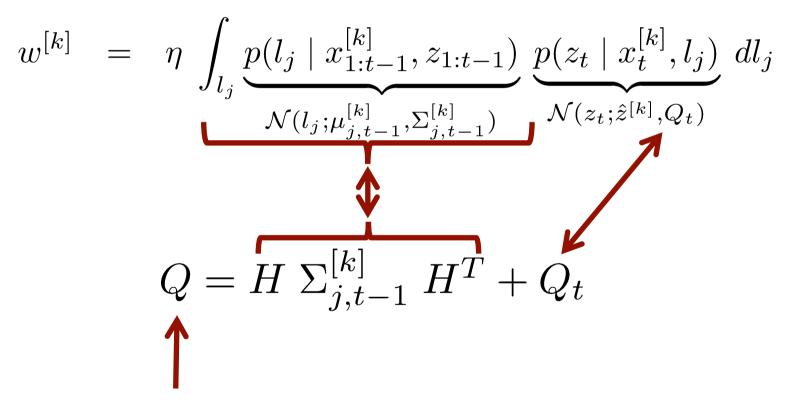
= $\eta \int_{l_j} p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, l_j) p(l_j \mid x_{1:t}^{[k]}, z_{1:t-1}) dl_j$
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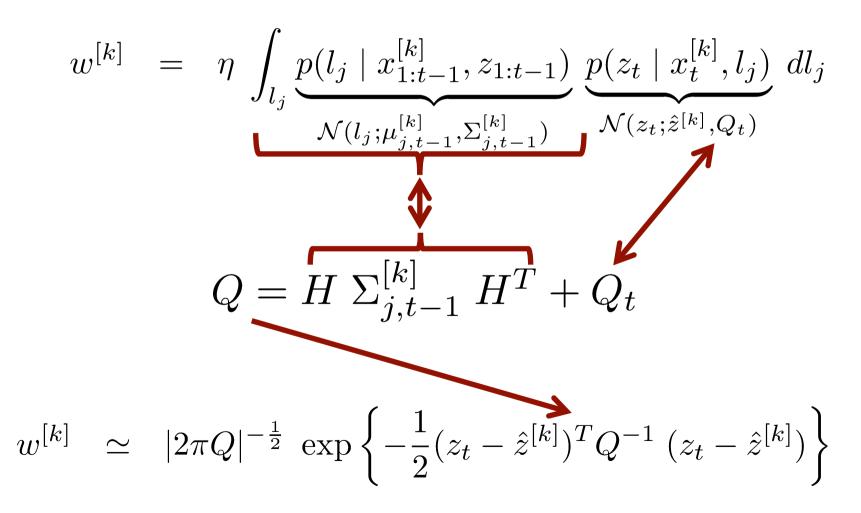
= $\eta \int_{l_j} p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, l_j) p(l_j \mid x_{1:t}^{[k]}, z_{1:t-1}) dl_j$
= $\eta \int_{l_j} \underbrace{p(z_t \mid x_t^{[k]}, l_j)}_{\mathcal{N}(z_t; \hat{z}^{[k]}, Q_t)} \underbrace{p(l_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(l_j; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} dl_j$

This leads to



measurement covariance (pose uncertainty of I and measurement noise)

This leads to

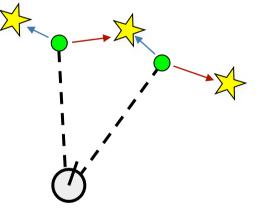


FastSLAM 1.0 – Part 2

11: 12:	else $\langle \mu_{j,t}^{[k]}, \Sigma_{j,t}^{[k]} \rangle = EKF-Update() // update landmark$
13:	$w^{[k]} = 2\pi Q ^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$
14:	end if
15:	for all unobserved features j' do
16:	$\langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} \rangle$ // leave unchanged
17:	endfor
18:	endfor
19:	$\mathcal{X}_{t} = \text{resample}\left(\left\langle x_{t}^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)$
20:	return \mathcal{X}_t

Data Association Problem

Which observation belongs to which landmark?



- More than one possible association
- Potential data associations depend on the pose of the robot

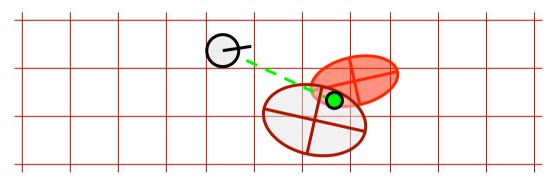
Particles Support for Multi-Hypotheses Data Association

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- Decisions on a perparticle basis
- Robot pose error is factored out of data association decisions

 $\overline{\mathbf{x}}$

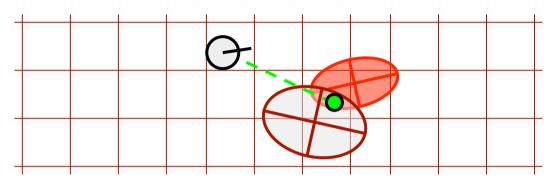
Per-Particle Data Association



Was the observation generated by the **red** or by the **brown** landmark?

P(observation|red) = 0.3 P(observation|brown) = 0.7

Per-Particle Data Association

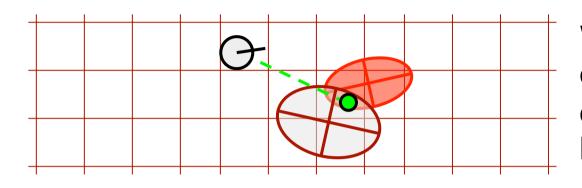


Was the observation generated by the **red** or by the **brown** landmark?

P(observation|red) = 0.3 P(observation|brown) = 0.7

- Two options for per-particle data association
 - Pick the most probable match
 - Pick an random association weighted by the observation likelihoods
- If the probability for an assignment is too low, generate a new landmark

Per-Particle Data Association

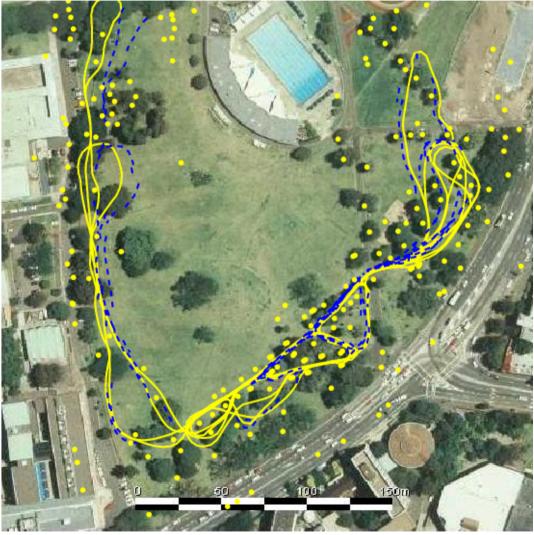


Was the observation generated by the **red** or by the **brown** landmark?

- Multi-modal belief
- Pose error is factored out of data association decisions
- Simple but effective data association
- Big advantage of FastSLAM over EKF

Results – Victoria Park

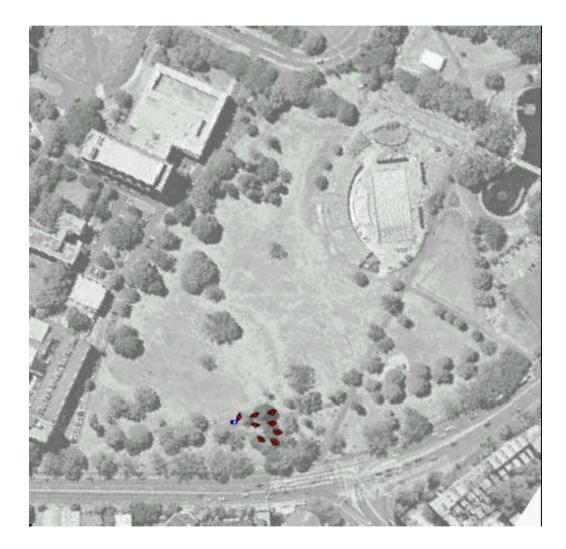
- 4 km traverse
- < 2.5 m RMS position error
- 100 particles



Blue = GPS Yellow = FastSLAM

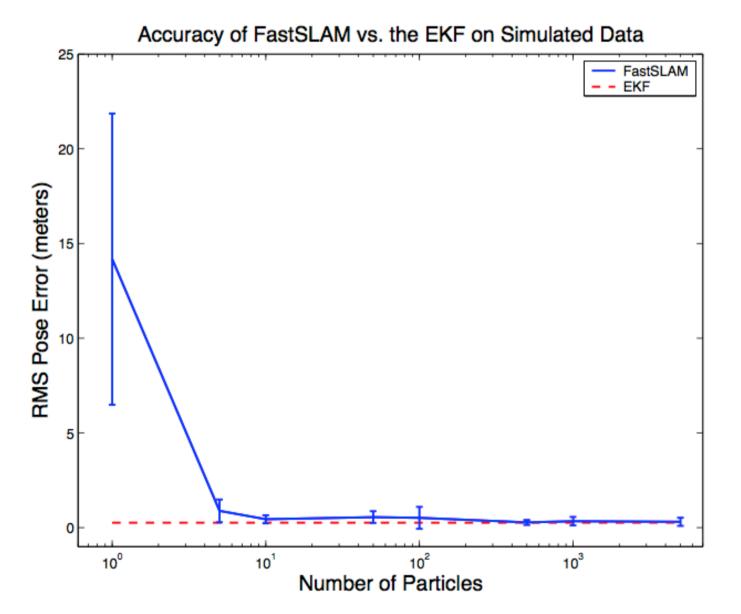
Courtesy: Mike Montemerlo 50

Results – Victoria Park (Video)

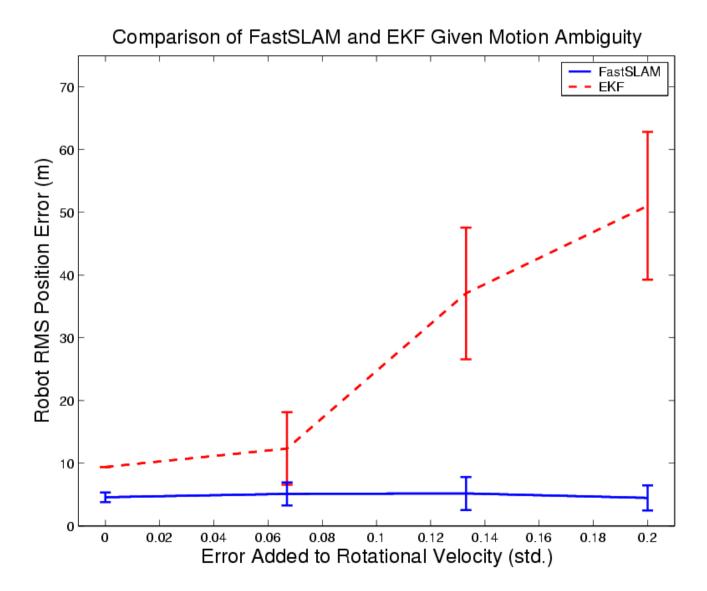


Courtesy: Mike Montemerlo

Results (Sample Size)



Results (Motion Uncertainty)



FastSLAM 1.0 Summary

- Use a particle filter to model the belief
- Factors the SLAM posterior into lowdimensional estimation problems
- Model only the robot's path by sampling
- Compute the landmarks given the path
- Per-particle data association
- No robot pose uncertainty in the perparticle data association

FastSLAM Complexity – Simple Implementation

 Update robot particles based on the control

 $\mathcal{O}(N)$

 $\mathcal{O}(N)$

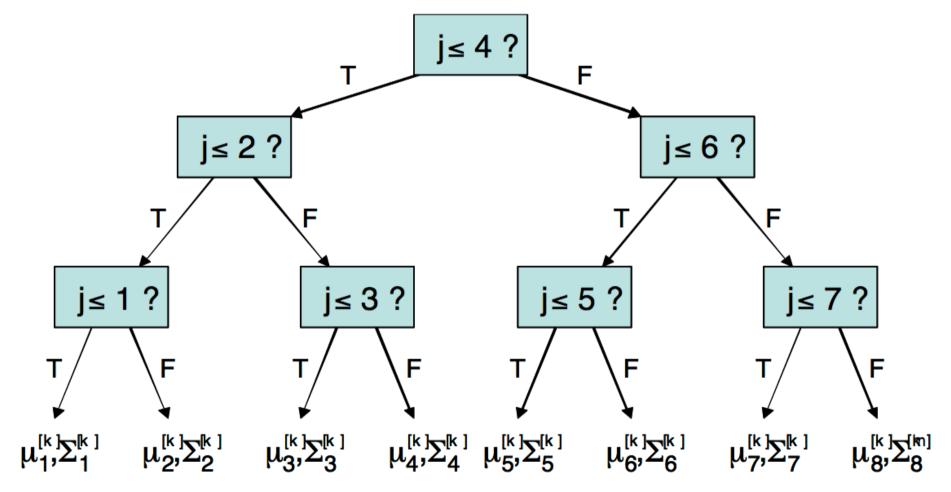
 Incorporate an observation into the Kalman filters

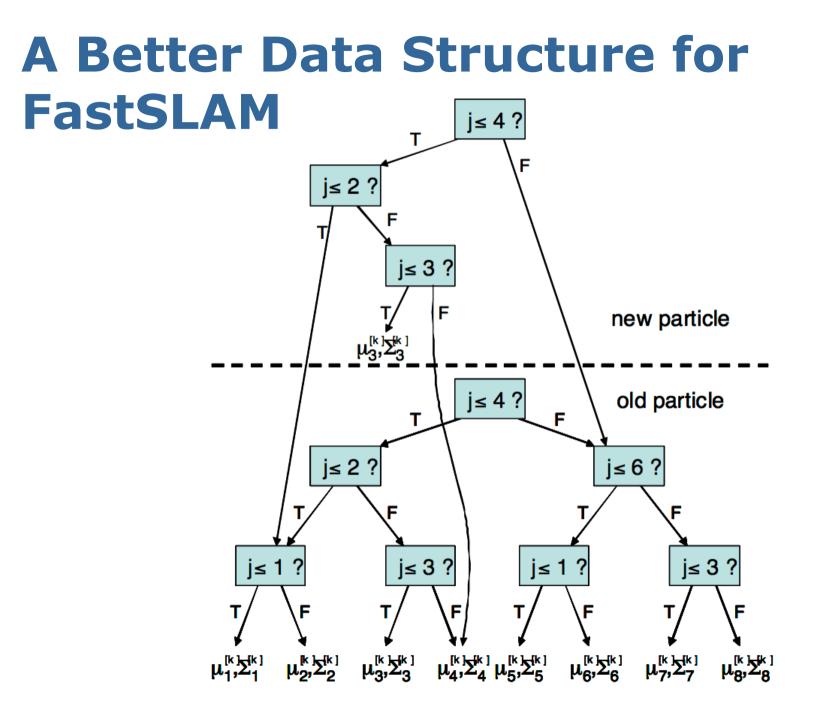
Resample particle set

N = Number of particles M = Number of map features $\mathcal{O}(NM)$

 $\mathcal{O}(NM)$

A Better Data Structure for FastSLAM





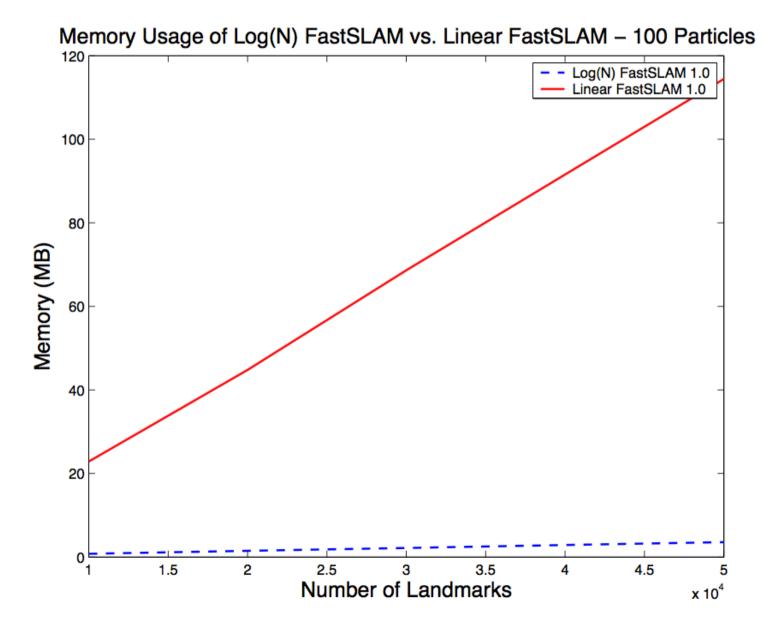
FastSLAM Complexity

- Update robot particles based on the control
- $\mathcal{O}(N)$
- Incorporate an observation $\ \mathcal{O}(N\log M)$ into the Kalman filters
- Resample particle set

 $\mathcal{O}(N \log M)$

N = Number of particles M = Number of map features $\mathcal{O}(N \log M)$

Memory Complexity



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FastSLAM 1.0

 FastSLAM 1.0 uses the motion model as the proposal distribution

 $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$

 Is there a better distribution to sample from?

[Montemerlo et al., 2002] 60

FastSLAM 1.0 to FastSLAM 2.0

 FastSLAM 1.0 uses the motion model as the proposal distribution

 $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$

- FastSLAM 2.0 considers also the measurements during sampling
- Especially useful if an accurate sensor is used (compared to the motion noise)

FastSLAM 2.0 (Informally)

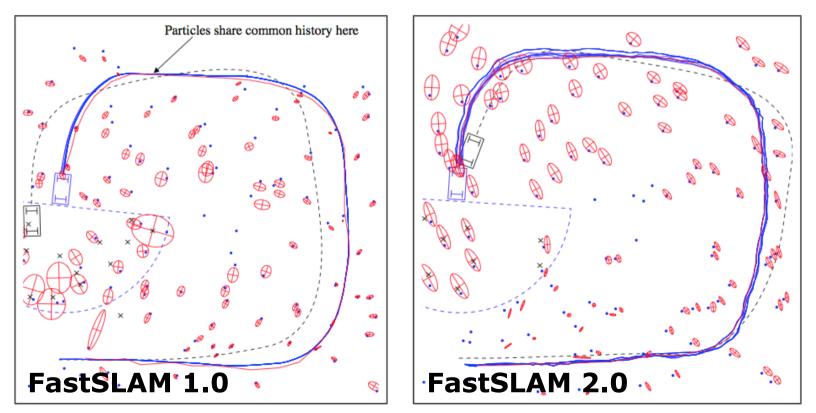
FastSLAM 2.0 samples from

$$x_t^{[k]} \sim p(x_t \mid x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t})$$

- Results in a more peaked proposal distribution
- Less particles are required
- More robust and accurate
- But more complex...

FastSLAM Problems

- How to determine the sample size?
- Particle deprivation, especially when closing (multiple) loops



FastSLAM Summary

- Particle filter-based SLAM
- Rao-Blackwellization: model the robot's path by sampling and compute the landmarks given the poses
- Allow for per-particle data association
- FastSLAM 1.0 and 2.0 differ in the proposal distribution
- Complexity $\mathcal{O}(N \log M)$

FastSLAM Results

- Scales well (1 million + features)
- Robust to ambiguities in the data association
- Advantages compared to the classical EKF approach (especially with nonlinearities)

Literature

FastSLAM

- Thrun et al.: "Probabilistic Robotics", Chapter 13.1-13.3 + 13.8 (see errata!)
- Montemerlo, Thrun, Kollar, Wegbreit: FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem, 2002
- Montemerlo and Thrun: Simultaneous Localization and Mapping with Unknown Data Association Using FastSLAM, 2003