Advanced Techniques for Mobile Robotics

Camera Calibration

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What is Camera Calibration?

- A camera projects 3D world points onto the 2D image plane
- **Calibration:** Find the quantities internal to the camera that affect this imaging process
  - Image center
  - Focal length
  - Lens distortion parameters

Why is Calibration Needed?

- Camera production errors
- Cheap lenses

Precise calibration is required for

- 3D interpretation of images
- Construction of world models
- Robot interaction with the world (hand-eye coordination)

Projective Space

- Projective space is an extension of the Euclidean space
- "Parallel lines intersect at infinity"

$$\mathbb{R}^n \to \mathbb{P}^n : (x_1, \ldots, x_n) \to (\lambda x_1, \ldots, \lambda x_n, \lambda) \in \mathbb{R}^{n+1} \setminus \mathbf{0}_{n+1}$$

- Here, equivalence is defined up to scale:

$$\hat{\mathbf{x}} \sim \hat{\mathbf{y}} \Leftrightarrow \exists \lambda \in \mathbb{R} \setminus \{0\} : \hat{\mathbf{x}} = \lambda \hat{\mathbf{y}}$$

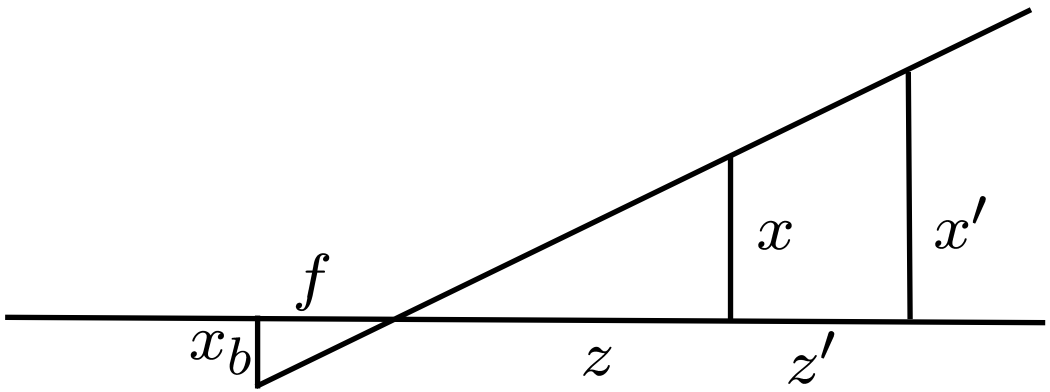
- Special case: Projective Plane $\mathbb{P}^2$
- A linear transformation within $\mathbb{P}^2$ is called a homography

Homography

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \sim \underbrace{\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}}_{\boxed{\text{Homography } \mathbf{H}}} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

- $\mathbf{H}$ has 9-1(scale invariance)=8 DoF
- A pair of points gives us 2 equations
- Therefore, we need at least 4 point correspondences for calculating a homography

Simple Pinhole Camera Model



$$\frac{x_b}{f} = \frac{x}{z} = \frac{x'}{z'}$$

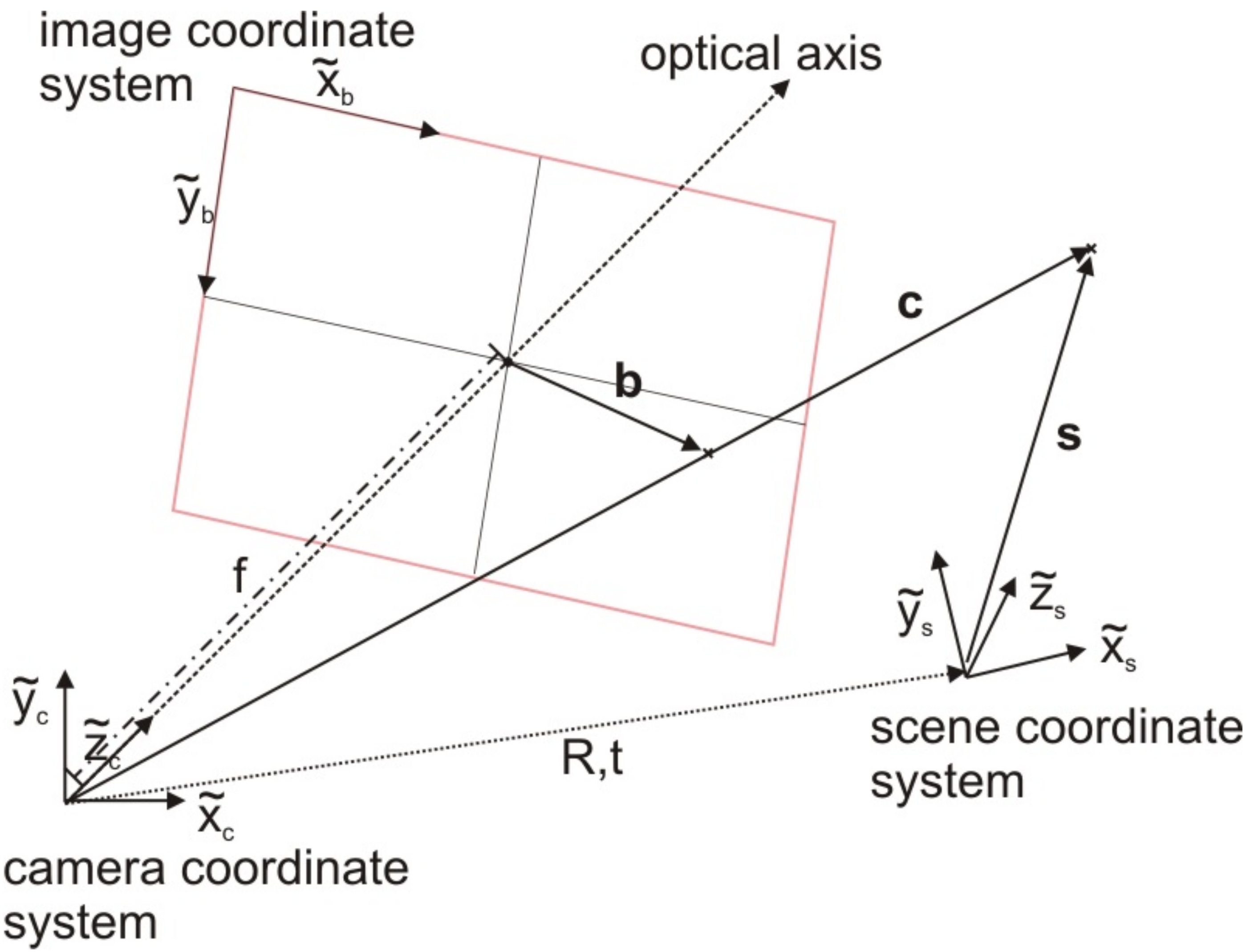
$$x_b = f\frac{x}{z} = f\frac{x'}{z'}$$

Pinhole Camera Model

- Perspective transformation using homogeneous coordinates:

$$\mathbf{b} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \underbrace{\begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Intrinsic camera parameters}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{pmatrix}}_{\text{Extrinsic camera parameters}} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Pinhole Camera Model



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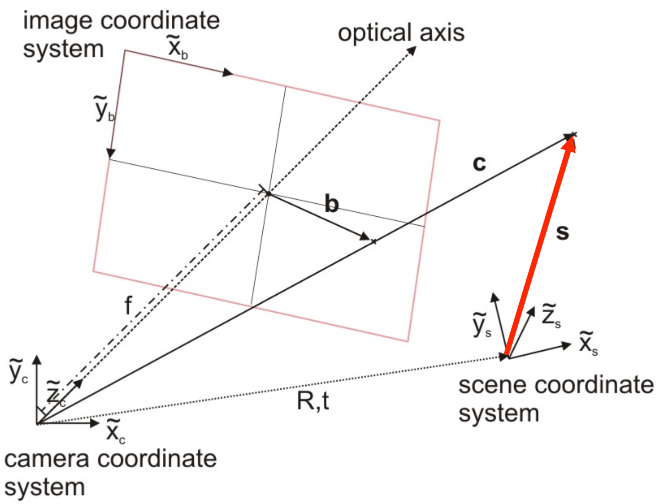
world/scene coordinate system



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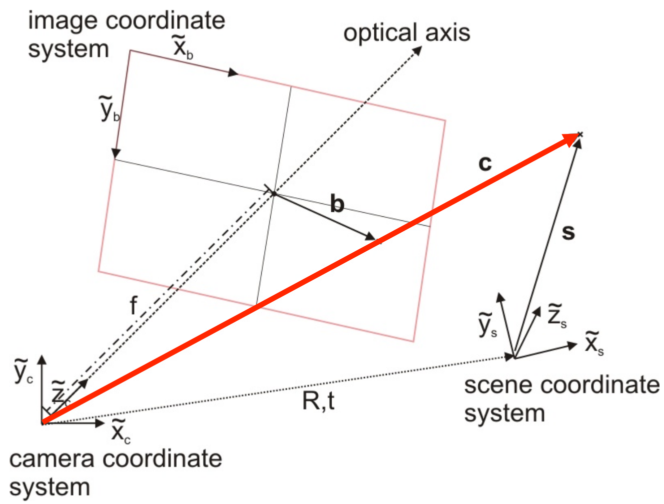
$\downarrow$

camera coordinate system



Pinhole Camera Model

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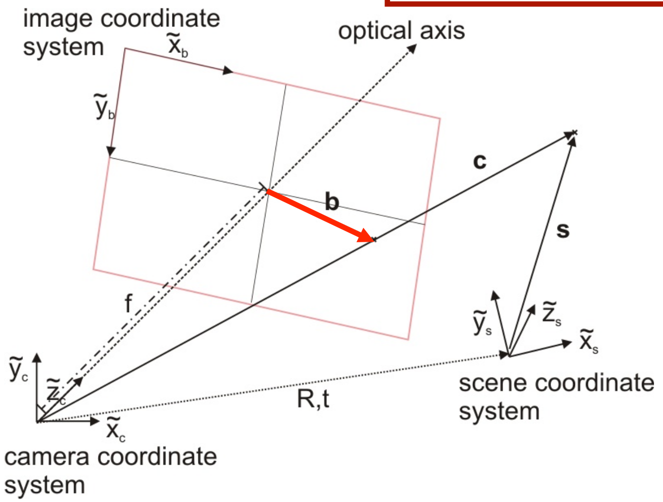
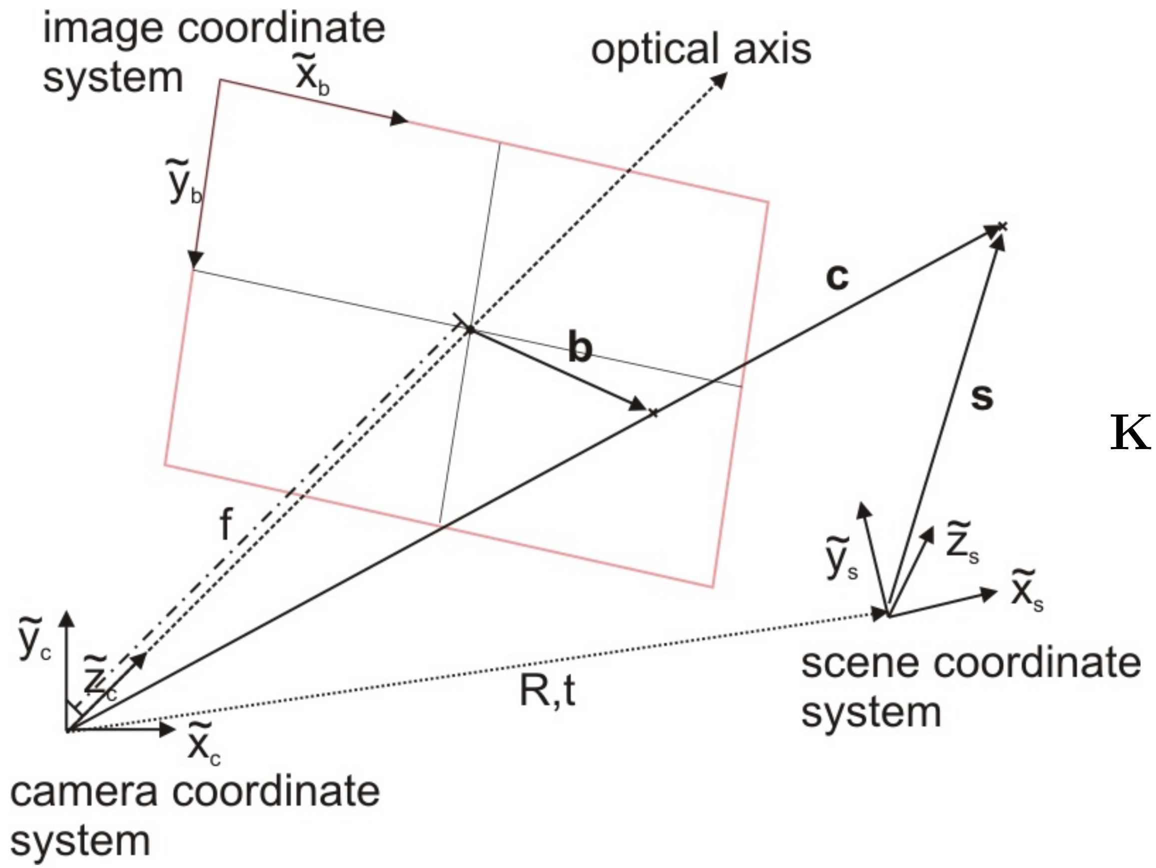
$\downarrow$

image coordinate system



Pinhole Camera Model

- Interpretation of intrinsic camera parameters:



$$\mathrm{K} = \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

Pinhole Camera Model

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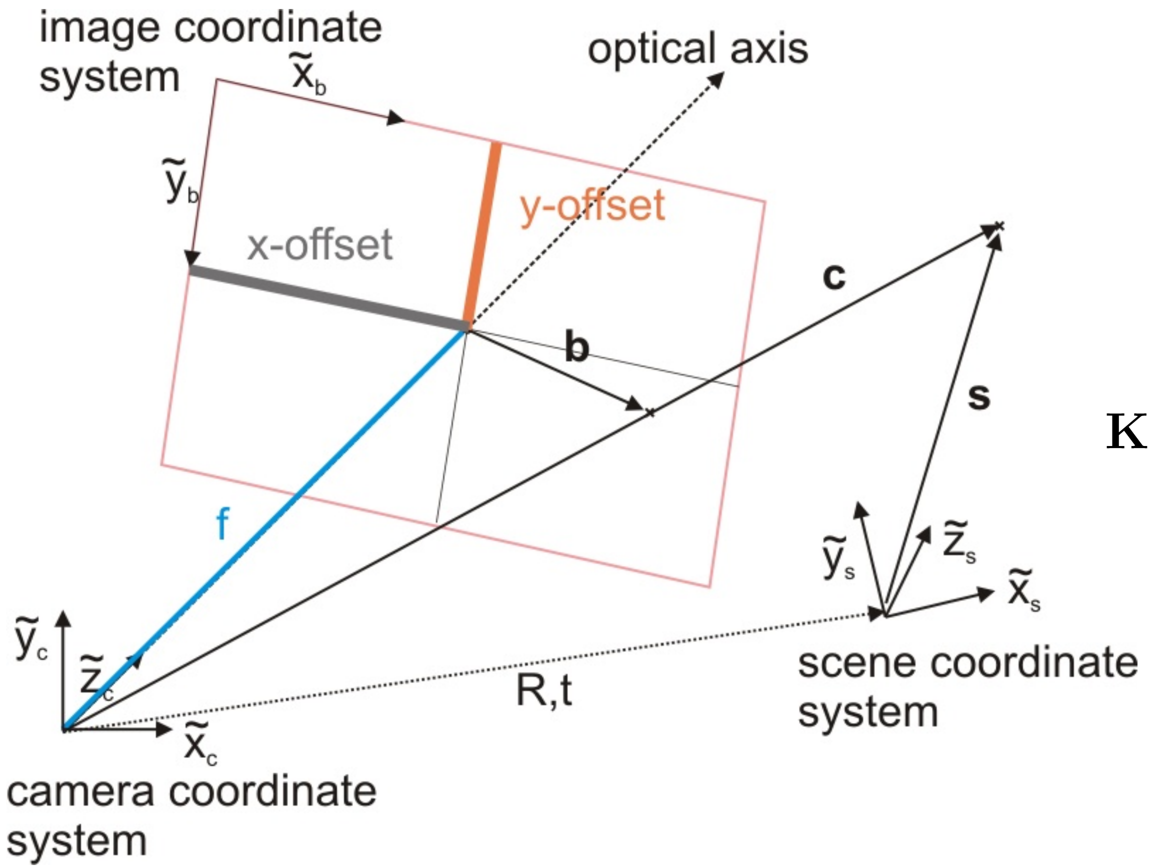

focal length **x-offset**

$$\mathrm{K} = \begin{pmatrix} \boxed{f_x} & 0 & \boxed{c_x} \\ 0 & \boxed{f_y} & \boxed{c_y} \\ 0 & 0 & 1 \end{pmatrix}$$

y-offset

Camera Calibration

- Calculate intrinsic parameters (later: and lens distortion parameters) from a series of images
  - 2D camera calibration
  - 3D camera calibration
  - Self calibration

Camera Calibration

- Calculate intrinsic parameters (later: and lens distortion parameters) from a series of images
  - **2D camera calibration**
  - 3D camera calibration
  - Self calibration

needs an external pattern

2D Camera Calibration

- Use a 2D pattern (e.g., a checkerboard)

- Size and structure of the pattern is known

Trick for 2D Camera Calibration

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- Trick: set the world coordinate system to the corner of the checkerboard

Trick for 2D Camera Calibration

- Use a 2D pattern (e.g., a checkerboard)

- Trick: set the world coordinate system to the corner of the checkerboard
- Now: All points on the checkerboard lie on one plane!

Trick for 2D Camera Calibration

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ \boxed{0} \\ 1 \end{pmatrix}$$

- Since all points lie on a plane, their z component is 0 in world coordinates

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- Since all points lie on a plane, their z component is 0 in world coordinates
- Thus, we can delete the 3rd column of the extrinsic parameter matrix

Simplified Form for 2D Camera Calibration

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

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Setting Up the Equations

$$\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \underbrace{\begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{K}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{pmatrix}}_{(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})}$$

$$\Rightarrow \quad (\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \mathbf{K}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})$$

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$$(\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \mathbf{K}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})$$

$$\Rightarrow \quad \mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1, \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2$$

Exploit Constraints

$$\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \underbrace{\begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{K}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{pmatrix}}_{(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})}$$

$$(\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \mathbf{K}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})$$

$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1, \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2$$

- Note that $(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ form an orthonormal basis, thus: $\mathbf{r}_1^T \mathbf{r}_2 = 0, \quad \|r_1\| = \|r_2\| = 1$

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$$\xrightarrow{\mathbf{r}_1^T\mathbf{r}_2 = 0} \quad \mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2 = 0$$

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$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1, \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2$$

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

$$\xrightarrow{\|\mathbf{r}_1\| = \|\mathbf{r}_2\| = 1} \quad \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2$$

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$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2$$

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 - \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

Use Both Equations

$$\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \underbrace{\begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{K}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{pmatrix}}_{(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})}$$

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Equations from Constraints

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0 \qquad (1)$$

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 - \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0 \qquad (2)$$

- $\mathrm{B} := \mathrm{K}^{-T} \mathrm{K}^{-1}$ is symmetric and positive definite but only defined up to a scale factor

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- $\mathrm{B} := \mathbf{K}^{-T}\mathbf{K}^{-1}$ is symmetric and positive definite but only defined up to a scale factor

- Thus: $\mathrm{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}$

Note: K can be calculated from B using Cholesky factorization (chol$(B) = AA^T$ **i.e.** $A = \mathrm{K}^{-T}$**)**

Build System of Equations

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- Define: $\mathrm{b} = (b_{11}, b_{12}, b_{13}, b_{22}, b_{23}, b_{33}) \qquad (3)$

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- Define: $\mathrm{b} = (b_{11}, b_{12}, b_{13}, b_{22}, b_{23}, b_{33}) \qquad (3)$
- (1)-(3) can be used to construct a system of equations in form of $\mathrm{Vb} = 0$

The Matrix V

- Setting up the matrix $\mathbf{V}$

$$\mathbf{V} = \begin{pmatrix} \mathbf{v}_{12}^T \\ \mathbf{v}_{11}^T - \mathbf{v}_{22}^T \end{pmatrix}$$

with

$$\begin{aligned} \mathbf{v}_{ij} \;=\; & (h_{i1}h_{j1}, h_{i1}h_{j2} + h_{i2}h_{j1}, h_{i2}h_{j2}, \\ & h_{i3}h_{j1} + h_{i1}h_{j3}, h_{i3}h_{j2} + h_{i2}h_{j3}, h_{i3}h_{j3})^T \end{aligned}$$

- For one image, we obtain $\begin{pmatrix} \mathbf{v}_{12}^T \\ \mathbf{v}_{11}^T - \mathbf{v}_{22}^T \end{pmatrix} \mathbf{b} = \mathbf{0}$

- For multiple, we stack the matrices to one 2n x 6 matrix

$$\begin{array}{r} \text{image 1} \rightarrow \\ \\ \\ \text{image n} \rightarrow \\ \\ \end{array} \begin{pmatrix} \mathbf{v}_{12}^T \\ \mathbf{v}_{11}^T - \mathbf{v}_{22}^T \\ \cdots \\ \mathbf{v}_{12}^T \\ \mathbf{v}_{11}^T - \mathbf{v}_{22}^T \end{pmatrix} \mathbf{b} = \mathbf{0}$$

Direct Linear Transformation

- Each plane gives us two equations
- Since $\mathbf{B}$ has 6 degrees of freedom, we need at least 3 different views of a plane

- We need at least 4 points per plane
- Solve $\mathbf{V}\mathbf{b} = 0$

Solving the Linear System

- The system $\mathbf{V}\mathbf{b} = 0$ has a trivial solution
- This system has the trivial solution $\mathbf{b}^* = 0$ which will not lead to a valid matrix B
- Solution: impose constraint $\|\mathbf{b}\| = 1$

In Reality…

- Real measurements are corrupted with noise

➡ Find the maximum likelihood solution that minimizes the least-squares error

$$\mathbf{b}^* = \underset{\mathbf{b}}{\text{argmin}} \, \|\mathbf{V}\mathbf{b}\| \quad \text{with} \; \|\mathbf{b}\| = 1$$

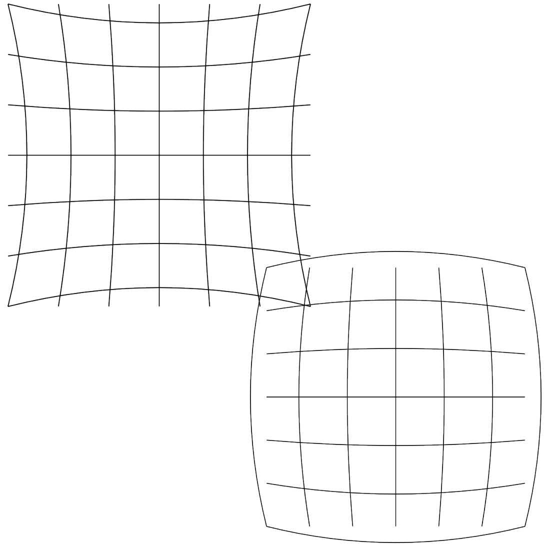
In Reality…

- Real measurements are corrupted with noise
- Find the maximum likelihood solution that minimizes the least-squares error

$$\mathbf{b}^* = \underset{\mathbf{b}}{\text{argmin}} \, \|\mathbf{V}\mathbf{b}\| \quad \text{with } \|\mathbf{b}\| = 1$$

- Minimize $L(\mathbf{b}, \lambda) = \|\mathbf{V}\mathbf{b}\|^2 + \lambda(1 - \|\mathbf{b}\|^2)$
- Compute derivative, set to zero, …
- Results in $(\mathbf{V}^T\mathbf{V})\mathbf{b} = \lambda\mathbf{b}$
- Eigenvalue/eigenvector problem

Lens Distortion Model



Non-linear effects:
- Radial distortion
- Tangential distortion

- Compute the corrected image point:

(1) $$x' = x/z \qquad y' = y/z$$

(2) $$x'' = x'(1 + k_1 r^2 + k_2 r^4) + 2p_1 x' y' + p_2(r^2 + 2x'^2)$$
$$y'' = y'(1 + k_1 r^2 + k_2 r^4) + p_1(r^2 + 2y'^2) + 2p_2 x' y'$$

where $r^2 = x'^2 + y'^2$ k_1, k_2 : radial distortion coefficients

p_1, p_2 : tangential distortion coefficients

(3) $$u = f_x \cdot x'' + c_x \qquad v = f_y \cdot y'' + c_y$$

Error Minimization

- Lens distortion can be calculated by minimizing a non-linear error function

$$\min_{(\mathbf{K},\kappa,\mathbf{R}_i,\mathbf{t}_i)} \sum_i \sum_j \|\mathbf{x}_{ij} - \widehat{x}(\mathbf{K},\kappa,\mathbf{R}_i,\mathbf{t}_i;\mathbf{X}_{ij})\|^2$$

...linearize to obtain a quadratic function, compute derivative, set it to 0, solve linear system, iterate...

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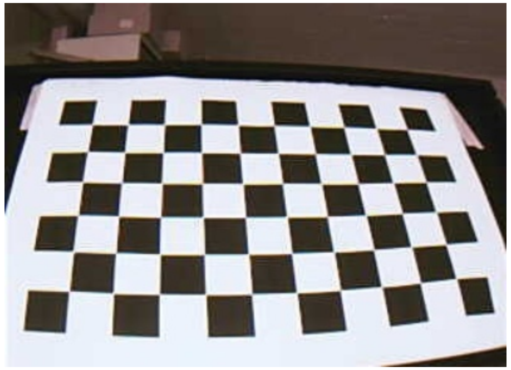
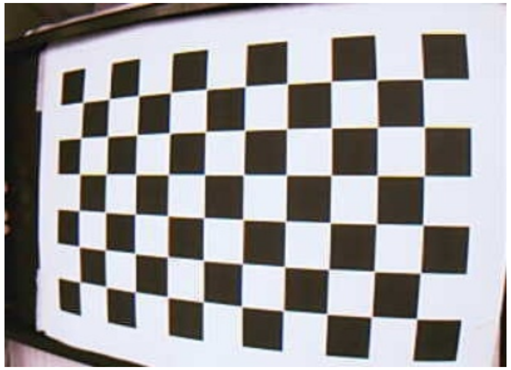
- In practice: estimation of the lens distortion parameters κ using techniques such as Levenberg-Marquardt
- The parameters obtained by the linear function are used as starting values

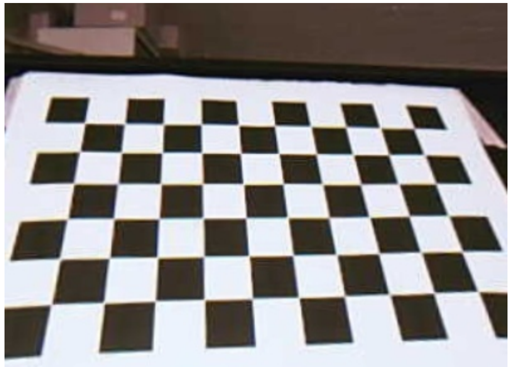
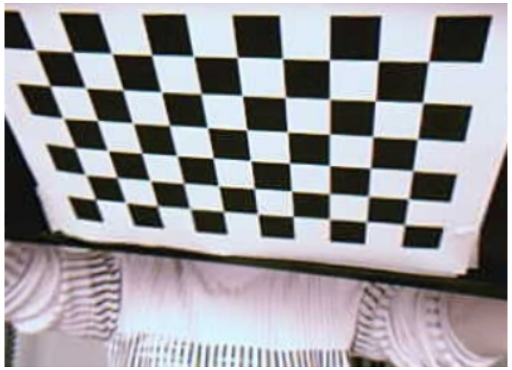
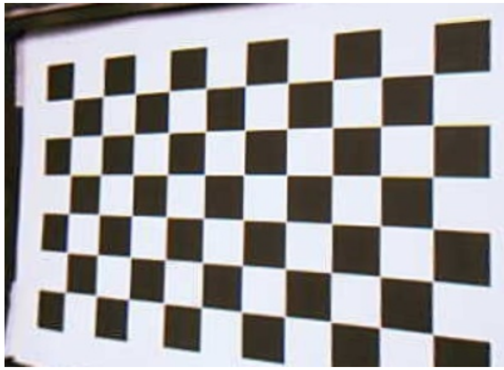
Results: Webcam

- Before calibration:





- After calibration:

Summary

- Pinhole camera model
- Non-linear model for lens distortion
- Approach to 2D camera calibration that
  - accurately determines the model parameters
  - is easy to realize