Introduction to Mobile Robotics

Iterative Closest Point Algorithm (ICP)

UNI FREIBURG

Mapping With Raw Odometry

Courtesy: Dirk Hähnel

Slides by Cyrill Stachniss

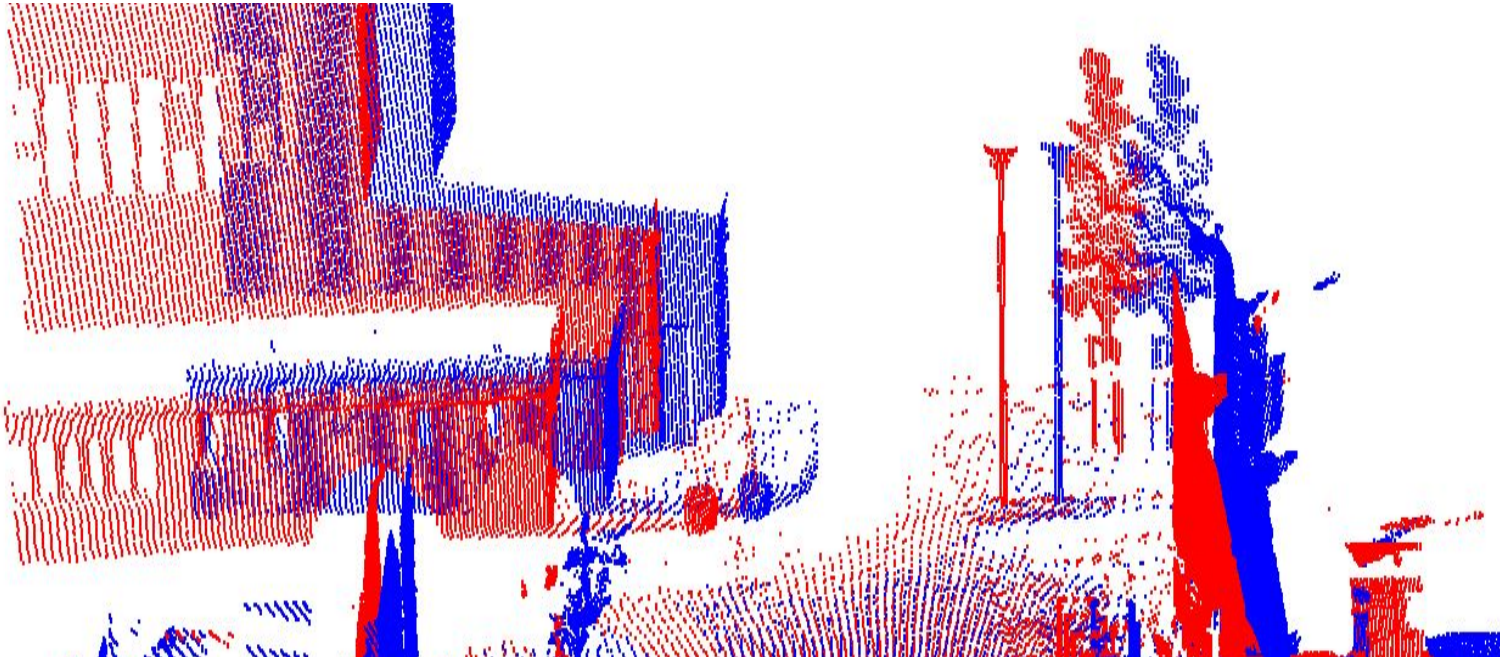
Motivation

- Motion is noisy
- Assuming known poses fails!
- Often, the sensor is rather precise

- Scan-matching tries to incrementally align two scans or a map to a scan, without revising the past/map

Slides by Cyrill Stachniss

Example: Aligning Two 3D Maps



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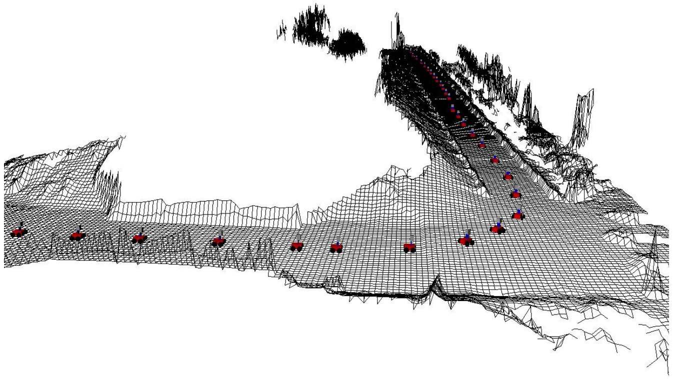
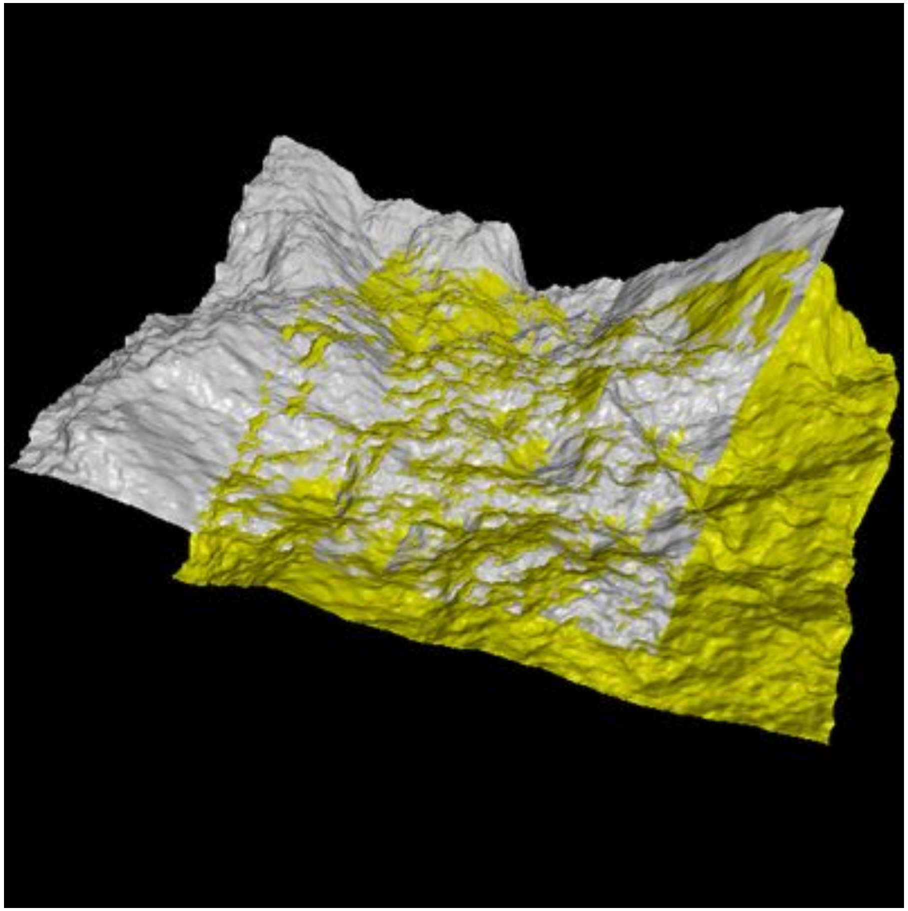
Motivation





Goal: Find local transformation to align points

The Problem

- Given two corresponding point sets:

$$X = \{x_1, ..., x_{N_x}\}$$

$$P = \{p_1, ..., p_{N_p}\}$$

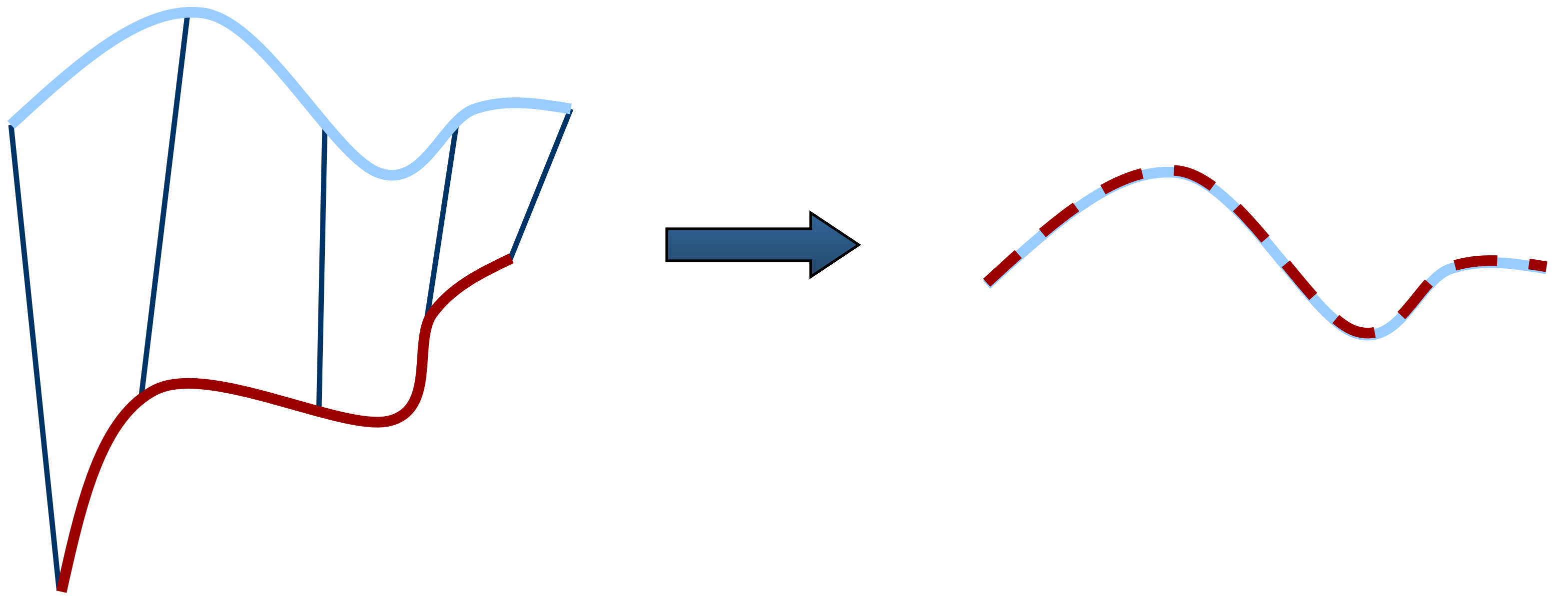
- Wanted: Translation *t* and rotation *R* that minimize the sum of the squared errors:

$$E(R,t) = \frac{1}{N_p} \sum_{i=1}^{N_p} ||x_i - Rp_i - t||^2$$

Here, x_i and p_i are corresponding points

Key Idea

- If the correct correspondences are known, the correct relative rotation/translation can be calculated in **closed form**



Center of Mass

$$\mu_x = \frac{1}{N_x} \sum_{i=1}^{N_x} x_i \qquad \text{and} \qquad \mu_p = \frac{1}{N_p} \sum_{i=1}^{N_p} p_i$$

are the centers of mass of the two point sets

Idea:

- Subtract the corresponding center of mass from every point in the two point sets before calculating the transformation
- The resulting point sets are:

$$X' = \{x_i - \mu_x\} = \{x'_i\} \qquad \text{and}$$
$$P' = \{p_i - \mu_p\} = \{p'_i\}$$

Singular Value Decomposition

Let $W = \sum_{i=1}^{N_p} x'_i p'^T_i$

denote the singular value decomposition (SVD) of W by:

$$W = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V^T$$

where $U, V \in \mathbb{R}^{3\times 3}$ are unitary, and

$\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the singular values of W

SVD

Theorem (without proof):

If rank(W) = 3, the optimal solution of E(R,t) is unique and is given by:

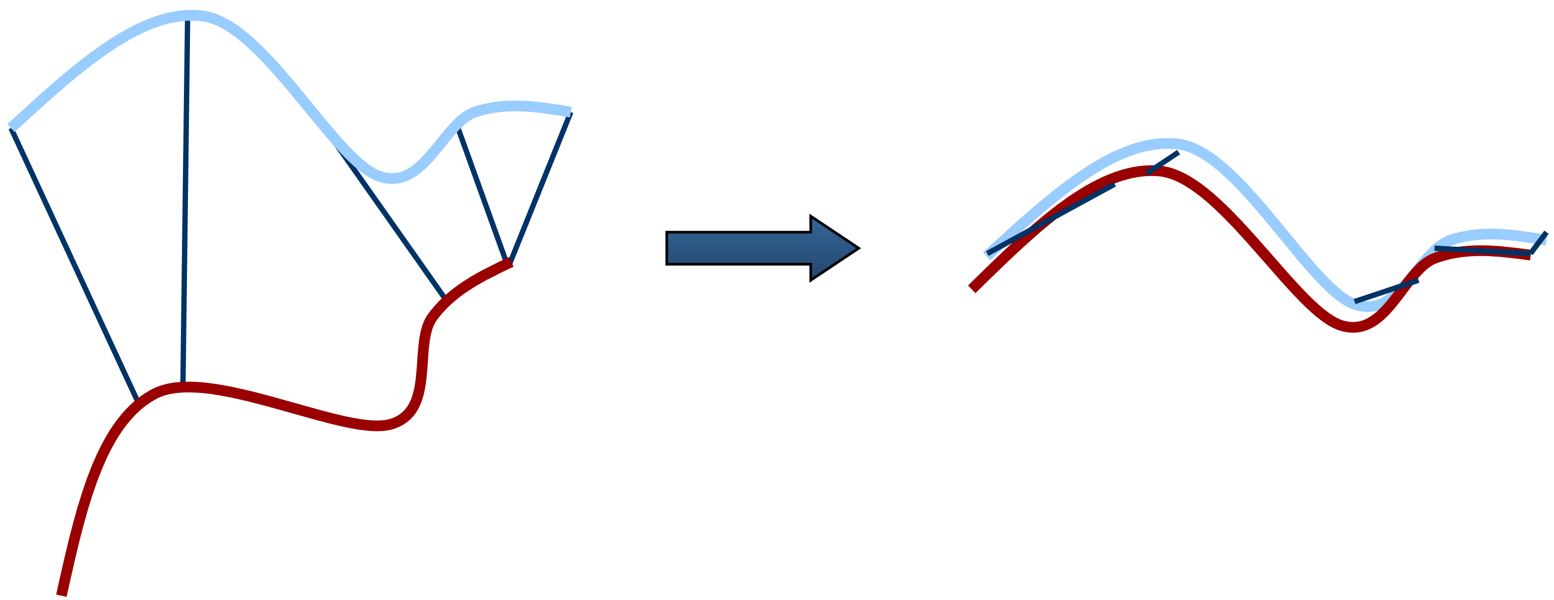
$$R = UV^T$$
$$t = \mu_x - R\mu_p$$

The minimal value of error function at (R,t) is:

$$E(R,t) = \sum_{i=1}^{N_p} (||x_i'||^2 + ||y_i'||^2) - 2(\sigma_1 + \sigma_2 + \sigma_3)$$

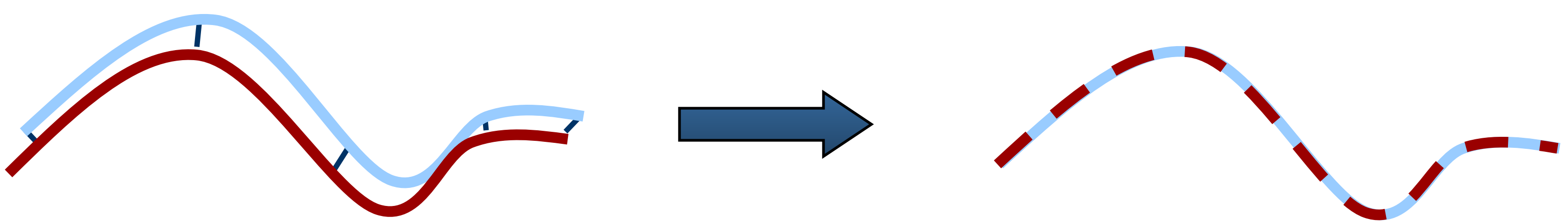
ICP with Unknown Data Association

- If the correct correspondences are **not known**, it is generally impossible to determine the optimal relative rotation and translation in one step



Iterative Closest Point (ICP) Algorithm

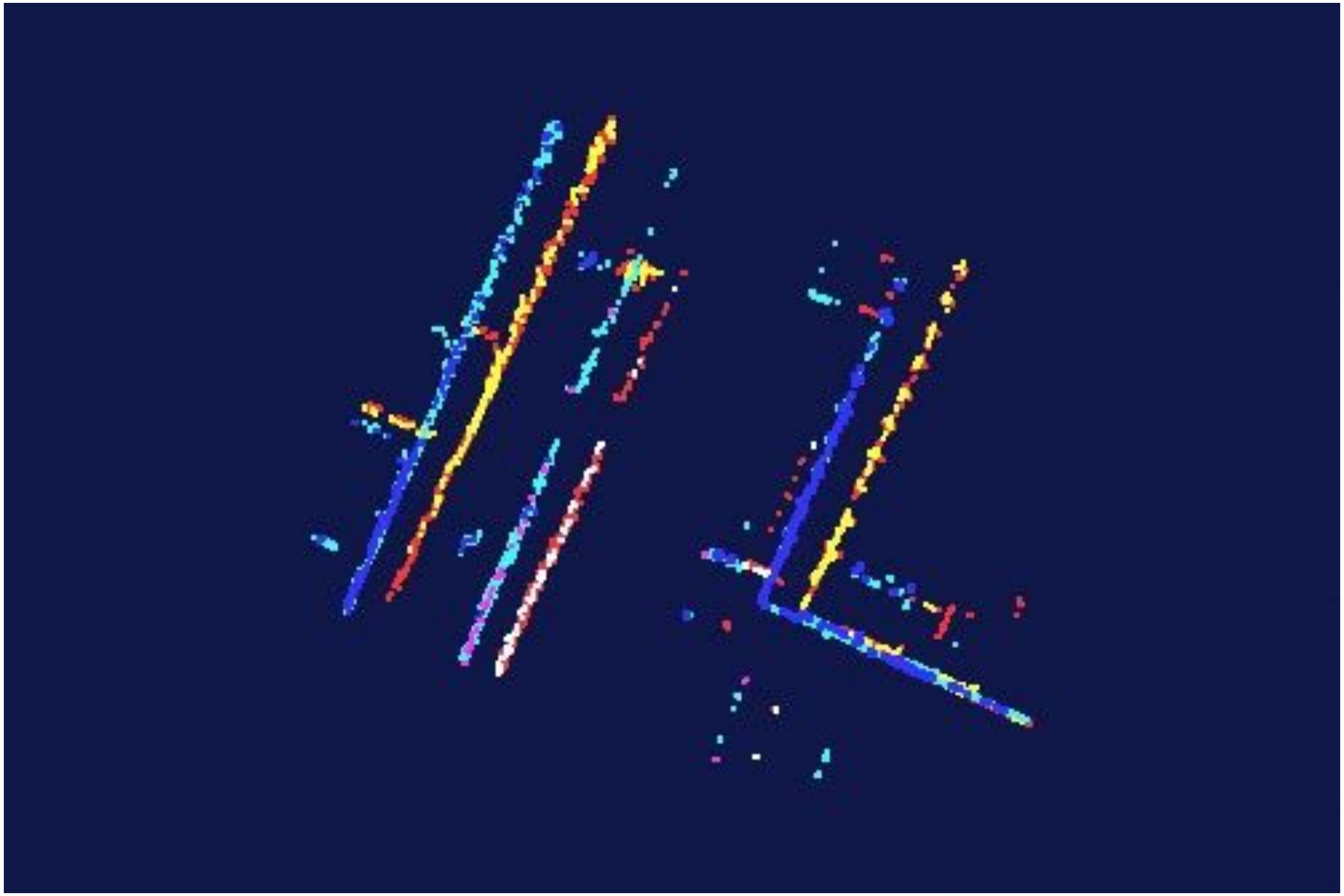
- Idea: Iterate to find alignment
- Iterative Closest Points
  [Besl & McKay 92]
- Converges if starting positions are "close enough"



Basic ICP Algorithm

- Determine corresponding points
- Compute rotation R, translation t via SVD
- Apply R and t to the points of the set to be registered
- Compute the error $E(R,t)$
- If error decreased and error > threshold
  - Repeat these steps
  - Stop and output final alignment, otherwise

ICP Example



ICP Variants

Variants on the following stages of ICP have been proposed:

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs

Performance of Variants

- Various aspects of performance:
  - Speed
  - Stability (local minima)
  - Tolerance wrt. noise and outliers
  - Basin of convergence
    (maximum initial misalignment)

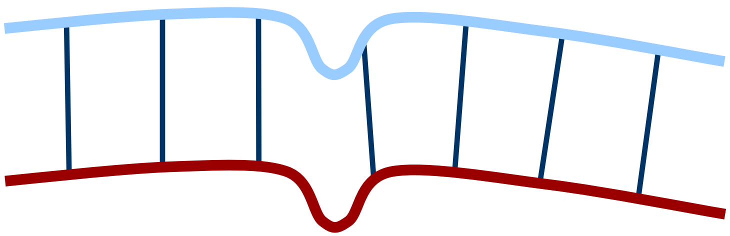
ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs

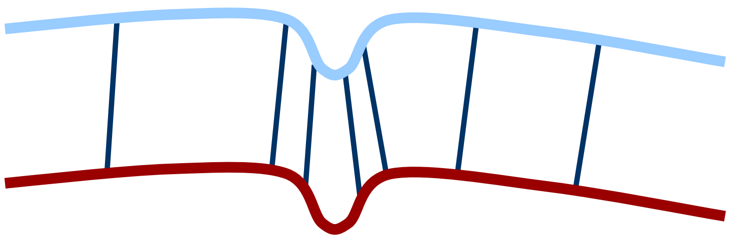
Selecting Source Points

- Use all points
- Uniform sub-sampling
- Random sampling
- Feature based sampling
- Normal-space sampling
  (Ensure that samples have normals distributed as uniformly as possible)

Normal-Space Sampling



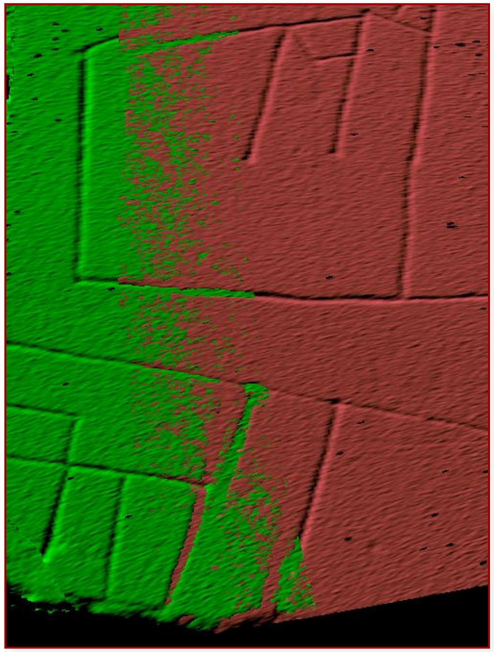
uniform sampling



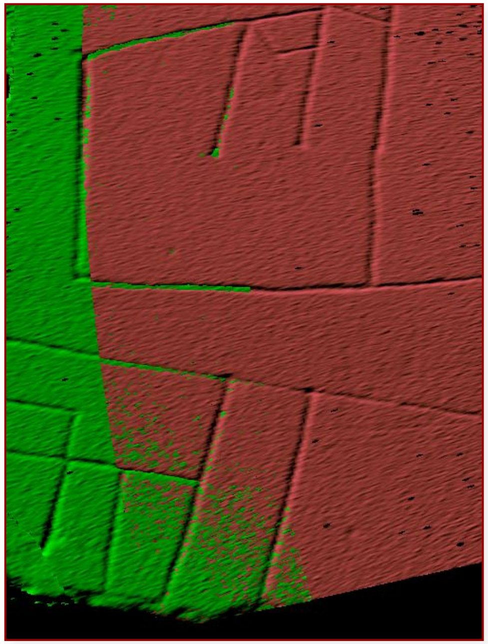
normal-space sampling

Comparison

- Normal-space sampling better for mostly smooth areas with sparse features
  [Rusinkiewicz et al., 01]



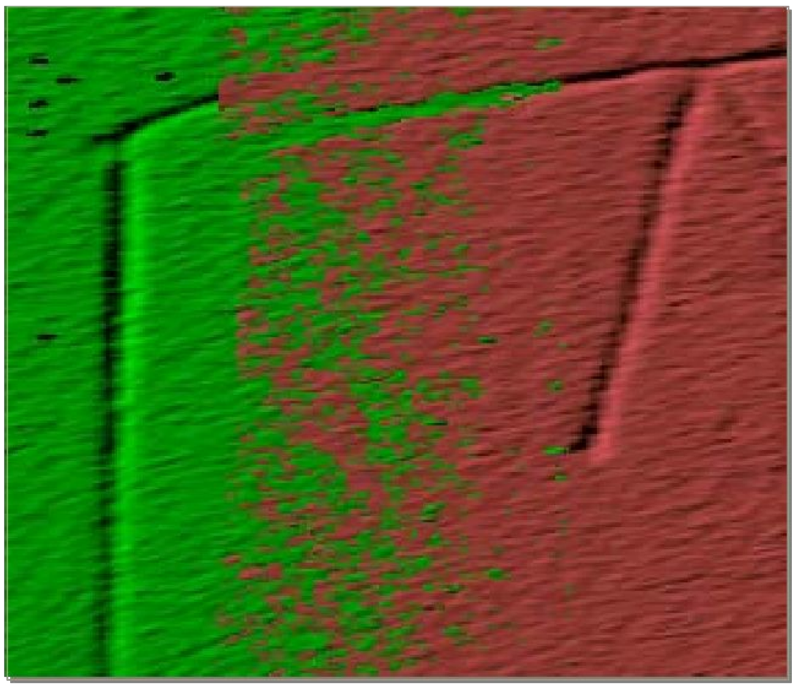
Random sampling



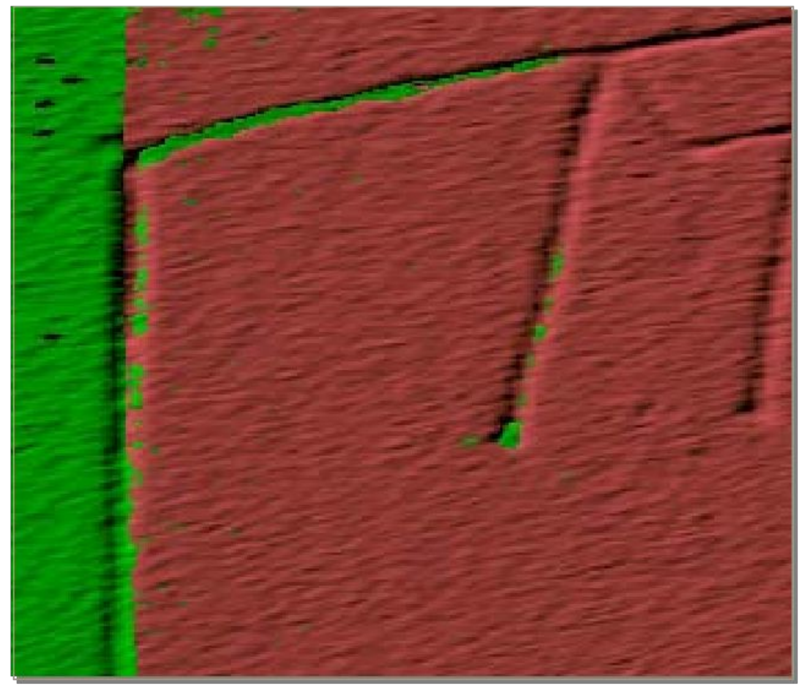
Normal-space sampling

Comparison

- Normal-space sampling better for mostly smooth areas with sparse features
  [Rusinkiewicz et al., 01]



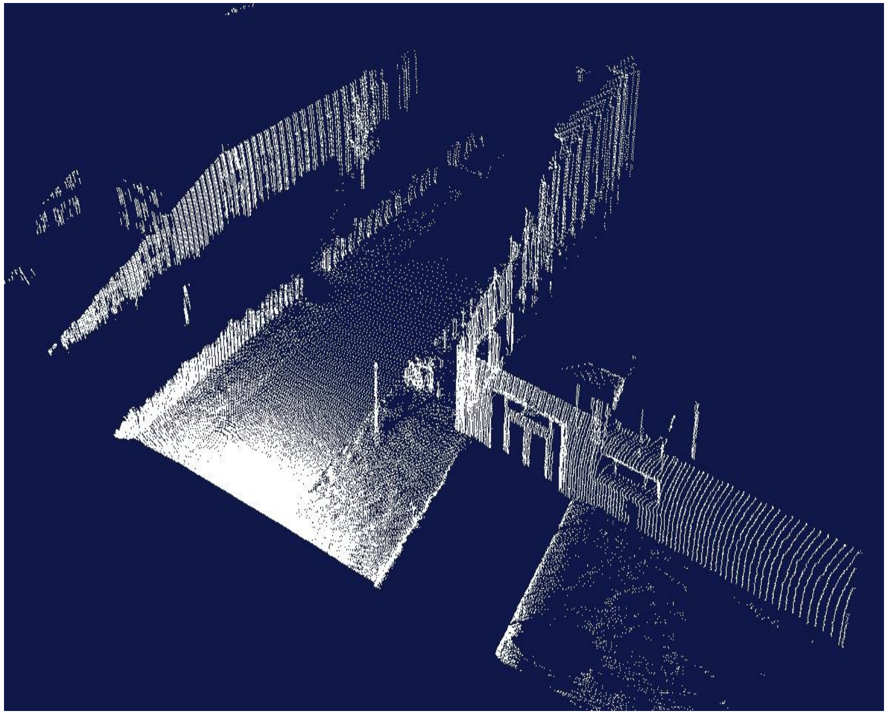
Random sampling



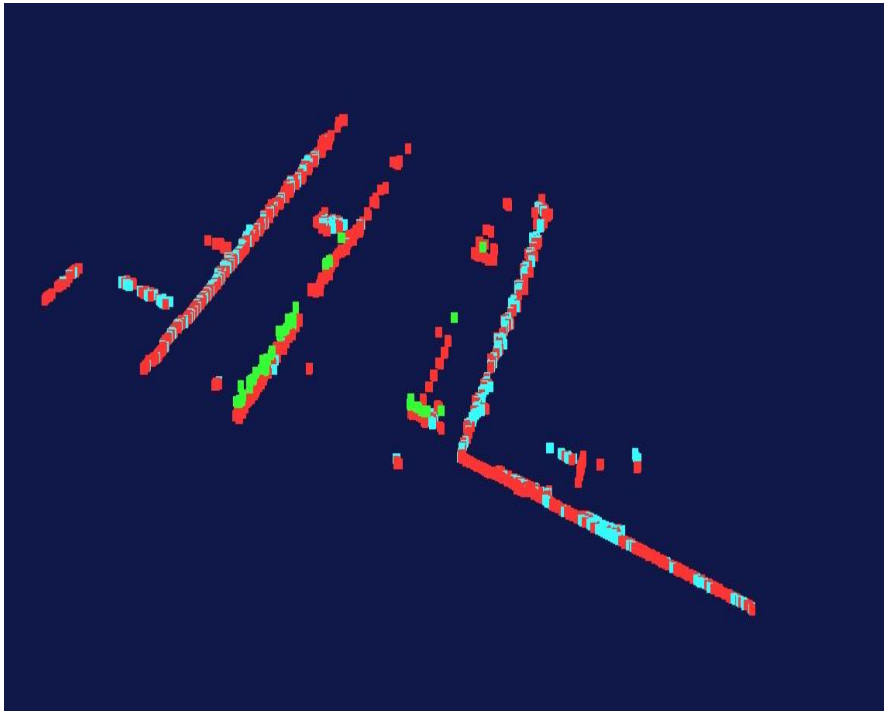
Normal-space sampling

Feature-Based Sampling

- Try to find "important" points
- Decreases the number of correspondences to find
- Higher efficiency and higher accuracy
- Requires preprocessing



3D Scan (~200.000 Points)



Extracted Features (~5.000 Points)

ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
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4. Rejecting certain (outlier) point pairs

Weighting

- Select a set of points for each set
- Match the selected points of the two sets
- **Weight the corresponding pairs**
- E.g., assign lower weights for points with higher point-point distances
- Determine transformation that minimizes the error function

ICP Variants

1. Point subsets (from one or both point sets)
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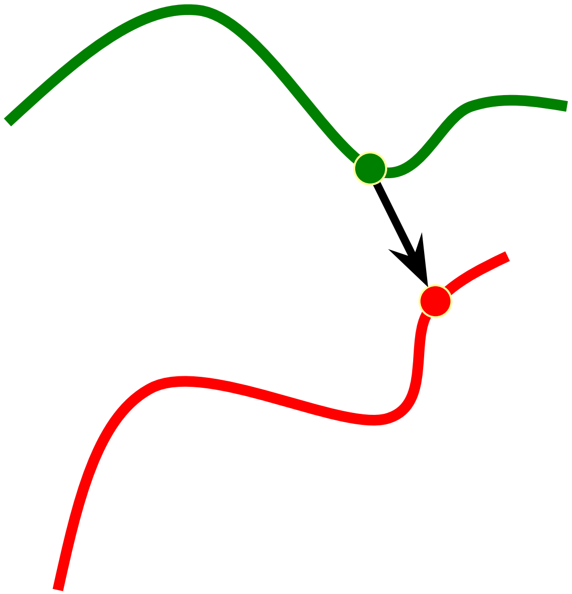
Data Association

- Has greatest effect on convergence and speed
- Matching methods:
  - Closest point
  - Normal shooting
  - Closest compatible point
  - Projection-based

Closest-Point Matching

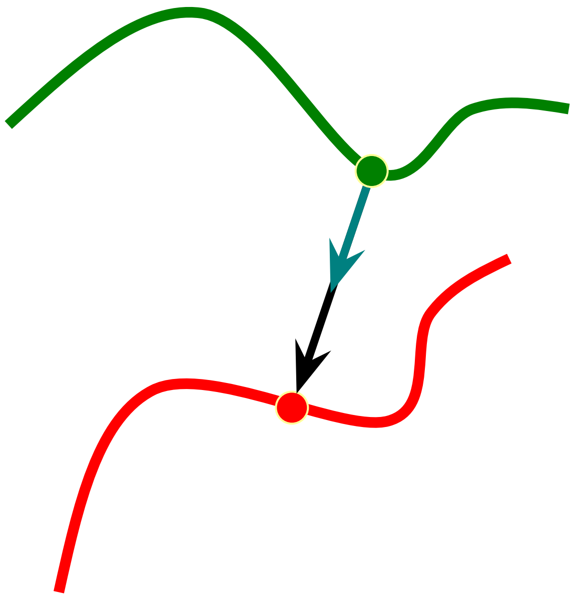
- Find closest point in other the point set



Generally stable, but slow convergence

Normal Shooting

- Project along normal, intersect other point set



Slightly better convergence results than closest point for smooth structures, worse for noisy or complex structures

Closest Compatible Point

- Improves the two previous variants by considering the **compatibility** of the points
- Only match compatible points
- Compatibility can be based on
  - Normals
  - Colors
  - Curvature
  - Higher-order derivatives
  - Other local features

Point-to-Plane Error Metric

- Solved using standard nonlinear least squares methods (e.g., Levenberg-Marquardt method [Press92]).
- Each iteration generally slower than the point-to-point version, however, often significantly better convergence rates [Rusinkiewicz01]
- Using point-to-plane distance instead of point-to-point lets flat regions slide along each other [Chen & Medioni 91]

ICP Variants

1. Point subsets (from one or both point sets)
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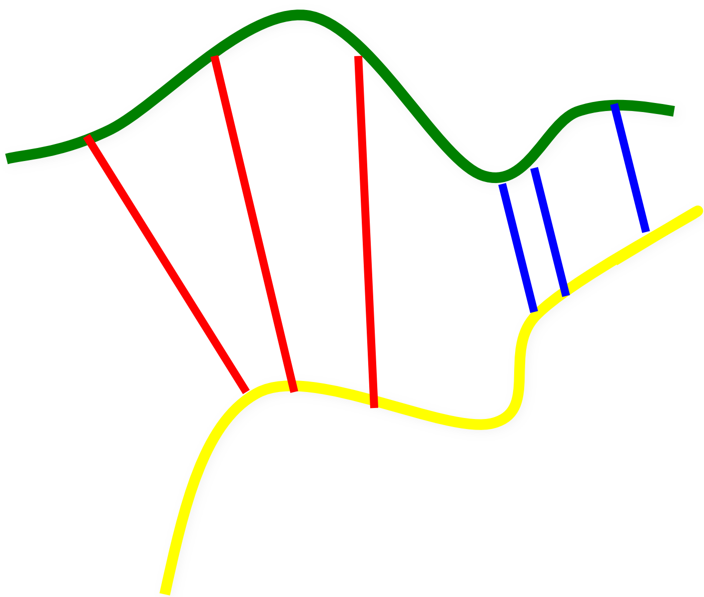
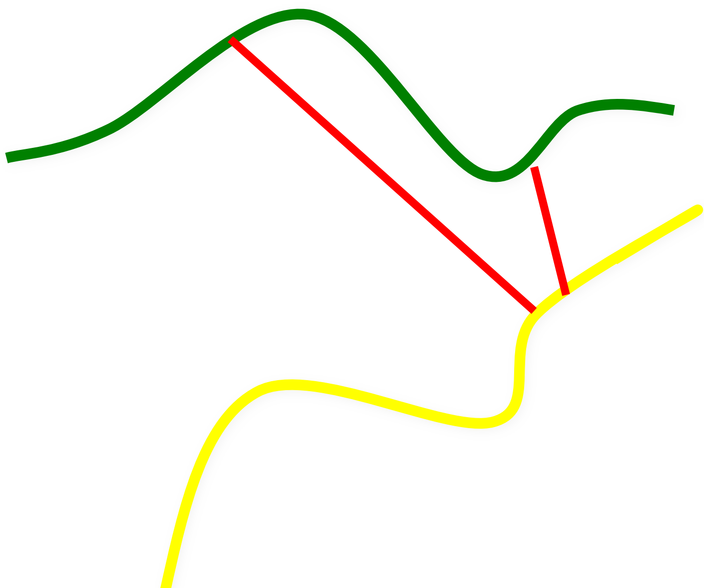
Rejecting (Outlier) Point Pairs

- Corresponding points with point to point distance higher than a given threshold



Rejecting (Outlier) Point Pairs

- Corresponding points with point to point distance higher than a given threshold
- Rejection of pairs that are not consistent with their neighboring pairs [Dorai 98]



Rejecting (Outlier) Point Pairs

- Corresponding points with point to point distance higher than a given threshold
- Rejection of pairs that are not consistent with their neighboring pairs [Dorai 98]
- Sort all correspondences with respect to their error and delete the worst *t*%, Trimmed ICP (TrICP) [Chetverikov et al. 02]
  - *t* is used to estimate the overlap
  - Problem: Knowledge about the overlap is necessary or has to be estimated

Summary: ICP Algorithm

- Potentially sample Points
- Determine corresponding points
- Potentially weight / reject pairs
- Compute rotation R, translation t (e.g. SVD)
- Apply R and t to all points of the set to be registered
- Compute the error $E(R,t)$
- If error decreased and error > threshold
  - Repeat to determine correspondences etc.
  - Stop and output final alignment, otherwise

ICP Summary

- ICP is a powerful algorithm for calculating the displacement between scans
- The major problem is to determine the correct data associations
- Convergence speed depends on point matched points
- Given the correct data associations, the transformation can be computed efficiently using SVD
- ICP does not always converge