Particle Filter in Brief Robot Mapping Non-parametric, recursive Bayes filter Posterior is represented by a set of FastSLAM – Feature-based SLAM weighted samples with Particle Filters Not limited to Gaussians Proposal to draw new samples Weight to account for the differences **Cyrill Stachniss** between the proposal and the target Works well in low-dimensional spaces AIS Autonomous Intelligent 1 2

Particle Filter Algorithm

1. Sample the next particle set using the proposal distribution

 $x_t^{[i]} \sim proposal(x_t \mid \ldots)$

2. Compute the importance weights

 $w_t^{[i]} = \frac{target(x_t^{[i]})}{proposal(x_t^{[i]})}$

3. Resampling: "Replace unlikely samples by more likely ones"

Particle Representation

A set of weighted samples

 $\mathcal{X} = \left\{ \left\langle x^{[i]}, w^{[i]} \right\rangle \right\}_{i=1,\ldots,N}$

- Think of a sample as one hypothesis about the state
- For feature-based SLAM:

$$x = (x_{1:t}, l_{1,x}, l_{1,y}, \dots, l_{M,x}, l_{M,y})^T$$

pose landmarks

Dimensionality Problem

Particle filters are effective in low dimensional spaces as the likely volumes of the state space need to be covered with samples.

$$x = (x_{1:t}, l_{1,x}, l_{1,y}, \dots, l_{M,x}, l_{M,y})^T$$

high-dimensional

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If We Know the Poses of the Robot, Mapping is Easy!

 $\underline{x_{1:t}}, \underline{l_1}, \ldots, \underline{l_M}$

Can We Exploit Dependencies Between the Different Dimensions of the State Space?

 $x_{1:t}, l_{1}, \ldots, l_{M}$

Key Idea

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 $x_{1:t}, l_1, \ldots, l_M$

If we use the particle set only to model the robot's path, each sample is a path hypothesis. For each sample, we can then compute an individual map of landmarks

Rao-Blackwellization

 Factorization to exploit dependencies between variables:

 $p(a,b) = p(b \mid a) p(a)$

 If p(b | a) can be computed in closed form, represent only p(a) with samples and compute p(b | a) for every sample

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior
 poses map observations & movements

First introduced for SLAM by Murphy in 1999

 $p(x_{0:t}, l_{1:M} \mid z_{1:t}, u_{1:t})$

Rao-Blackwellization for SLAM





Rao-Blackwellization for SLAM



Revisit the Graphical Model



Landmarks are Conditionally Independent Given the Poses



<image><figure><complex-block><complex-block>

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

 $p(x_{0:t}, l_{1:M} \mid z_{1:t}, u_{1:t}) = p(x_{0:t} \mid z_{1:t}, u_{1:t}) \underline{p(l_{1:M} \mid x_{0:t}, z_{1:t})}$

Landmarks are conditionally independent given the poses



FastSLAM

- Proposed by Montemerlo et al. in 2002
- Each landmark is represented by a 2x2 EKF
- Each particle therefore has to maintain M individual EKFs



FastSLAM – Sensor Update



Particle #1 Image: mail of the second se

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FastSLAM – Sensor Update





FastSLAM 1.0 - Part 1

1:	: FastSLAM1.0_known_correspondence($z_t, c_t, u_t, \mathcal{X}_{t-1}$):		
2:	for $k = 1$ to N do // loop over all particles		
2: 3:	for $k = 1$ to N do // loop over all particles Let $\left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle, \ldots \right\rangle$ be particle k in \mathcal{X}_{t-1}		
4:	$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t) \qquad // \text{ sample pose}$		

Key Steps of FastSLAM 1.0

Extend the path posterior by sampling a new pose for each sample x_t^[k] ~ p(x_t | x_{t-1}^[k], u_t)
Compute particle weight w^[k] = |2πQ|^{-1/2} exp {-1/2(z_t - 2^[k])^TQ⁻¹(z_t - 2^[k])} measurement covariance
Update belief of observed landmarks (EKF update rule)
Resample 26

FastSLAM 1.0 – Part 1

1: 1	FastSLAM1.0_known_correspon	$\mathbf{ndence}(z_t, c_t, u_t, \mathcal{X}_{t-1})$:
2: 3:	for $k = 1$ to N do Let $\left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle \right.$	$//$ loop over all particles $,\ldots angle$ be particle k in \mathcal{X}_{t-1}
4:	$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$	// sample pose
5: 6:	$j = c_t$ if feature <i>j</i> never seen befor	// observed feature
7:	$\mu_{j,t}^{[k]} = h^{-1}(z_t, x_t^{[k]})$	// initialize mean
8: 9:	$egin{aligned} H &= h'(\mu_{j,t}^{[k]}, x_t^{[k]}) \ \Sigma_{j,t}^{[k]} &= H^{-1} \ Q_t \ (H^{-1})^T \ w^{[k]} &= p_0 \end{aligned}$	// calculate Jacobian // initialize covariance
10: 11:	$w^{[k]} = p_0$ else	// default importance weight

FastSLAM 1.0 – Part 2

11: else $\langle \mu_{j,t}^{[k]}, \Sigma_{j,t}^{[k]} \rangle = EKF\text{-}Update(\dots) // update \ landmark \\ w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$ 12:13:measurement cov. $Q = H \Sigma_{j,t-1}^{[k]} H^T + Q_t$ exp. observation 14: endiffor all unobserved features j' do $\langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} \rangle$ // leave unchanged endfor 15:16:17:endfor 18: $\mathcal{X}_{t} = \text{resample}\left(\left\langle x_{t}^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)$ 19:20:return \mathcal{X}_t

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FastSLAM in Action



Courtesy: Mike Montemerlo

11:	else	
12:	$ \begin{split} \hat{z}^{[k]} &= h(\mu_{j,t-1}^{[k]}, x_t^{[k]}) \\ H &= h'(\mu_{j,t-1}^{[k]}, x_t^{[k]}) \\ Q &= H \Sigma_{j,t-1}^{[k]} H^T + Q_t \\ K &= \Sigma_{j,t-1}^{[k]} H^T Q^{-1} \\ \mu_{j,t}^{[k]} &= \mu_{j,t-1}^{[k]} + K(z_t - \hat{z}^{[k]}) \end{split} $	// measurement prediction
13:	$H = h'(\mu_{i,t-1}^{[k]}, x_t^{[k]})$	// calculate Jacobian
14:	$Q = H \sum_{i,t-1}^{[k]} H^T + Q_t$	// measurement covariance
15:	$K = \sum_{i,t=1}^{[k]} H^T Q^{-1}$	// calculate Kalman gain
16:	$\mu_{j,t}^{[k]} = \mu_{j,t-1}^{[k]} + K(z_t - \hat{z}^{[k]})$	// update mean
17:	$\Sigma_{i,i}^{[\kappa]} = (I - K H) \Sigma_{i,i-1}^{[\kappa]}$	// update covariance
18:	$w^{[k]} = 2\pi Q ^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z})\right\}$	$(k^{[k]})^T$
		} // importance factor
19:	endif	,
20:	for all unobserved features j' do	
21:	$\langle \mu_{i',t}^{[k]}, \Sigma_{i',t}^{[k]} \rangle = \langle \mu_{i',t-1}^{[k]}, \Sigma_{i',t-1}^{[k]} \rangle$	// leave unchanged
23:	endfor	
24:	endfor	
25:	$\mathcal{X}_t = ext{resample} \left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} ight angle, \ldots ight.$	$\left(\left. , w^{[k]} \right\rangle_{k=1,\ldots,N} \right)$
26:	return \mathcal{X}_t	········/

The Importance Weight

- Derivation of the importance weight
- Based on the importance sampling principle

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$



$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

=
$$\frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_{t}^{[k]} \mid x_{t-1}, u_{t}) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

Bayes rule + factorization

$$\begin{split} {}^{[k]} &= \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})} \\ &= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) \ p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})} \\ &= \frac{\eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) \ p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)} \\ &\frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})} \end{split}$$

The Importance Weight

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) \ p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) \ p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$\frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

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The Importance Weight

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} = \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) \\ = \eta \ \int_{l_j} p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, l_j) \ p(l_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dl_j$$

The Importance Weight

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$= \frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$
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The Importance Weight

 Integrating over the pose of the observed landmark leads to

$$\begin{split} w^{[k]} &= \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) \\ &= \eta \ \int_{l_j} p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, l_j) \ p(l_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dl_j \\ &= \eta \ \int_{l_j} p(z_t \mid x_t^{[k]}, l_j) \ p(l_j \mid x_{1:t-1}^{[k]}, z_{1:t-1}) \ dl_j \end{split}$$

The Importance Weight

 Integrating over the pose of the observed landmark leads to

$$\begin{split} w^{[k]} &= \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) \\ &= \eta \ \int_{l_j} p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, l_j) \ p(l_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dl_j \\ &= \eta \ \int_{l_j} \underbrace{p(z_t \mid x_t^{[k]}, l_j)}_{\mathcal{N}(z_t; \hat{z}^{[k]}, Q_t)} \underbrace{p(l_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(l_j; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} \ dl_j \end{split}$$

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The Importance Weight

This leads to

$$w^{[k]} = \eta \int_{l_{j}} \underbrace{p(l_{j} \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(l_{j}; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} \underbrace{p(z_{t} \mid x_{t}^{[k]}, l_{j})}_{\mathcal{N}(z_{t}; \hat{z}^{[k]}, Q_{t})} dl_{j}$$

$$Q = H \sum_{j,t-1}^{[k]} H^{T} + Q_{t}$$

$$w^{[k]} \simeq |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_{t} - \hat{z}^{[k]})^{T}Q^{-1}(z_{t} - \hat{z}^{[k]})\right\}$$

$$43$$

The Importance Weight

This leads to



FastSLAM 1.0 - Part 2

11: 12:	else $\langle \mu_{j,t}^{[k]}, \Sigma_{j,t}^{[k]} \rangle = EKF$ -Update() // update landmark
13:	$w^{[k]} = 2\pi Q ^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$
14:	endif
15:	for all unobserved features j' do
16:	$\langle \mu_{i',t}^{[k]}, \Sigma_{i',t}^{[k]} \rangle = \langle \mu_{i',t-1}^{[k]}, \Sigma_{i',t-1}^{[k]} \rangle$ // leave unchanged
17:	endfor
18:	endfor
19:	$\mathcal{X}_t = ext{resample}\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N} \right)$
20:	return \mathcal{X}_t

Data Association Problem

Which observation belongs to which landmark?



- More than one possible association
- Potential data associations depend on the pose of the robot

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Per-Particle Data Association



Was the observation generated by the **red** or by the **brown** landmark?

P(observation|red) = 0.3 P(observation|brown) = 0.7

Particles Support for Multi-Hypotheses Data Association

 Decisions on a perparticle basis

 Robot pose error is factored out of data association decisions

∽

Per-Particle Data Association



- Two options for per-particle data association
 - Pick the most probable match
 - Pick an random association weighted by the observation likelihoods
- If the probability for an assignment is too low, generate a new landmark

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Per-Particle Data Association



Was the observation generated by the **red** or by the **brown** landmark?

- Multi-modal belief
- Pose error is factored out of data association decisions
- Simple but effective data association
- Big advantage of FastSLAM over EKF

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Results – Victoria Park

- 4 km traverse
- < 2.5 m RMS position error
- 100 particles



Blue = GPS Yellow = FastSLAM

Courtesy: Mike Montemerlo 50

Results – Victoria Park (Video)



Courtesy: Mike Montemerlo

Results (Sample Size)





FastSLAM Complexity – Simple Implementation

- Update robot particles $\mathcal{O}(N)$ based on the control
- Incorporate an observation $\mathcal{O}(N)$ into the Kalman filters
- Resample particle set
 - N = Number of particles M = Number of map features

A Better Data Structure for FastSLAM



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 $\mathcal{O}(NM)$

 $\mathcal{O}(NM)$



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Memory Complexity



FastSLAM 1.0

 FastSLAM 1.0 uses the motion model as the proposal distribution

 $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$

 Is there a better distribution to sample from?

FastSLAM 1.0 to FastSLAM 2.0

 FastSLAM 1.0 uses the motion model as the proposal distribution

 $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$

- FastSLAM 2.0 considers also the measurements during sampling
- Especially useful if an accurate sensor is used (compared to the motion noise)

[Montemerlo et al., 2003] 61

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FastSLAM 2.0 (Informally)

FastSLAM 2.0 samples from

 $x_t^{[k]} \sim p(x_t \mid x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t})$

- Results in a more peaked proposal distribution
- Less particles are required
- More robust and accurate
- But more complex...

[Montemerlo et al., 2003] 62

FastSLAM Problems

- How to determine the sample size?
- Particle deprivation, especially when closing (multiple) loops



FastSLAM Summary

- Particle filter-based SLAM
- Rao-Blackwellization: model the robot's path by sampling and compute the landmarks given the poses
- Allow for per-particle data association
- FastSLAM 1.0 and 2.0 differ in the proposal distribution
- Complexity $\mathcal{O}(N \log M)$

FastSLAM Results

- Scales well (1 million+ features)
- Robust to ambiguities in the data association
- Advantages compared to the classical EKF approach (especially with nonlinearities)

Literature

FastSLAM

- Thrun et al.: "Probabilistic Robotics", Chapter 13.1-13.3 + 13.8 (see errata!)
- Montemerlo, Thrun, Kollar, Wegbreit: FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem, 2002
- Montemerlo and Thrun: Simultaneous Localization and Mapping with Unknown Data Association Using FastSLAM, 2003