Robot Mapping

Short Introduction to Particle Filters and Monte Carlo Localization

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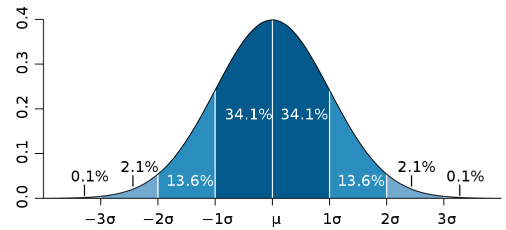
Gaussian Filters

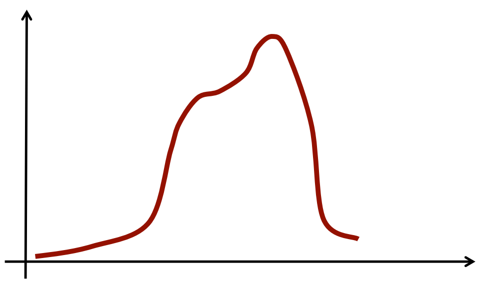
- The Kalman filter and its variants can only model **Gaussian distributions**

$$p(x) = \det(2\pi\Sigma)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$



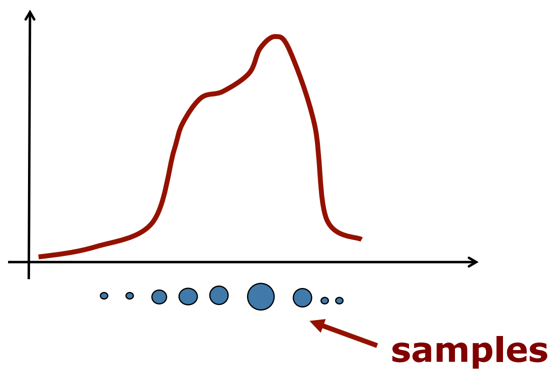
Motivation

- Goal: approach for dealing with **arbitrary distributions**



Key Idea: Samples

- Use **multiple samples** to represent arbitrary distributions



Particle Set

- Set of weighted samples

$$\mathcal{X} = \left\{ \left\langle x^{[i]}, w^{[i]} \right\rangle \right\}_{i=1,\dots,N}$$

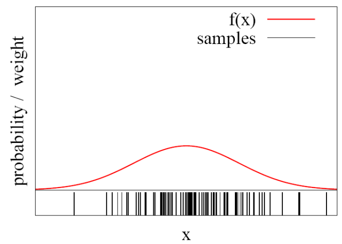
state hypothesis **importance weight**

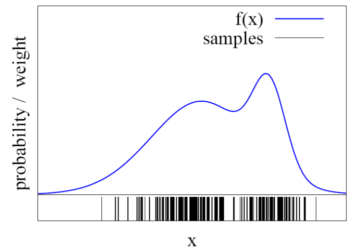
- The samples represent the posterior

$$p(x) = \sum_{i=1}^{N} w^{[i]} \, \delta_{x^{[i]}}(x)$$

Particles for Approximation

- Particles for function approximation



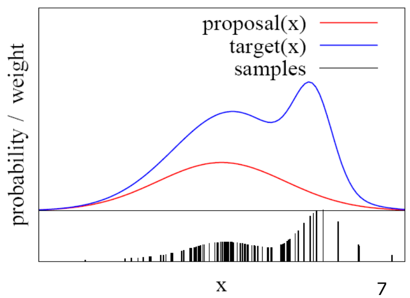


- The more particles fall into an interval, the higher its probability density

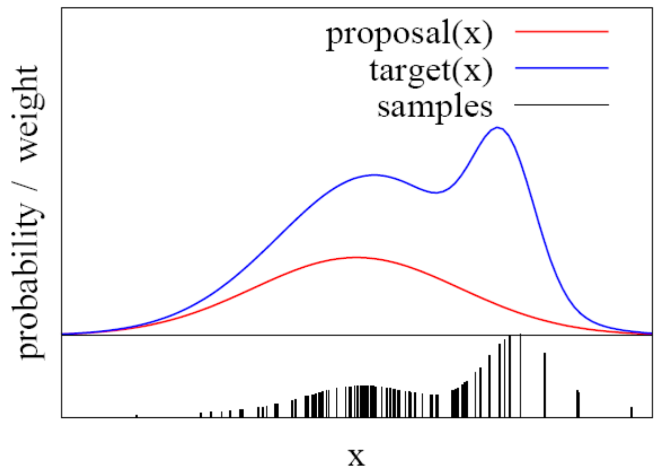
How to obtain such samples?

Importance Sampling Principle

- We can use a different distribution g to generate samples from f
- Account for the "differences between g and f" using a weight $w = f/g$
- target f
- proposal g
- Pre-condition: $f(x)>0 \rightarrow g(x)>0$



Importance Sampling Principle



Particle Filter

- Recursive Bayes filter
- Non-parametric approach
- Models the distribution by samples
- Prediction: draw from the proposal
- Correction: weighting by the ratio of target and proposal

The more samples we use, the better is the estimate!

Particle Filter Algorithm

1. Sample the particles using the proposal distribution

$$x_t^{[i]} \sim \pi(x_t \mid \ldots)$$

2. Compute the importance weights

$$w_t^{[i]} = \frac{target(x_t^{[i]})}{proposal(x_t^{[i]})}$$

3. Resampling: "Replace unlikely samples by more likely ones"

Particle Filter Algorithm

Particle_filter$(\mathcal{X}_{t-1}, u_t, z_t)$**:**

$\quad \bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

$\quad$ *for* $m = 1$ *to* M *do*

$\quad\quad$ *sample* $x_t^{[m]} \sim \pi(x_t)$

$\quad\quad$ $w_t^{[m]} = \frac{p(x_t^{[m]})}{\pi(x_t^{[m]})}$

$\quad\quad$ $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

$\quad$ *endfor*

$\quad$ *for* $m = 1$ *to* M *do*

$\quad\quad$ *draw* i *with probability* $\propto w_t^{[i]}$

$\quad\quad$ *add* $x_t^{[i]}$ *to* $\mathcal{X}_t$

$\quad$ *endfor*

$\quad$ *return* $\mathcal{X}_t$

Monte Carlo Localization

- Each particle is a pose hypothesis
- Proposal is the motion model

$$x_t^{[i]} \sim p(x_t \mid x_{t-1}, u_t)$$

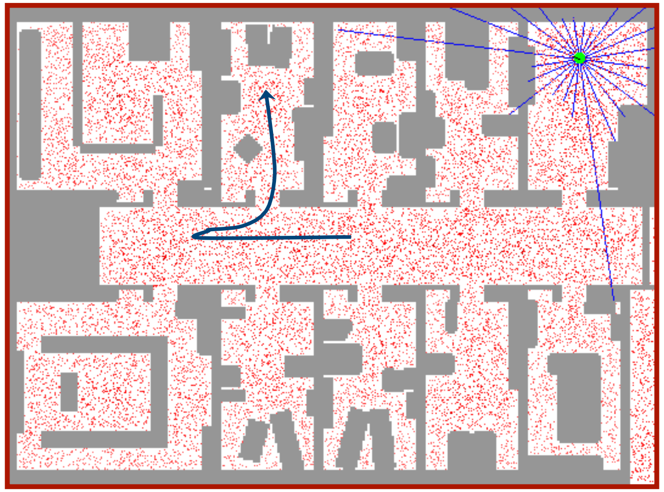
- Correction via the observation model

$$w_t^{[i]} = \frac{target}{proposal} \propto p(z_t \mid x_t, m)$$

Particle Filter for Localization

$$
\begin{array}{l}
\textbf{Particle\_filter}(\mathcal{X}_{t-1}, u_t, z_t)\textbf{:} \\
\quad \bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset \\
\quad \textit{for } m = 1 \textit{ to } M \textit{ do} \\
\qquad \textit{sample } x_t^{[m]} \sim \underline{p(x_t \mid u_t, x_{t-1}^{[m]})} \\
\qquad w_t^{[m]} = \underline{p(z_t \mid x_t^{[m]})} \\
\qquad \bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle \\
\quad \textit{endfor} \\
\quad \textit{for } m = 1 \textit{ to } M \textit{ do} \\
\qquad \textit{draw } i \textit{ with probability} \propto w_t^{[i]} \\
\qquad \textit{add } x_t^{[i]} \textit{ to } \mathcal{X}_t \\
\quad \textit{endfor} \\
\quad \textit{return } \mathcal{X}_t
\end{array}
$$

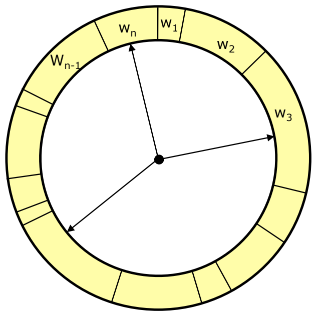
Application: Particle Filter for Localization (Known Map)



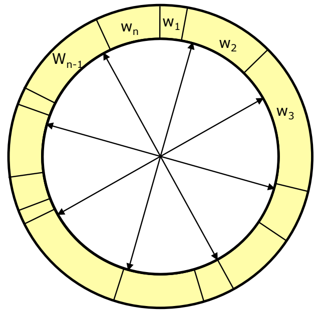
Resampling

- Survival of the fittest: "Replace unlikely samples by more likely ones"
- "Trick" to avoid that many samples cover unlikely states
- Needed as we have a limited number of samples

Resampling



- Roulette wheel
- Binary search
- O(n log n)



- Stochastic universal sampling
- Low variance
- O(n)

Low Variance Resampling

Low_variance_resampling($\mathcal{X}_t, \mathcal{W}_t$)**:**

$\bar{\mathcal{X}}_t = \emptyset$

$r = \text{rand}(0; M^{-1})$

$c = w_t^{[1]}$

$i = 1$

for $m = 1$ *to* M *do*

  $U = r + (m-1) \cdot M^{-1}$

  while $U > c$

    $i = i + 1$

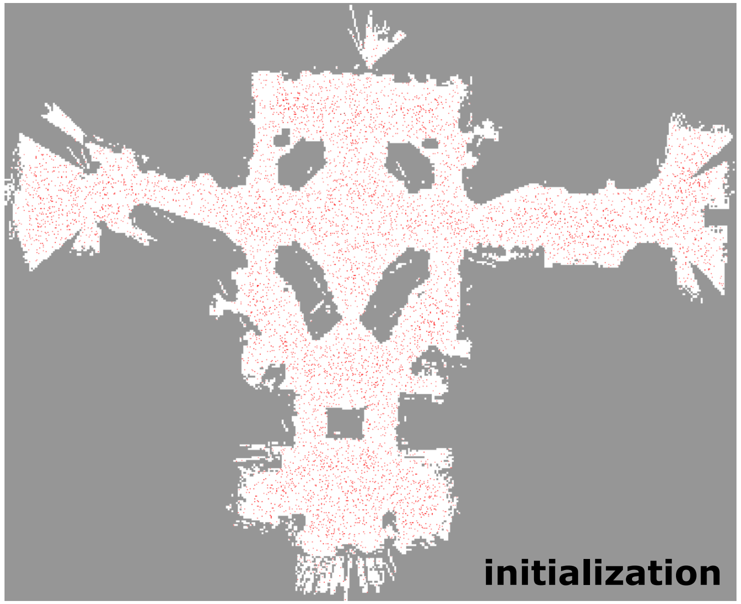
    $c = c + w_t^{[i]}$

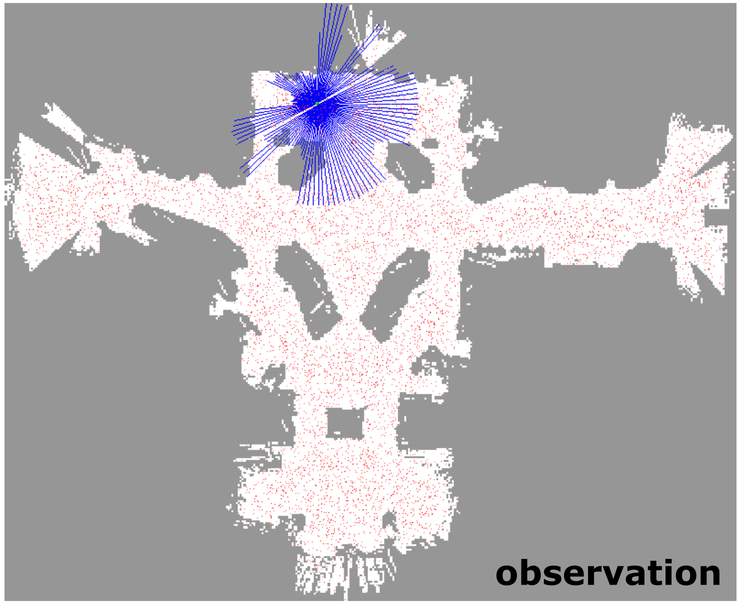
  endwhile

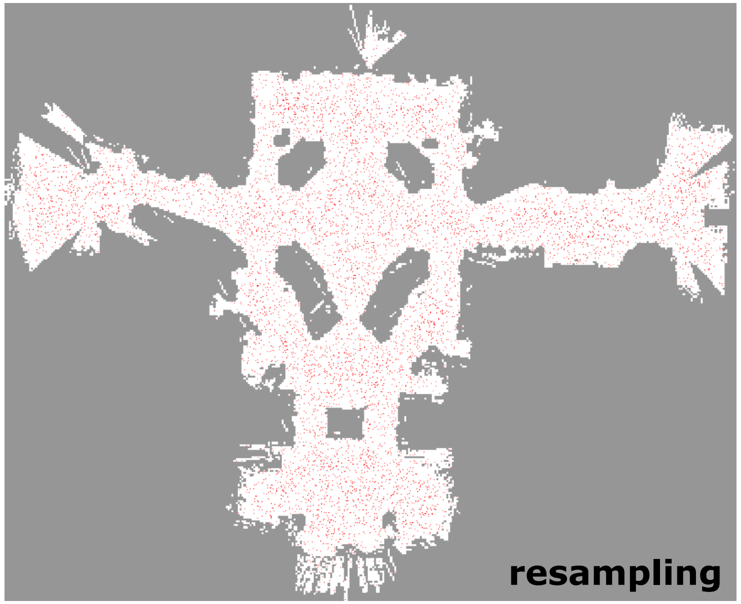
  add $x_t^{[i]}$ *to* $\bar{\mathcal{X}}_t$

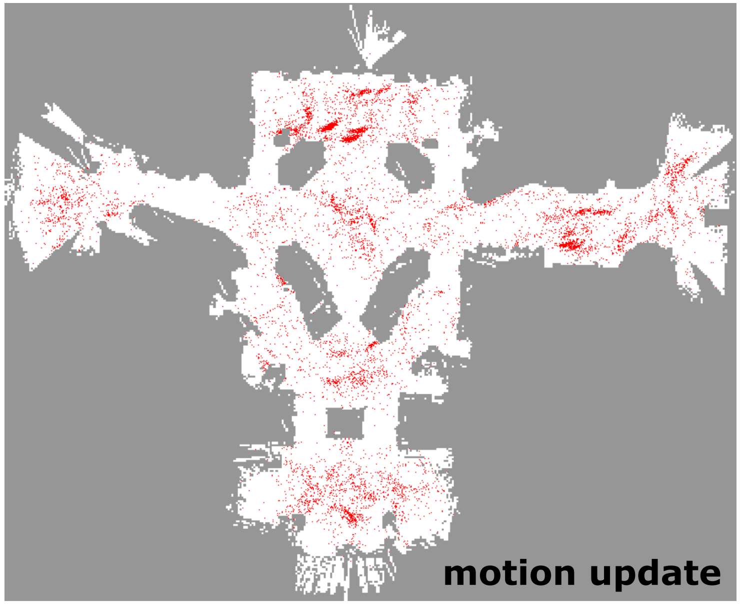
endfor

return $\bar{\mathcal{X}}_t$









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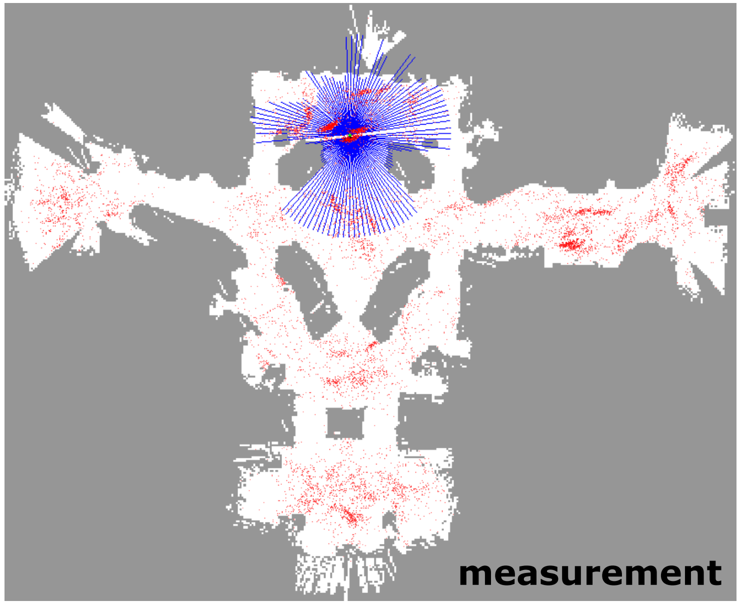

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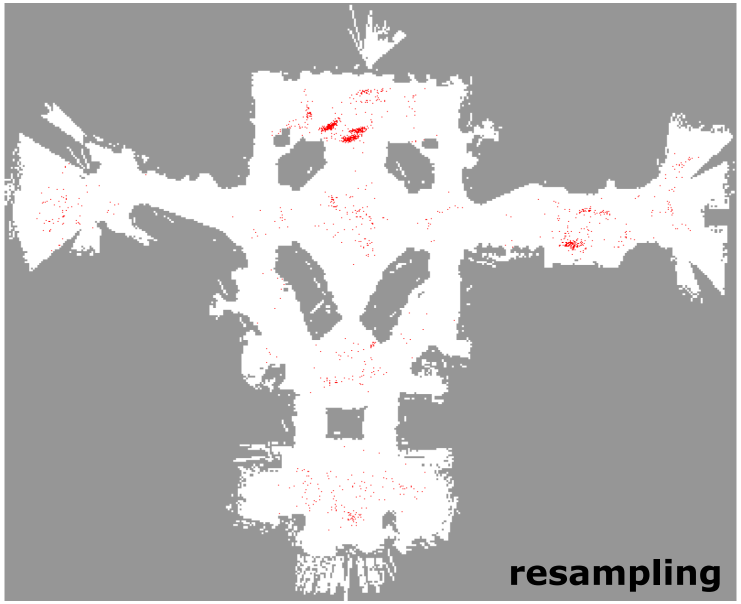

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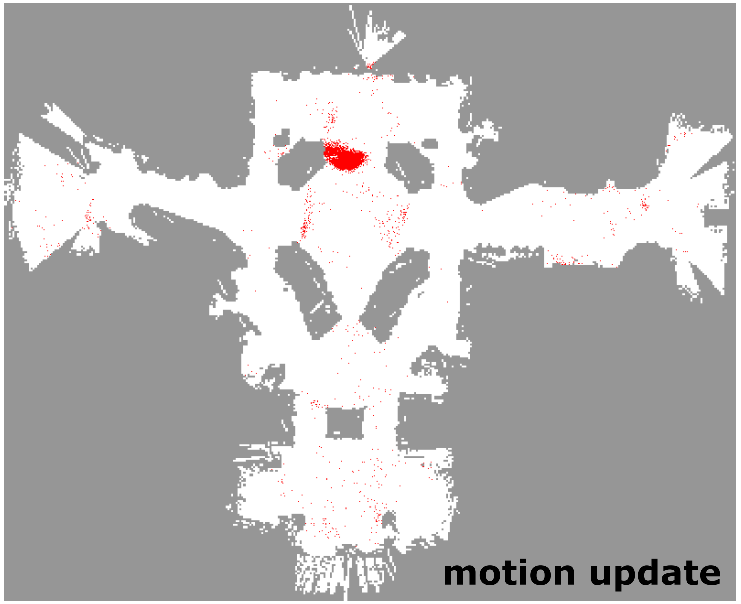
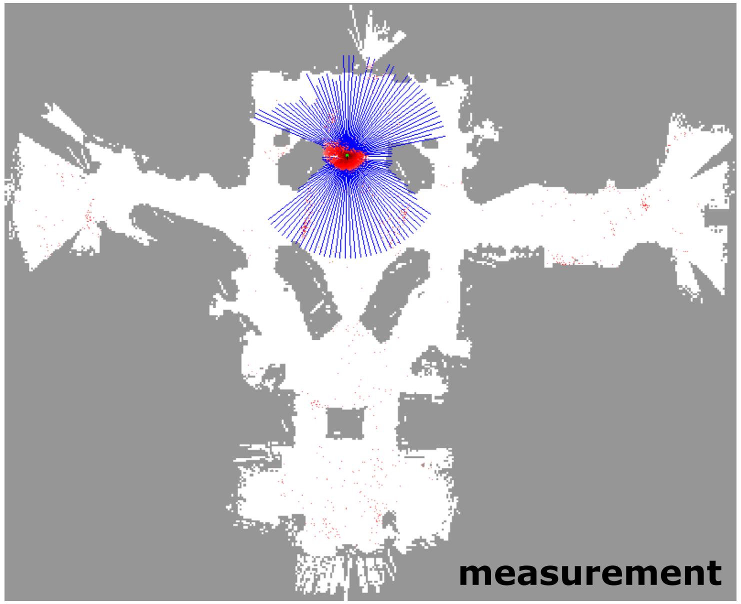
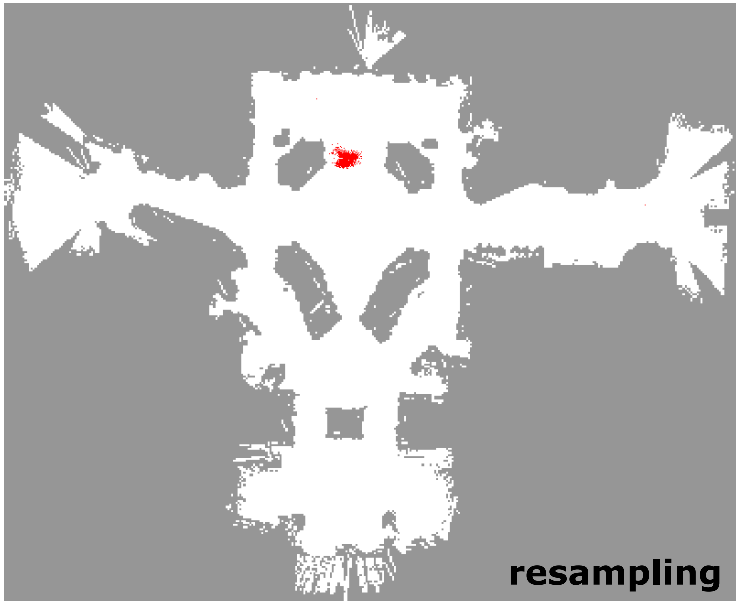
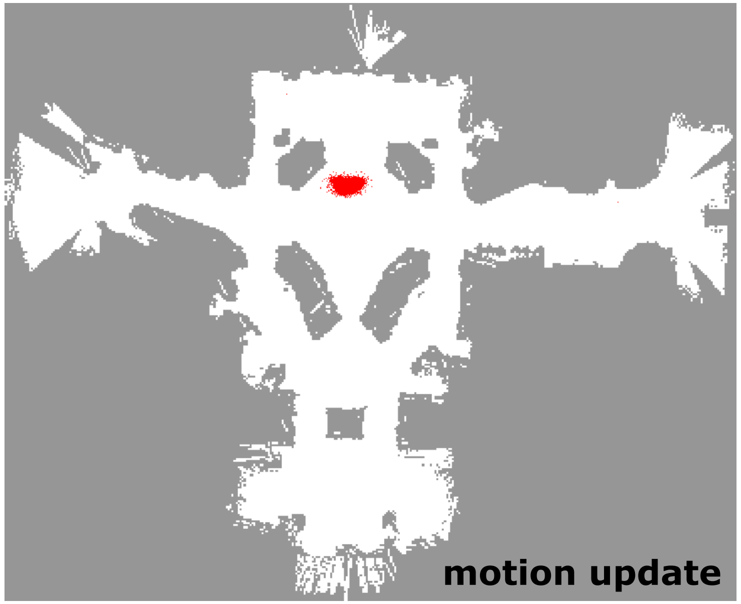

motion update


measurement


weight update


resampling



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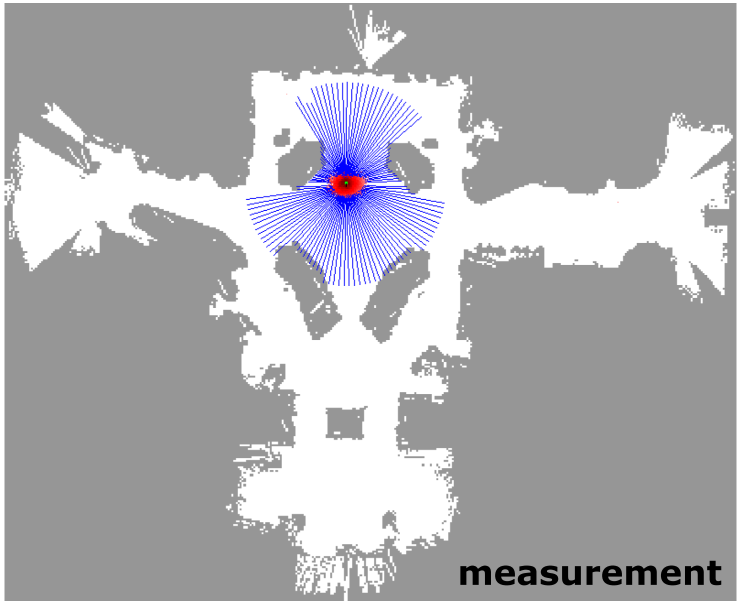

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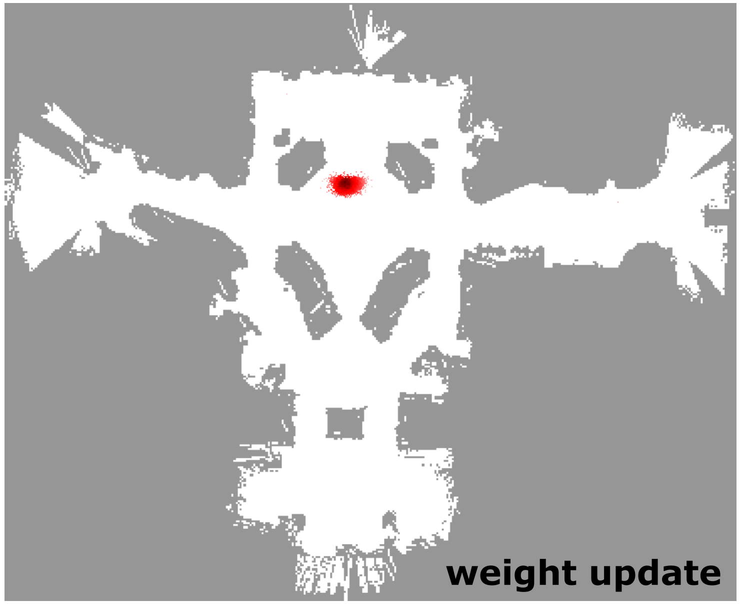

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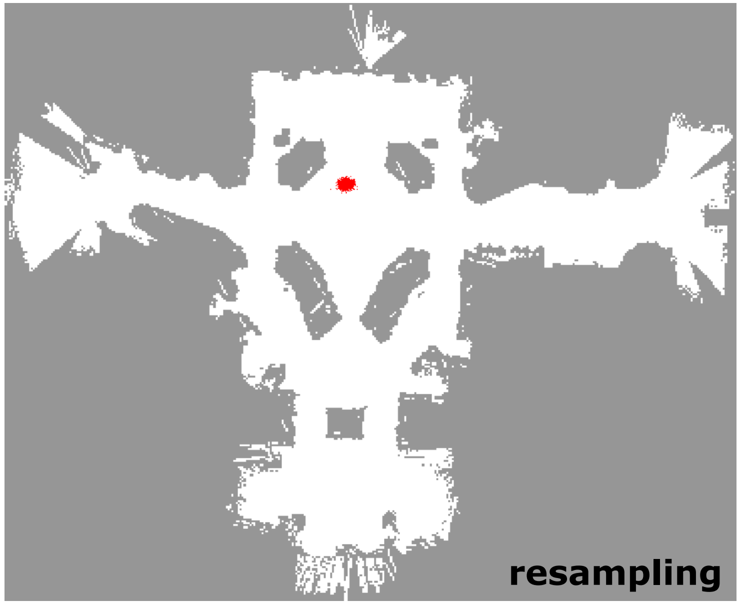

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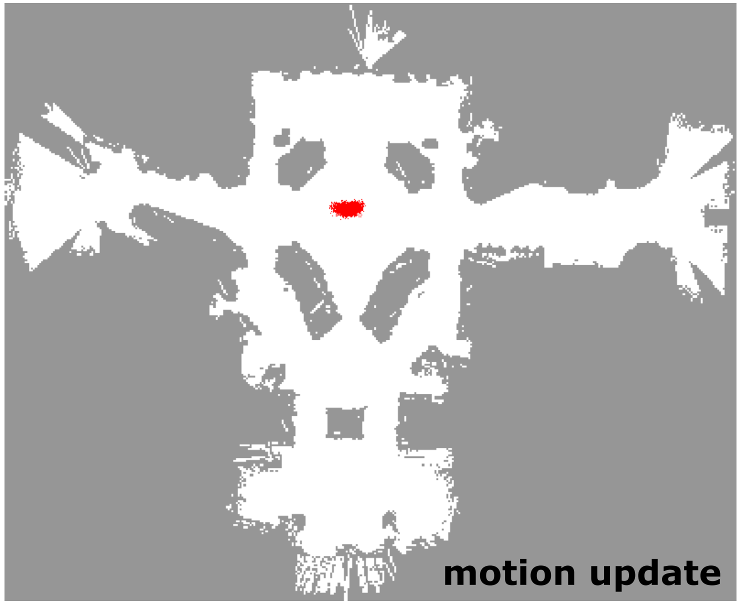

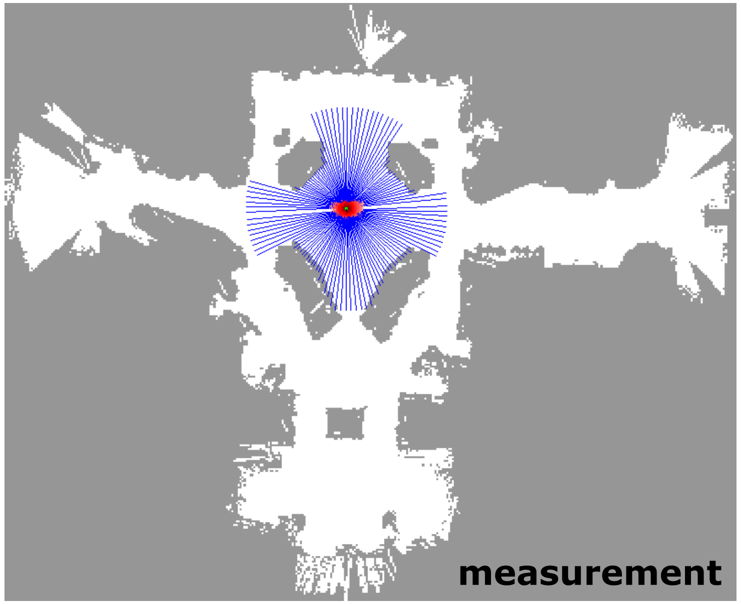

Summary – Particle Filters

- Particle filters are non-parametric, recursive Bayes filters
- Posterior is represented by a set of weighted samples
- Not limited to Gaussians
- Proposal to draw new samples
- Weight to account for the differences between the proposal and the target
- Work well in low-dimensional spaces

Summary – PF Localization

- Particles are propagated according to the motion model
- They are weighted according to the likelihood of the observation
- Called: Monte-Carlo localization (MCL)
- MCL is the gold standard for mobile robot localization today