

# Introduction to Mobile Robotics

## SLAM – Grid-based FastSLAM

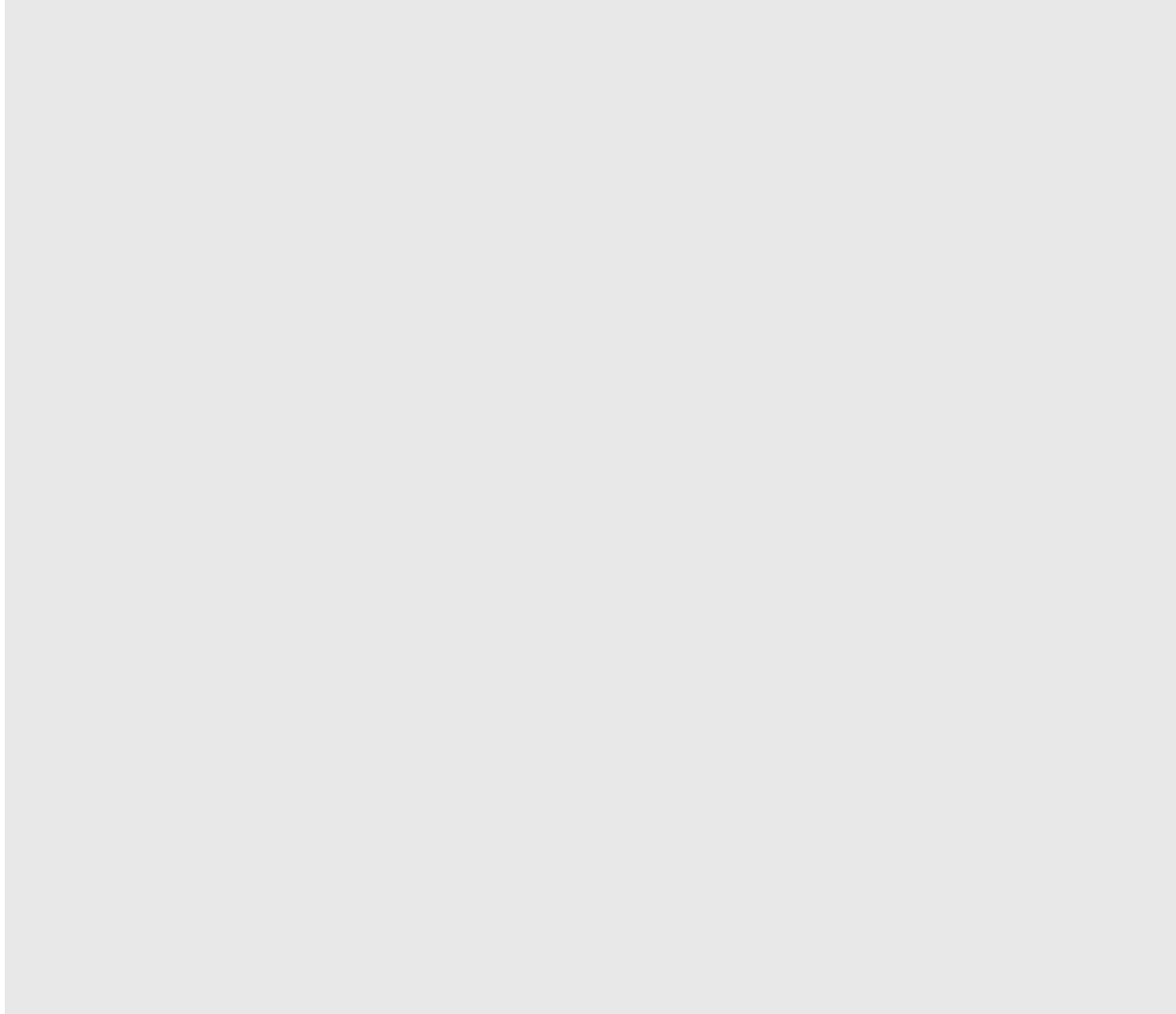
Daniel Büscher



# The SLAM Problem

- SLAM stands for simultaneous localization and mapping
- The task of building a map while estimating the pose of the robot relative to this map
- Why is SLAM hard?  
Chicken-or-egg problem:
  - a map is needed to localize the robot and
  - a pose estimate is needed to build a map

# Grid Mapping using Odometry

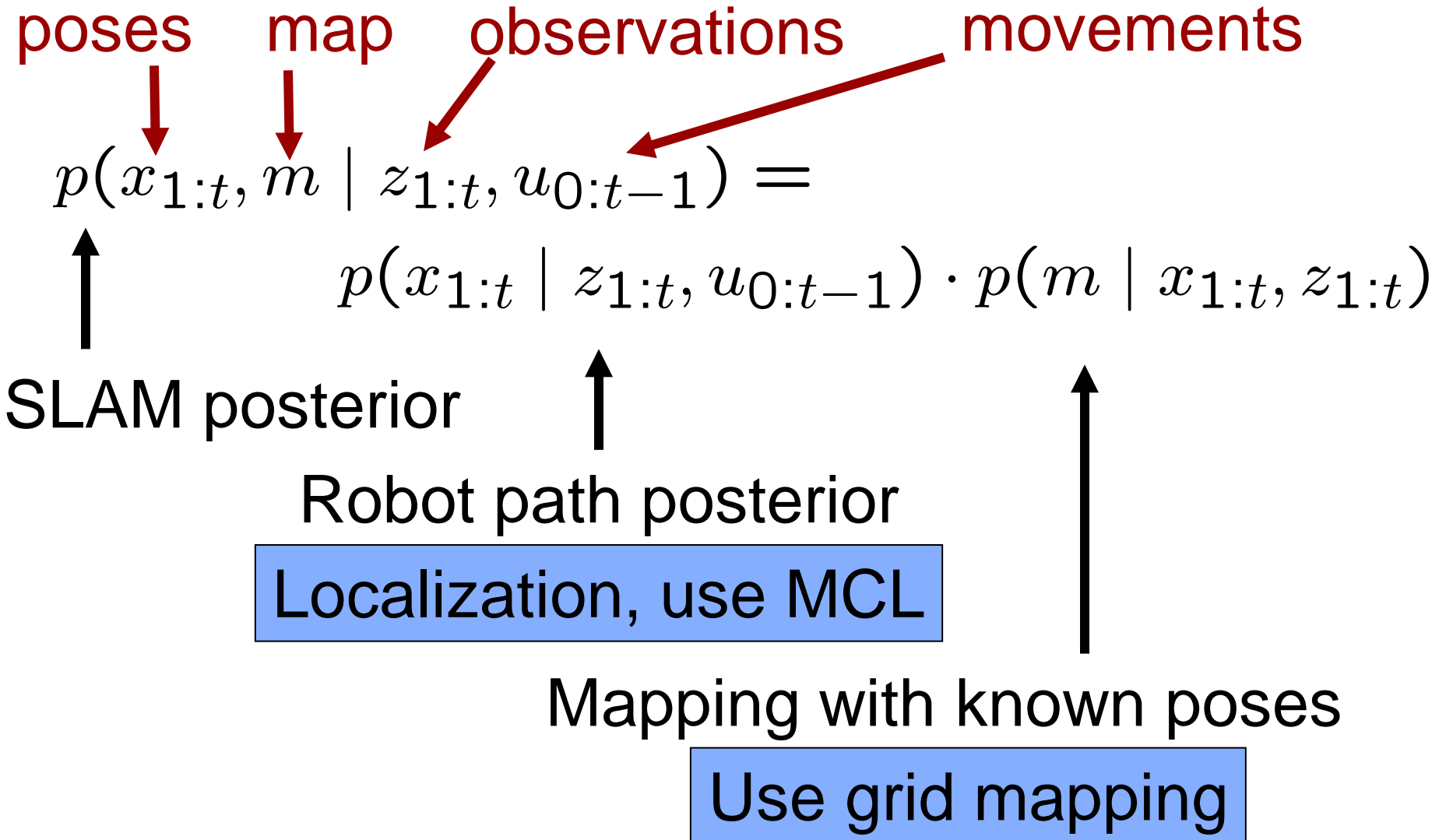


# 1. Grid-based SLAM

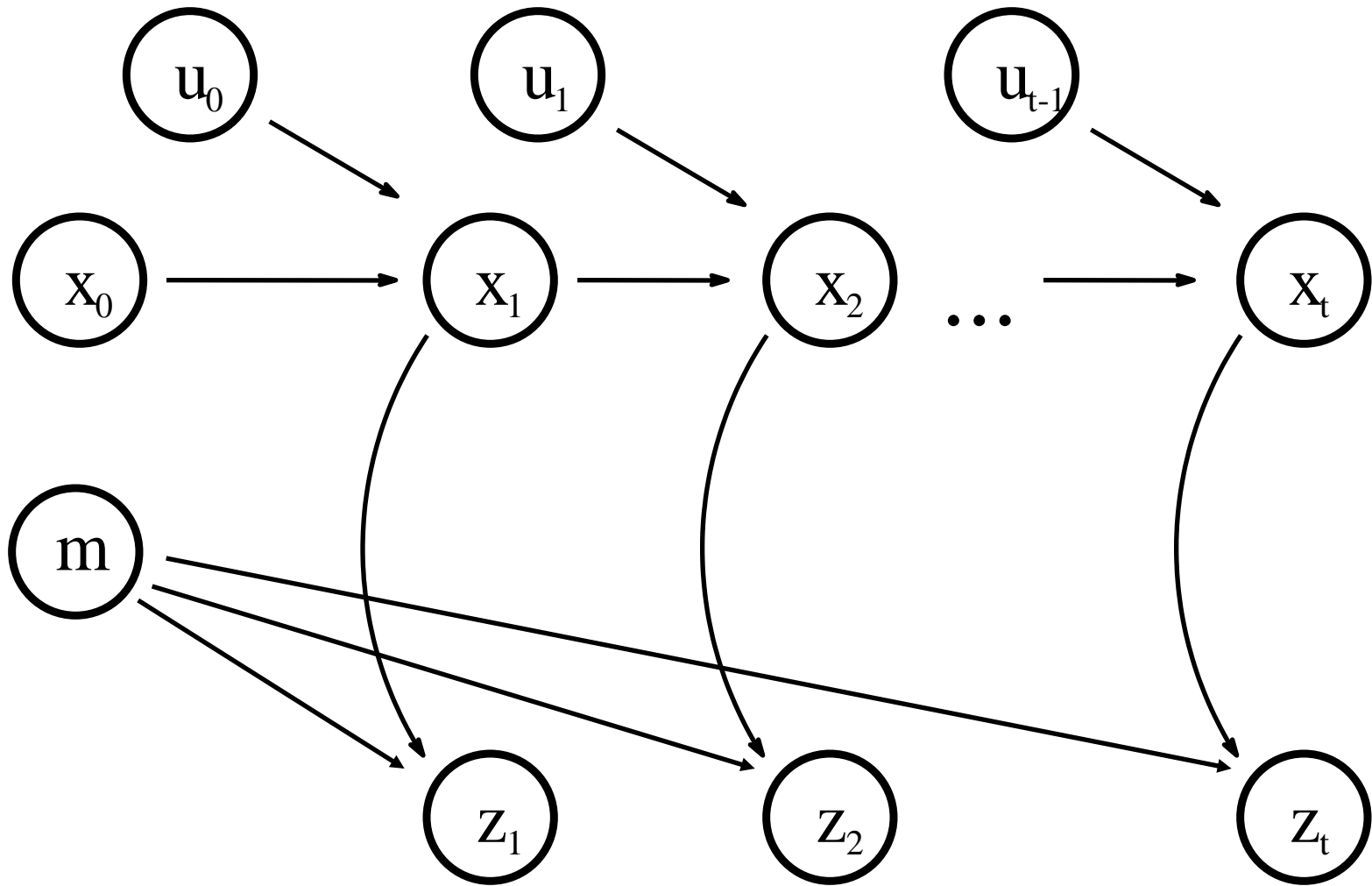
# Grid-based SLAM

- Can we solve the SLAM problem if no pre-defined landmarks are available?
- Can we use the ideas of FastSLAM to build grid maps?
- As with landmarks, the map depends on the poses of the robot during data acquisition
- If the poses are known, grid-based mapping is easy (“mapping with known poses”)

# Rao-Blackwellization



# A Graphical Model of Mapping with Rao-Blackwellized PFs

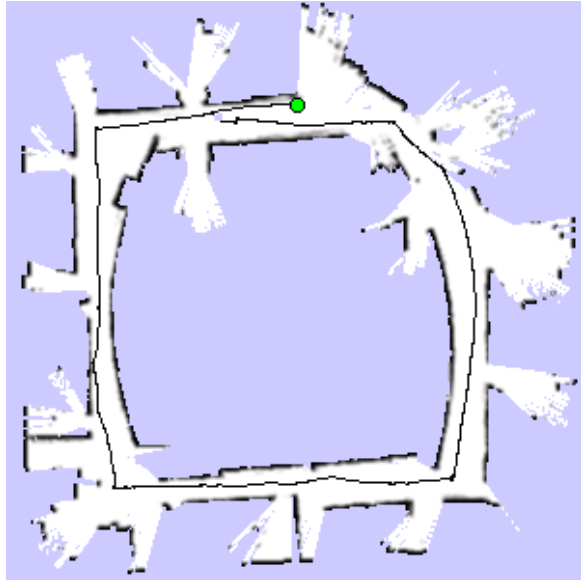


# Mapping with Rao-Blackwellized Particle Filters

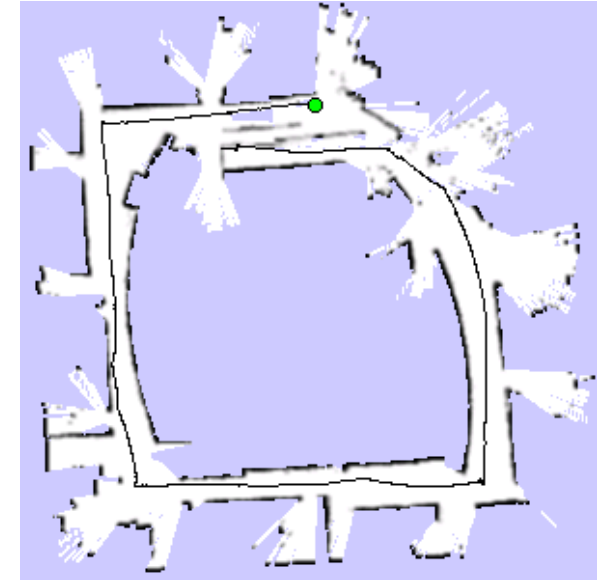
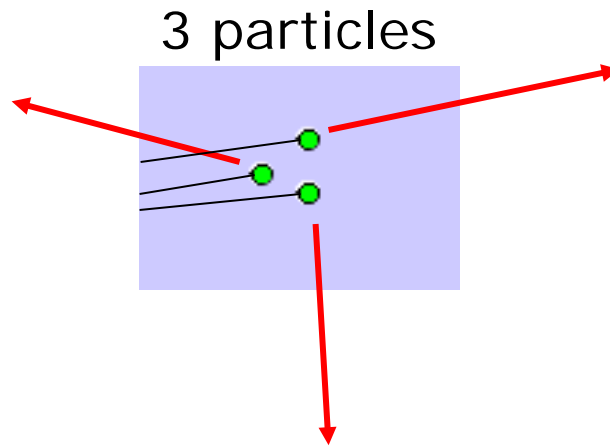
- Each particle represents a possible trajectory of the robot
- Each particle
  - maintains its own map and
  - updates it upon “mapping with known poses”
- Each particle survives with a probability proportional to the likelihood of the observations relative to its own map



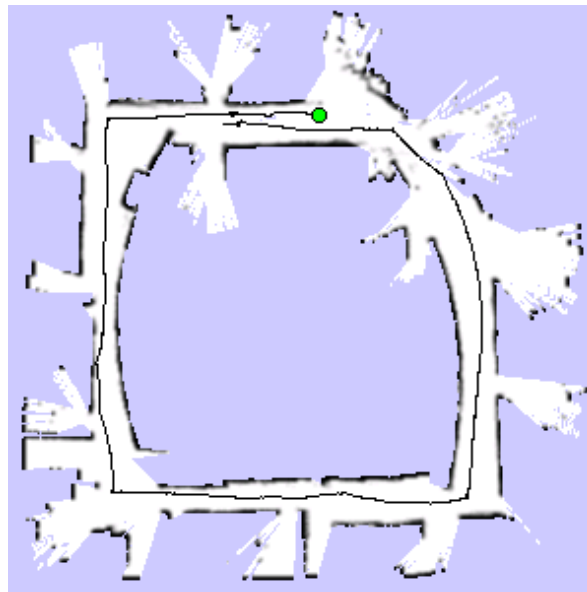
# Particle Filter Example



map of particle 1



map of particle 3



map of particle 2

## **2. Improved Proposal: Scan Matching**

# Problem

- Each map is quite big in case of grid maps
- Each particle maintains its own map, therefore, one needs to keep the number of particles small
- **Solution:**  
Compute better proposal distributions!
- **Idea:**  
Improve the pose estimate **before** applying the particle filter

# Pose Correction Using Scan Matching

Maximize the likelihood of the  $i$ -th pose and map relative to the  $(i-1)$ -th pose and map

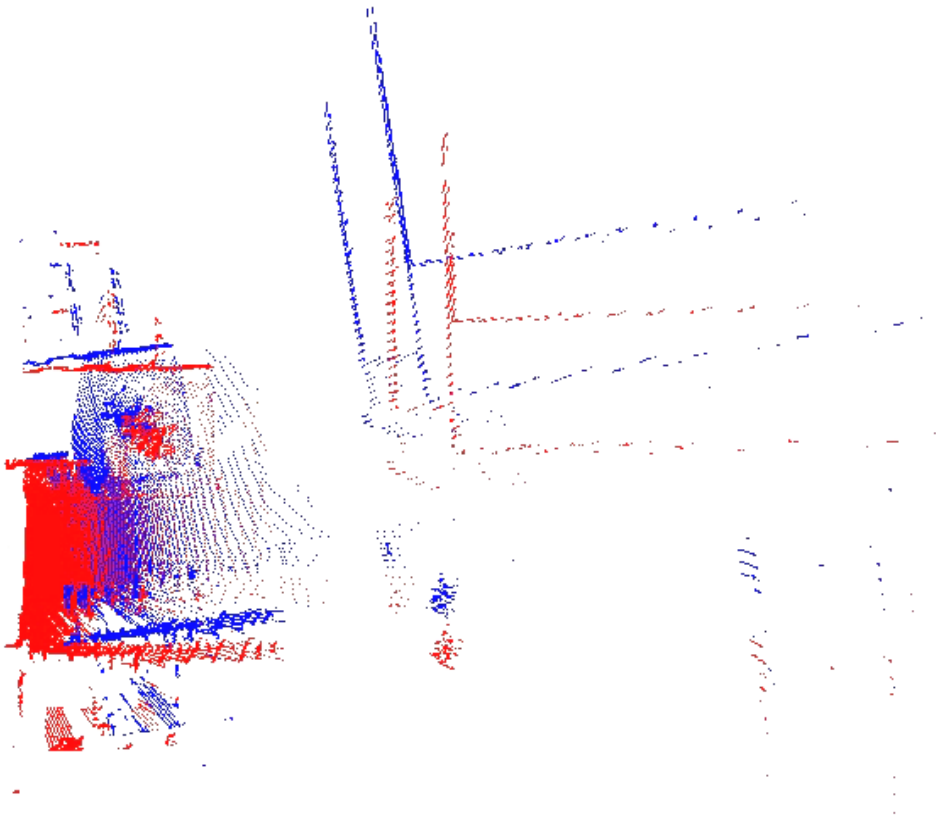
$$\hat{x}_t = \underset{x_t}{\operatorname{argmax}} \{ p(z_t \mid x_t, \hat{m}_{t-1}) \cdot p(x_t \mid u_{t-1}, \hat{x}_{t-1}) \}$$

current measurement

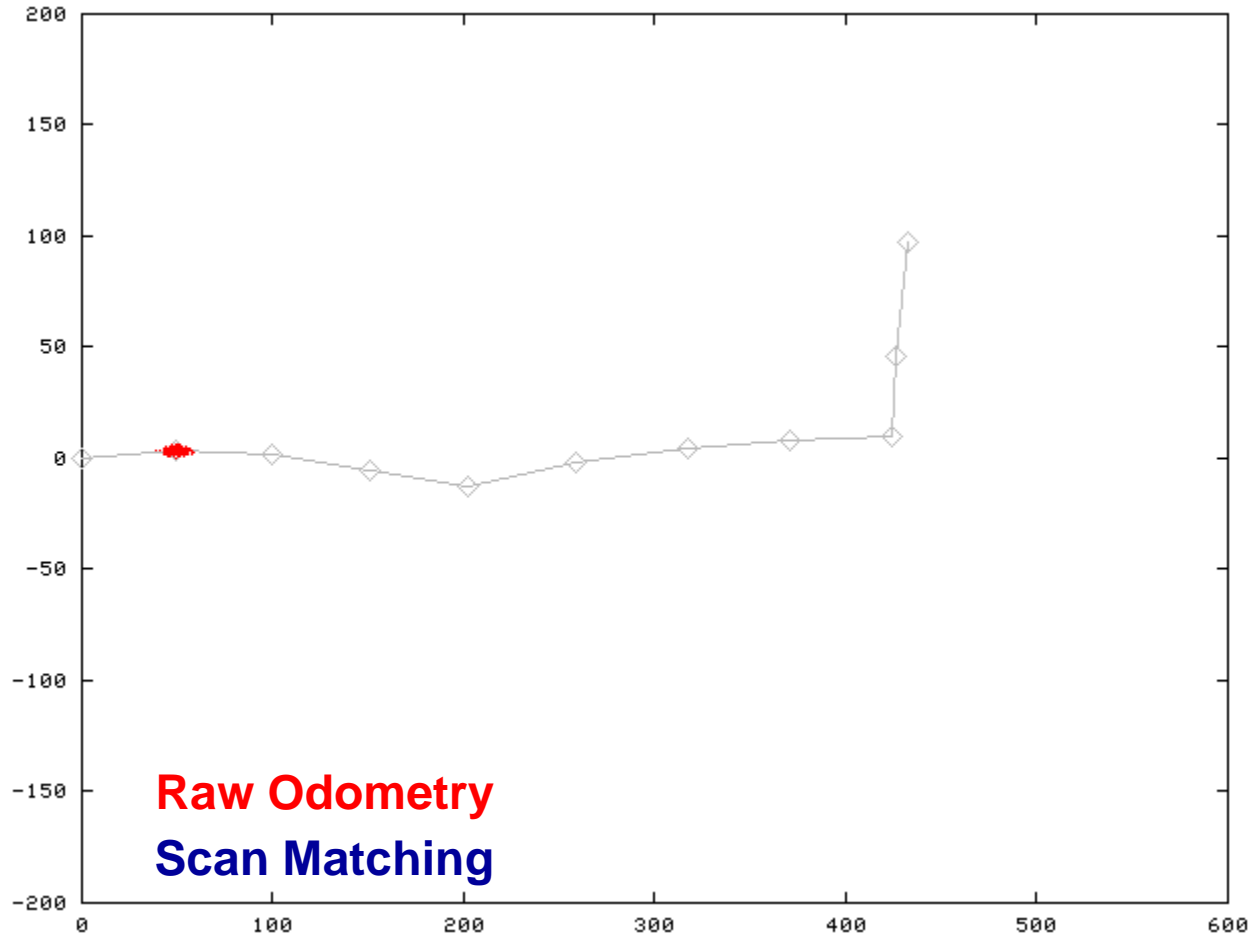
robot motion

map constructed so far

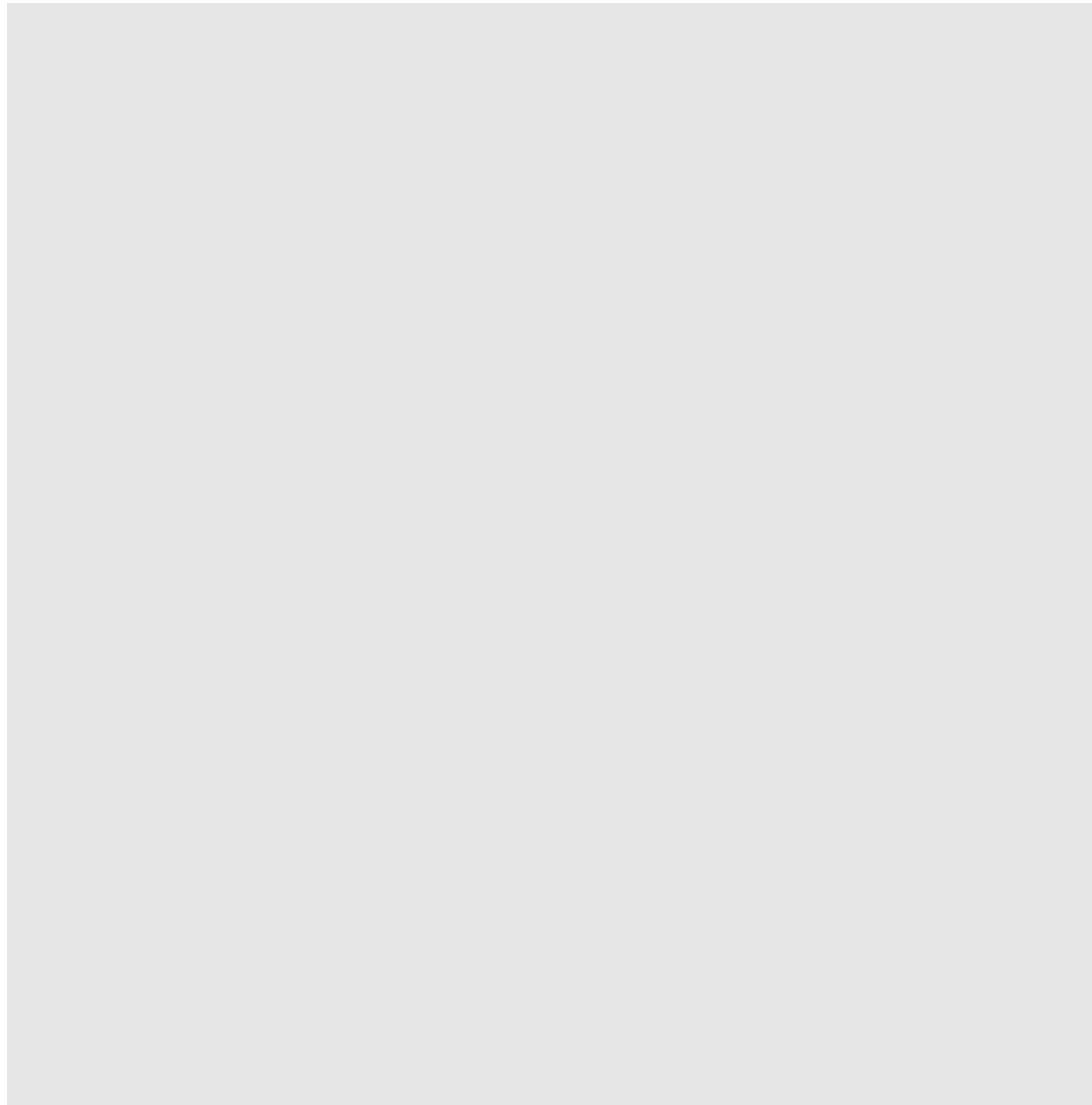
# Scan-Matching Example



# Motion Model for Scan Matching



# Mapping using Scan Matching

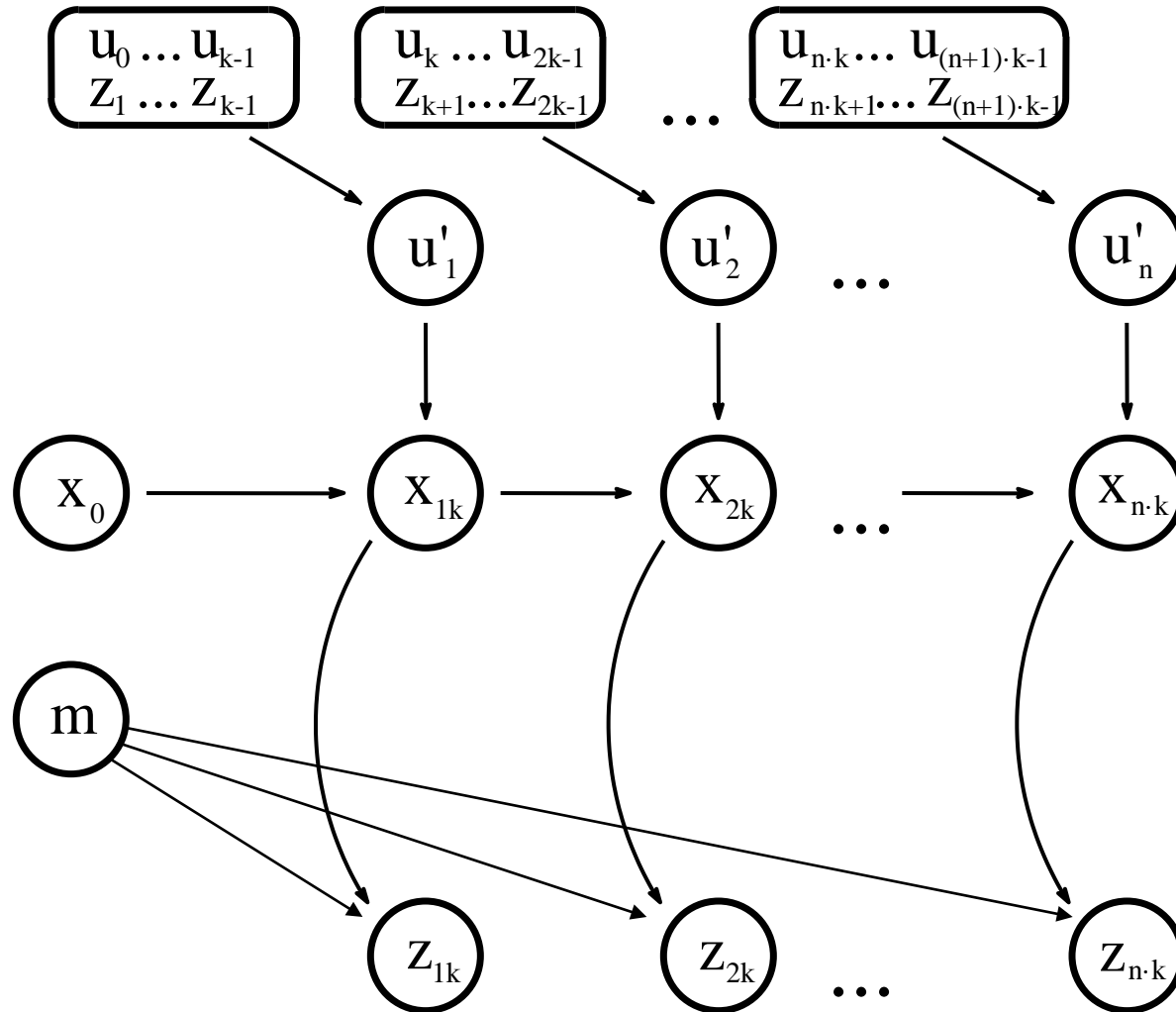


# FastSLAM with Improved Odometry

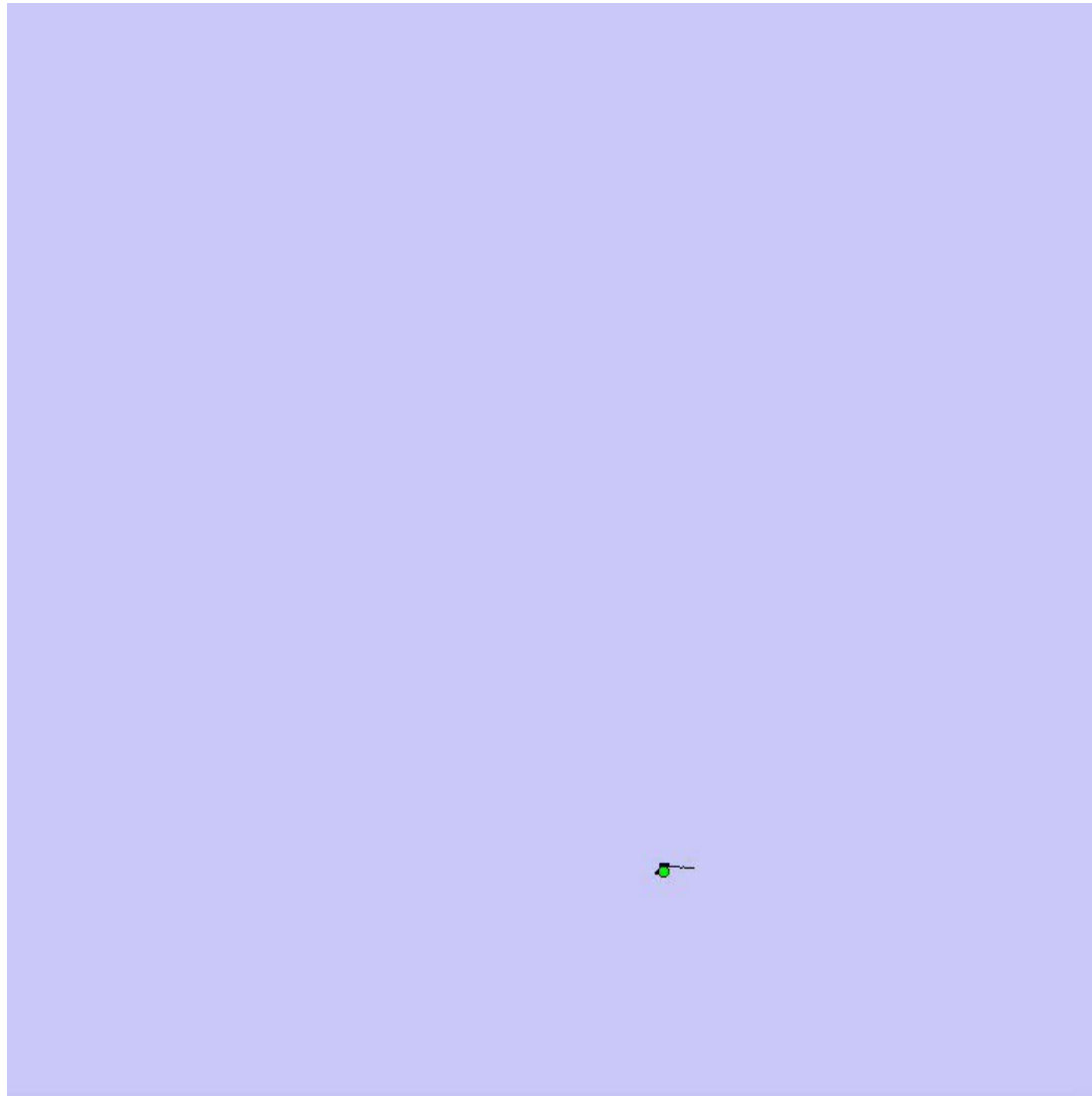
- Scan-matching provides a **locally consistent** pose correction
- Pre-correct short odometry sequences using scan-matching and use them as input to FastSLAM
- Fewer particles are needed, since the error in the input is smaller



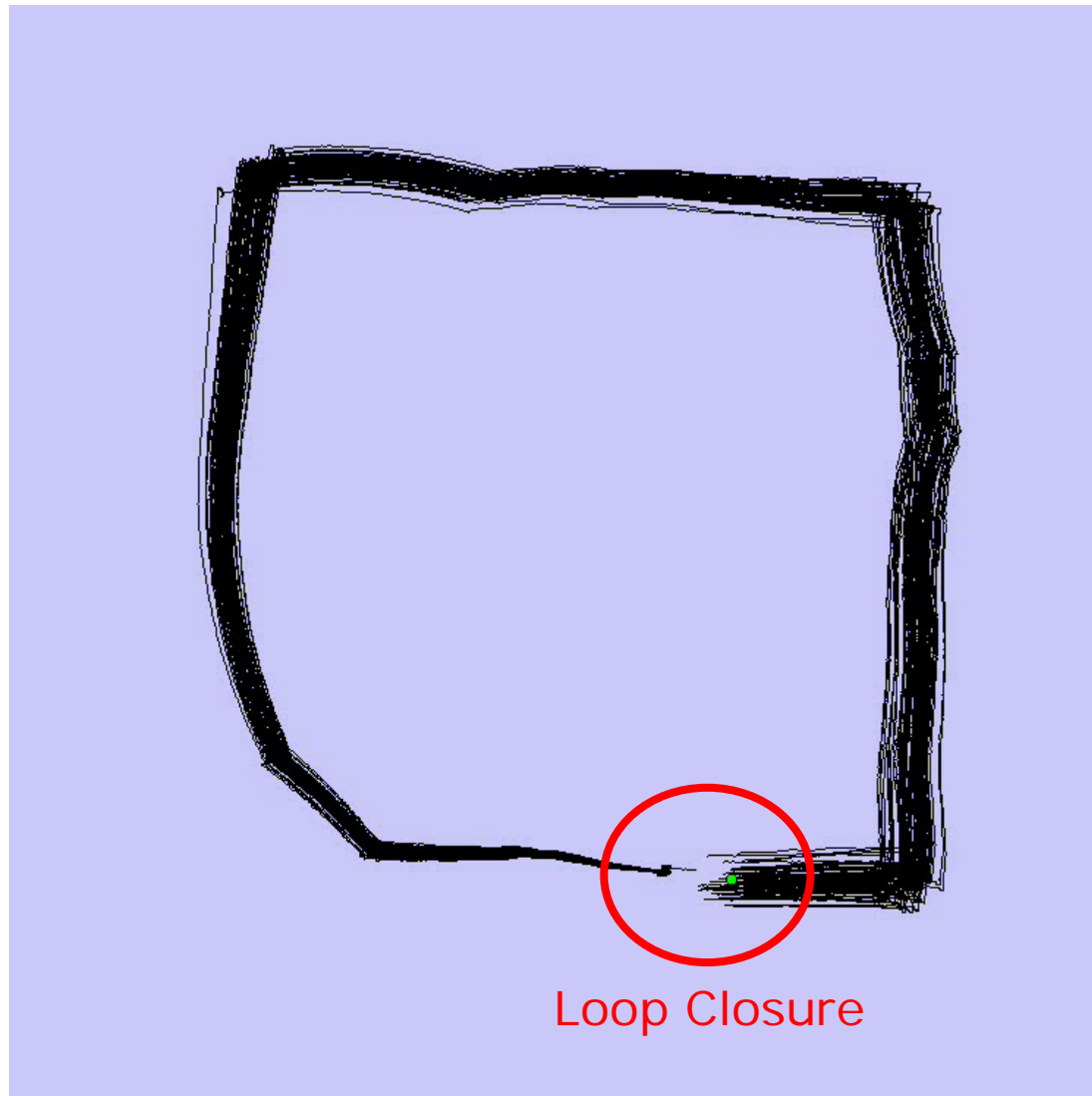
# Graphical Model for Mapping with Improved Odometry



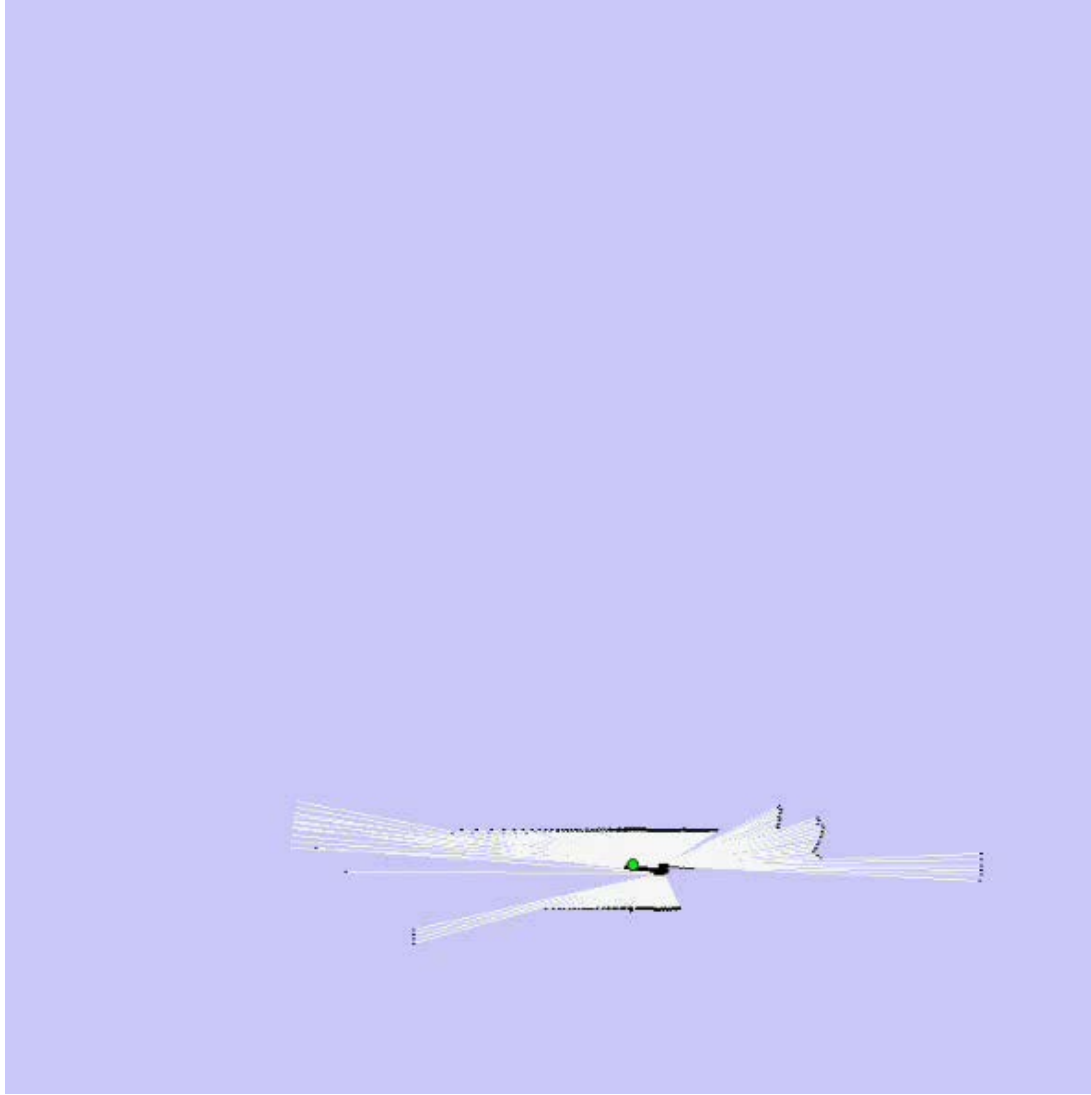
# FastSLAM with Scan-Matching



# FastSLAM with Scan-Matching

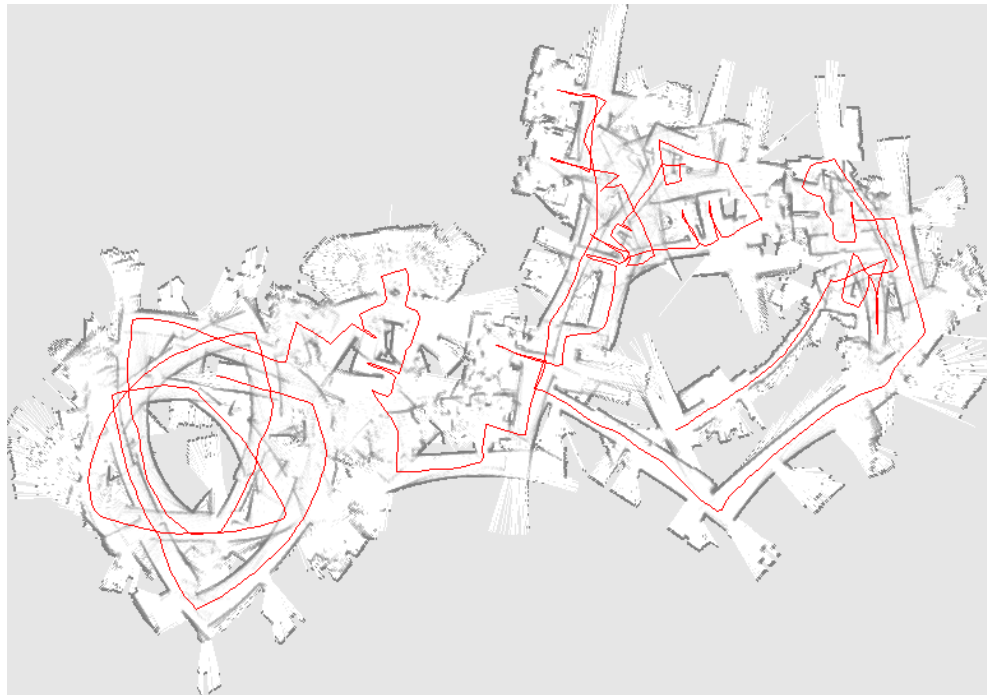


# FastSLAM with Scan-Matching



# FastSLAM without scan matching

- Same model for observations
- Odometry instead of scan matching as input
- Number of particles varying from 500 to 2,000
- Typical result often not usable:



# Conclusion (thus far ...)

- The presented approach is a highly efficient algorithm for SLAM combining ideas of scan matching and FastSLAM
- Scan matching is used to transform sequences of laser measurements into odometry measurements
- This version of grid-based FastSLAM can handle larger environments than before in “real time”

### **3. Improved Proposal: the “Optimal” Distribution**

# What's Next?

- Further reduce the number of particles
- Improved proposals will lead to more accurate maps
- Use the properties of our sensor when drawing the next generation of particles



# The Optimal Proposal Distribution

$$p(x_t | x_{t-1}^{(i)}, m^{(i)}, z_t, u_t) = \frac{p(z_t | x_t, m^{(i)}) p(x_t | x_{t-1}^{(i)}, u_t)}{\int p(z_t | x_t, m^{(i)}) p(x_t | x_{t-1}^{(i)}, u_t) dx_t}$$

↑  
Probability for pose given collected data

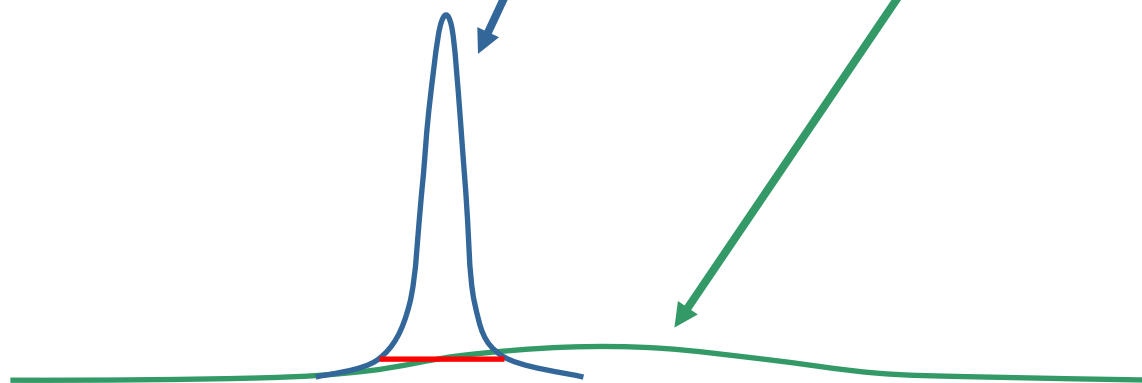
↑  
observation model

↑  
motion model

↑  
normalization

# The Optimal Proposal Distribution

$$p(x_t | x_{t-1}^{(i)}, m^{(i)}, z_t, u_t) = \frac{p(z_t | x_t, m^{(i)}) p(x_t | x_{t-1}^{(i)}, u_t)}{\int p(z_t | x_t, m^{(i)}) p(x_t | x_{t-1}^{(i)}, u_t) dx_t}$$



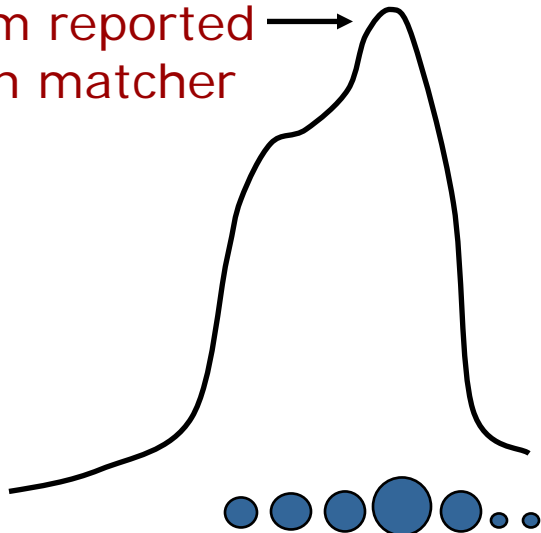
For lasers  $p(z_t | x_t, m^{(i)})$  is extremely peaked and dominates the product.

➔ We can safely approximate  $p(x_t | x_{t-1}^{(i)}, u_t)$  by a constant:  $p(x_t | x_{t-1}^{(i)}, u_t) \Big|_{x_t: p(z_t | x_t, m^{(i)}) > \epsilon} = c$

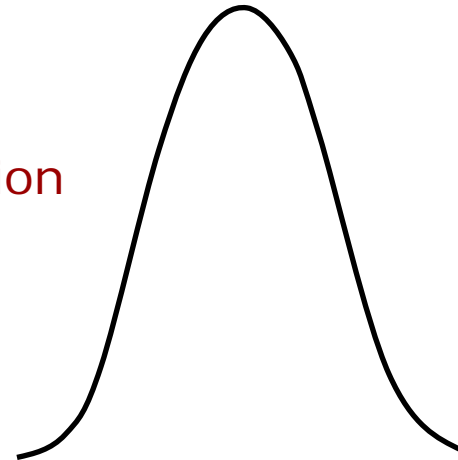
# Resulting Proposal Distribution

$$p(x_t | x_{t-1}^{(i)}, m^{(i)}, z_t, u_t) \simeq \frac{p(z_t | x_t, m^{(i)})}{\int_{x_t \in \{x | p(z_t | x, m^{(i)}) > \epsilon\}} p(z_t | x_t, m^{(i)}) dx_t}$$
$$\simeq \mathcal{N}(\mu^{(i)}, \Sigma^{(i)})$$

maximum reported  
by a scan matcher



Gaussian  
approximation



Draw next  
generation of  
samples

Sampled points around  
the maximum

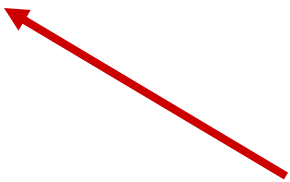
# Estimating the Parameters of the Gaussian for each Particle

$$\mu^{(i)} = \frac{1}{\eta} \sum_{j=1}^K x_j p(z_t | x_j, m^{(i)})$$

$$\Sigma^{(i)} = \frac{1}{\eta} \sum_{j=1}^K (x_j - \mu^{(i)})(x_j - \mu^{(i)})^T p(z_t | x_j, m^{(i)})$$

- $x_j$  are a set of sample points around the point  $x^*$  the scan matching has converged to.
- $\eta$  is a normalizing constant

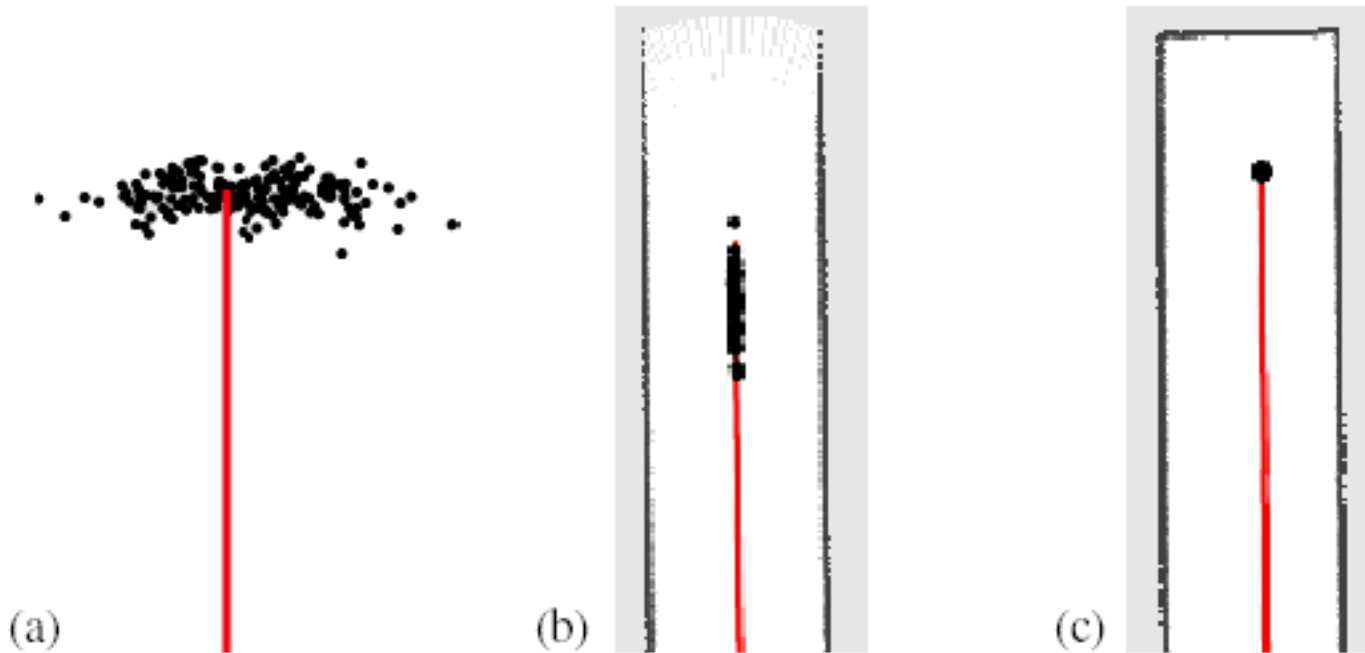
# Computing the Importance Weight

$$\begin{aligned}w_t^{(i)} &= w_{t-1}^{(i)} p(z_t | x_{t-1}^{(i)}, m^{(i)}, u_t) \\ &\approx w_{t-1}^{(i)} \int p(z_t | x_t, m^{(i)}) p(x_t | x_{t-1}^{(i)}, u_t) dx_t \\ &\approx w_{t-1}^{(i)} c \int_{x_t \in \{x | p(z_t | x, m^{(i)}) > \epsilon\}} p(z_t | x_t, m^{(i)}) dx_t \\ &\approx w_{t-1}^{(i)} c \sum_{j=1}^K p(z_t | x_j, m^{(i)})\end{aligned}$$


Sampled points around the maximum of the observation likelihood

# Improved Proposal

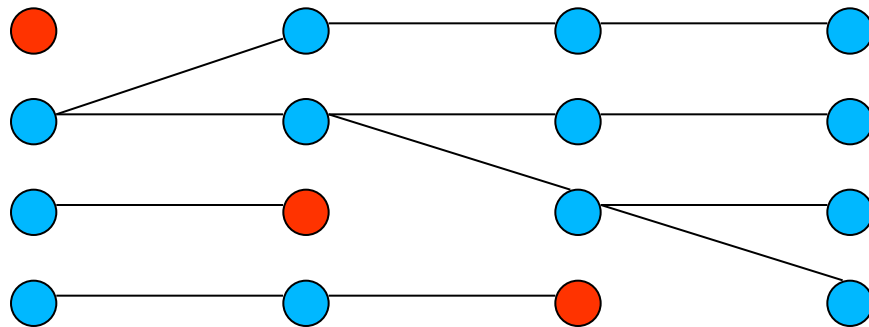
- The proposal adapts to the structure of the environment



## **4. Adaptive Resampling**

# Resampling

- Sampling from an improved proposal reduces the effects of resampling
- However, resampling at each step limits the “memory” of our filter
- Supposed we loose at each frame 25% of the particles, in the worst case we have a memory of only 4 steps.



**Goal: reduce the number of resampling actions**



# Selective Resampling

- Resampling is dangerous, since important samples might get lost (particle depletion problem)
- In case of suboptimal proposal distributions resampling is necessary to achieve convergence.
- Key question: When should we resample?

# Number of Effective Particles

$$n_{eff} = \frac{1}{\sum_i \left(w_t^{(i)}\right)^2}$$

- Assuming normalized particle weights that sum up to 1.0:  $\sum_{i=1}^n w_t^{(i)} = 1 \Rightarrow n_{eff} \in [1, n]$
- Empirical measure of how well the goal distribution is approximated by samples drawn from the proposal
- It describes “the variance of the particle weights”
- It is maximal for equal weights. In this case the distribution is close to the proposal

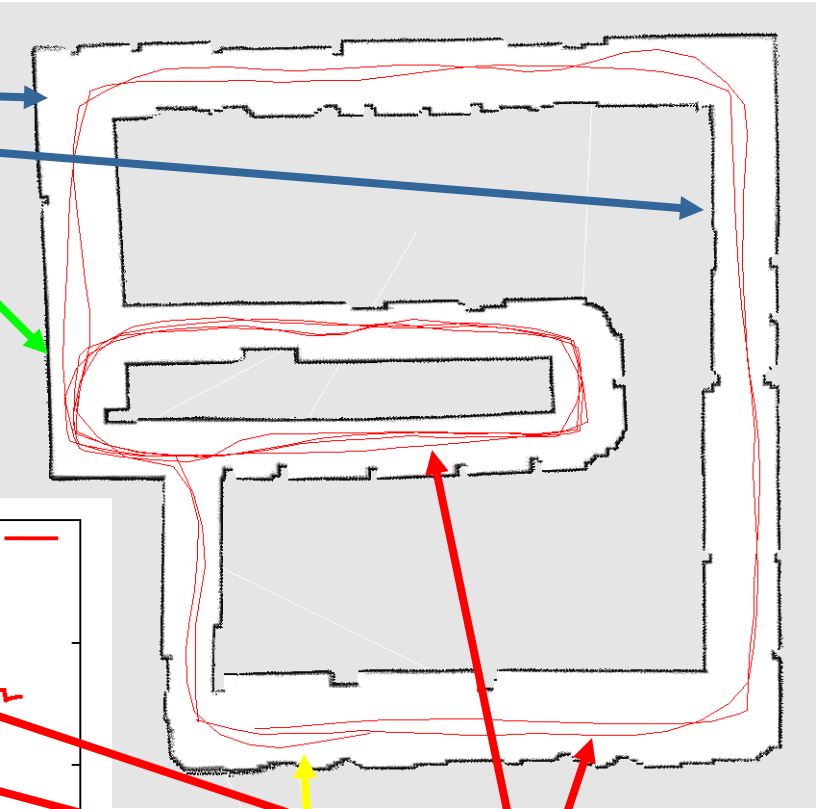
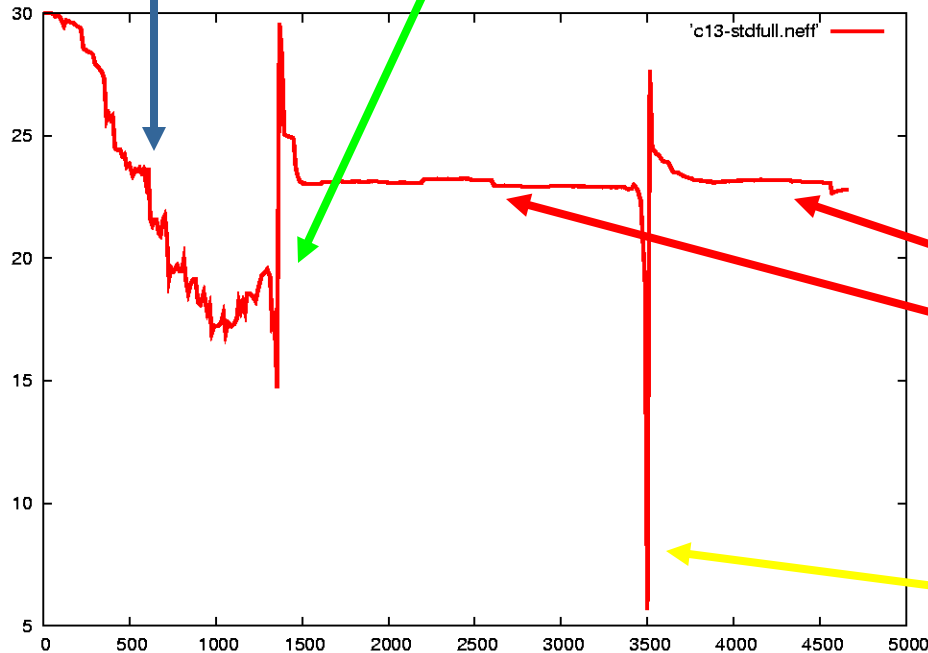
# Resampling with $n_{eff}$

- If our approximation is close to the proposal, no resampling is needed
- We only resample when  $n_{eff}$  drops below a given threshold, typically  $\frac{n}{2}$
- See [Doucet, '98; Arulampalam, '01]

# Typical Evolution of $n_{eff}$

visiting new areas

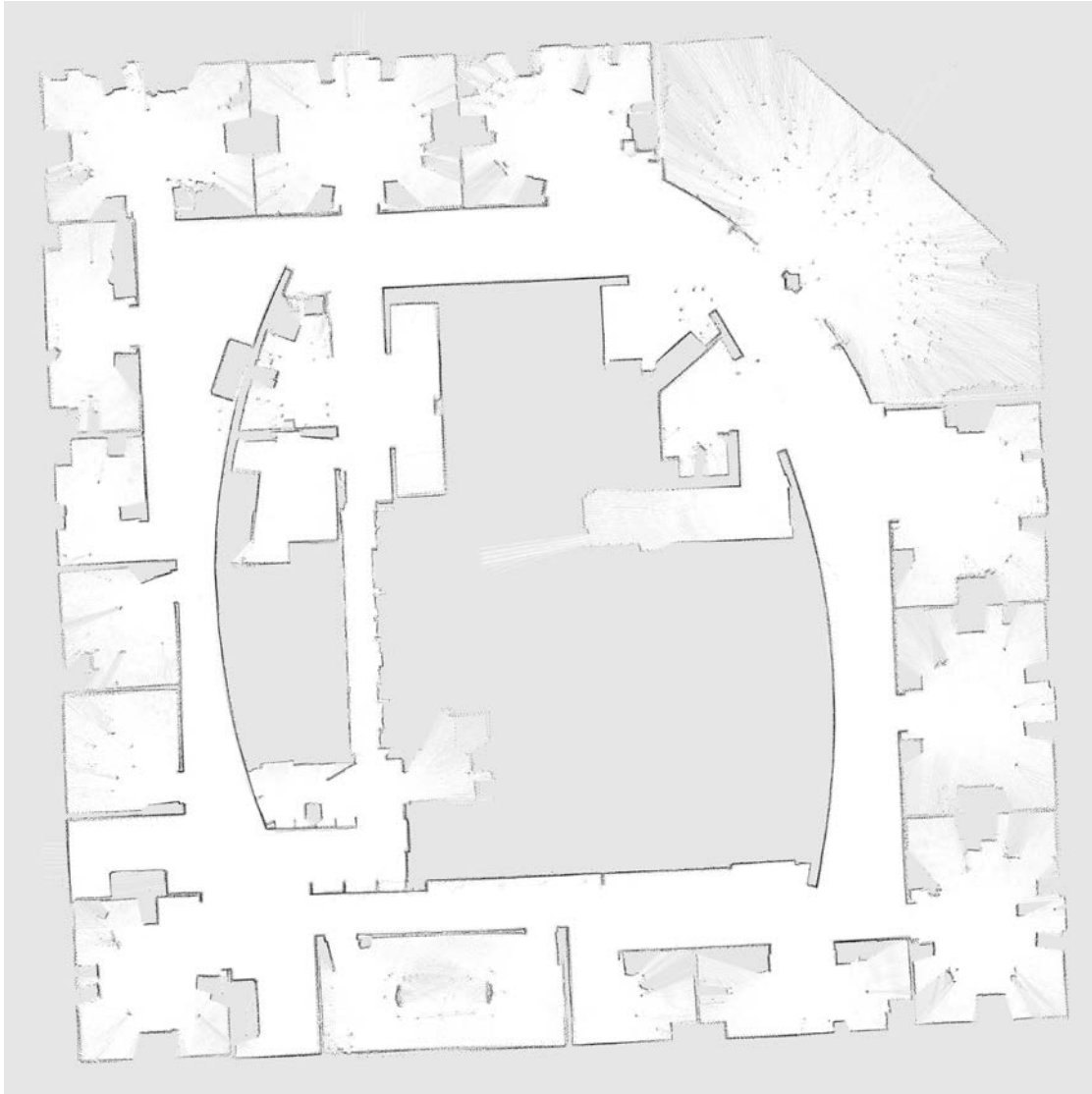
closing the first loop



visiting known areas

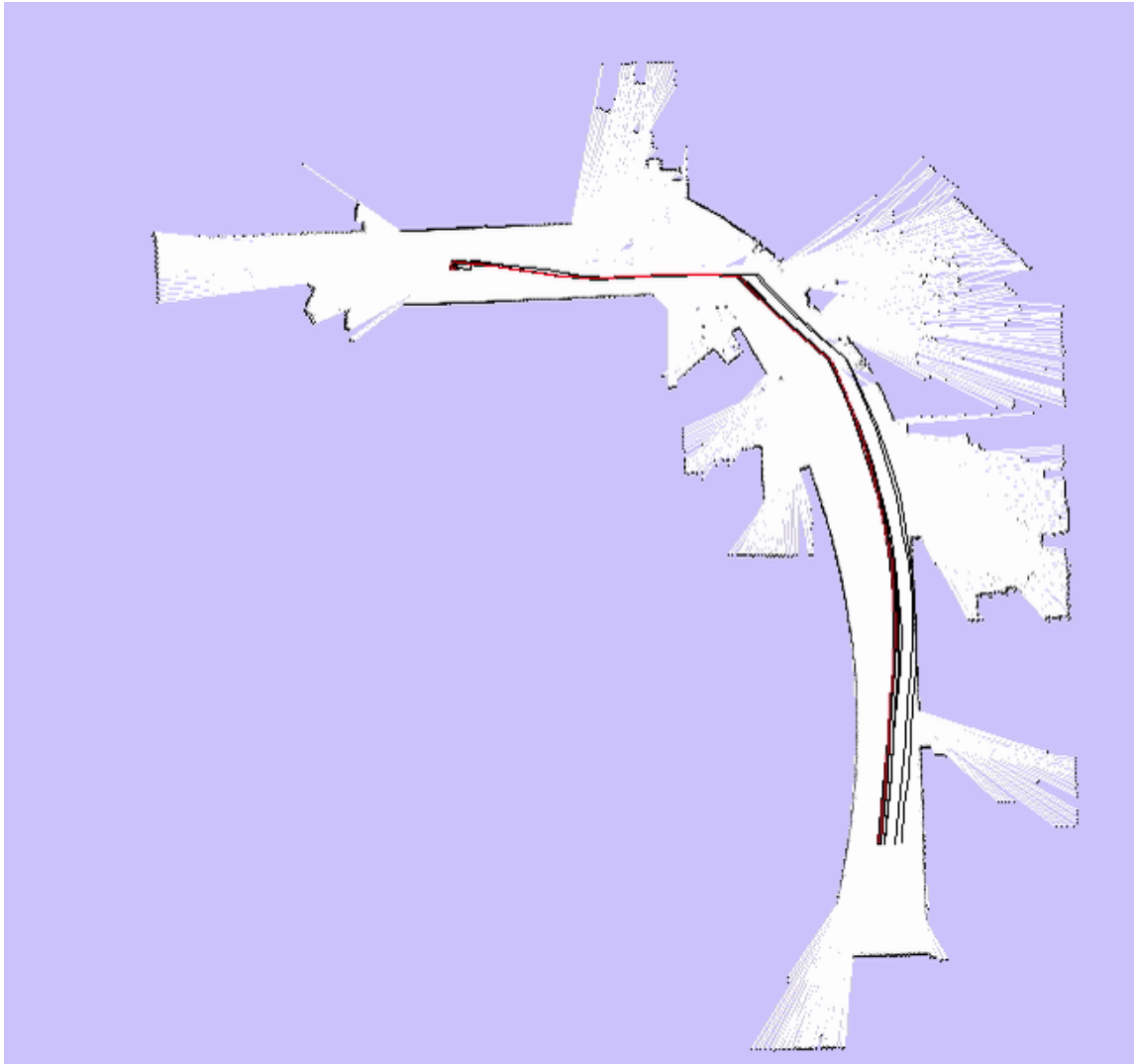
second loop closure

# Intel Lab



- **15 particles**
- four times faster than real-time P4, 2.8GHz
- 5cm resolution during scan matching
- 1cm resolution in final map

# Intel Lab



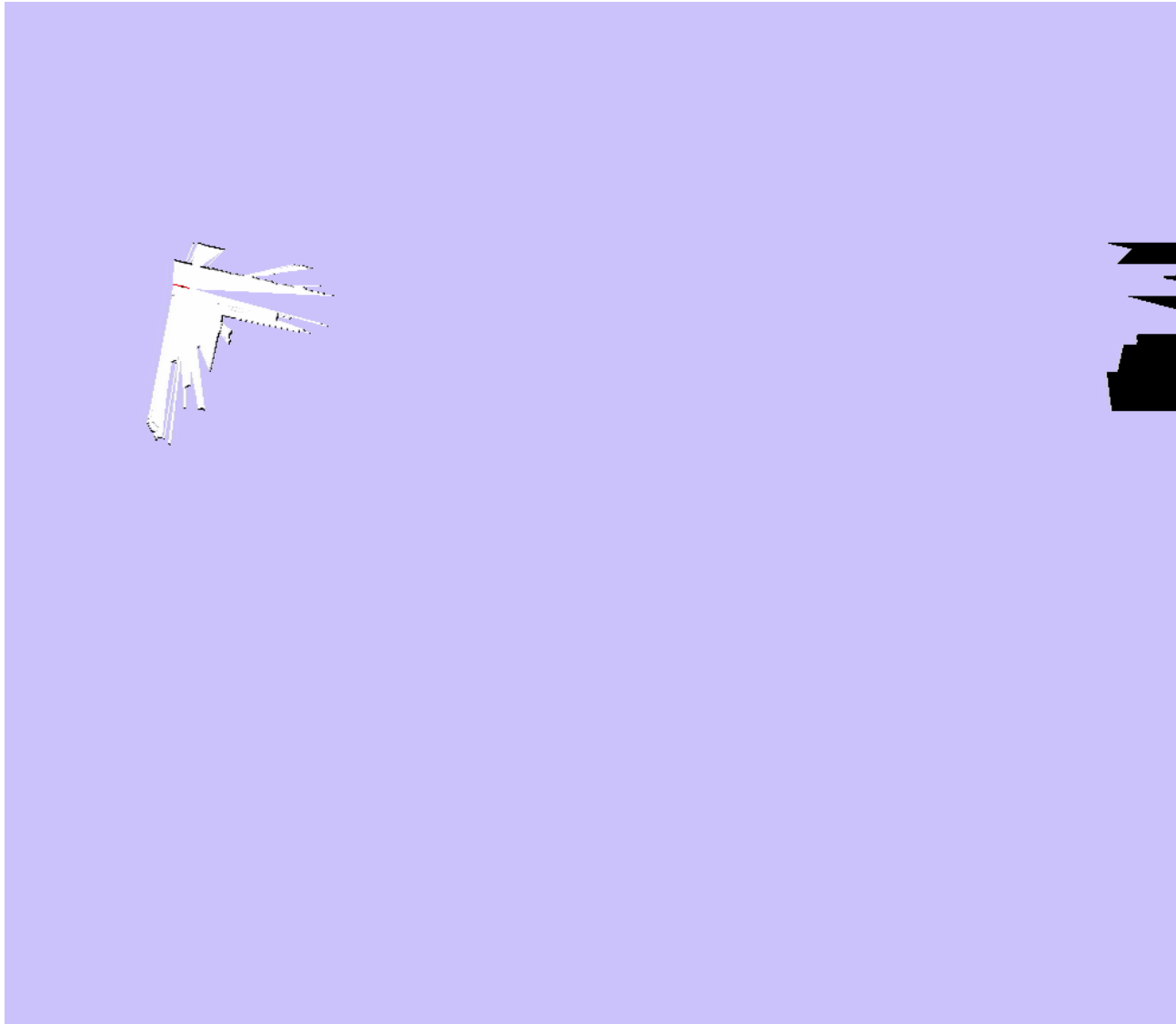
- **15 particles**
- Compared to FastSLAM with Scan-Matching, the particles are propagated closer to the true distribution

# Outdoor Campus Map



- **30 particles**
- 250x250m<sup>2</sup>
- 1.75 km (odometry)
- 20cm resolution during scan matching
- 30cm resolution in final map

# Outdoor Campus Map - Video



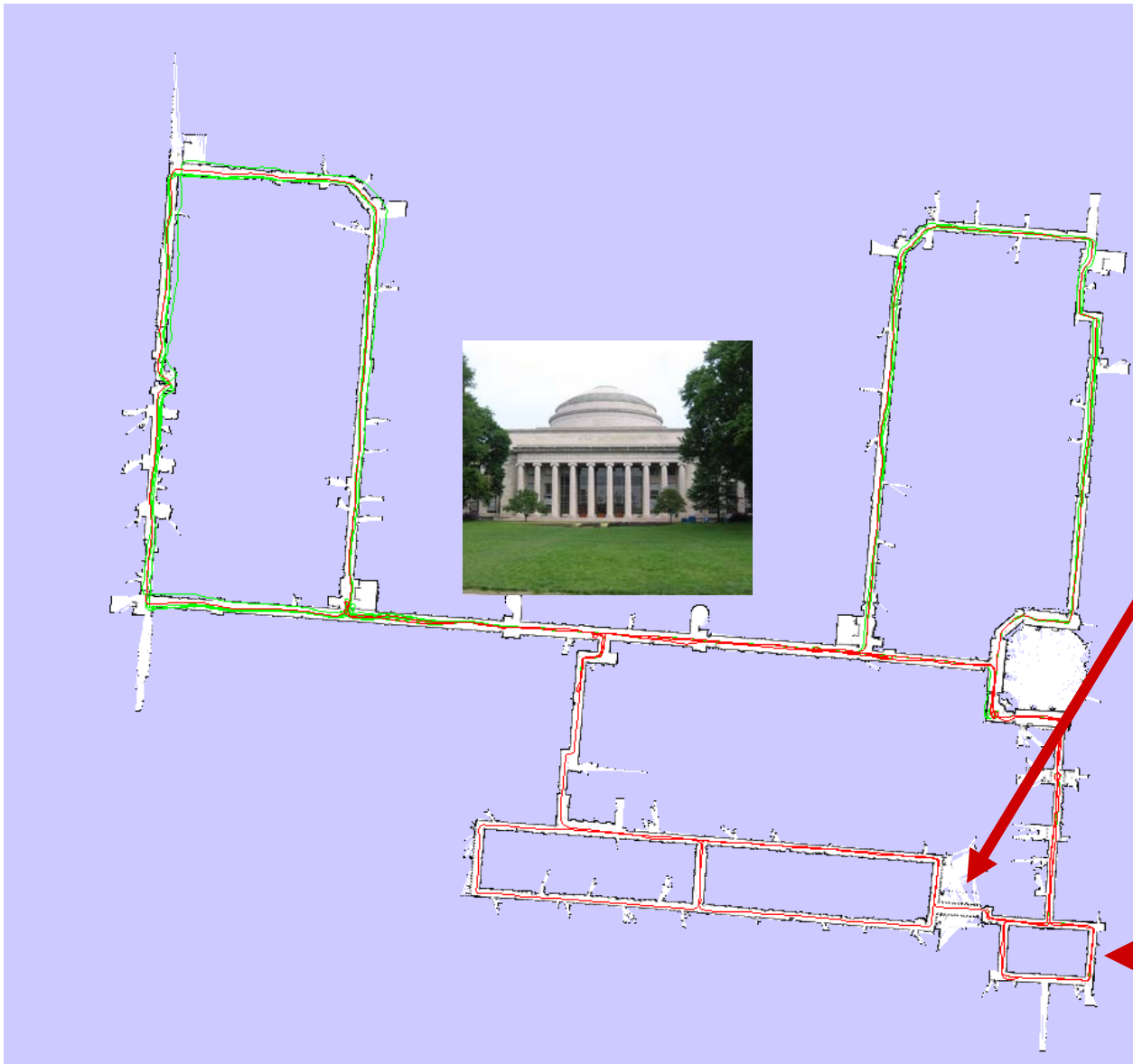


# MIT Killian Court

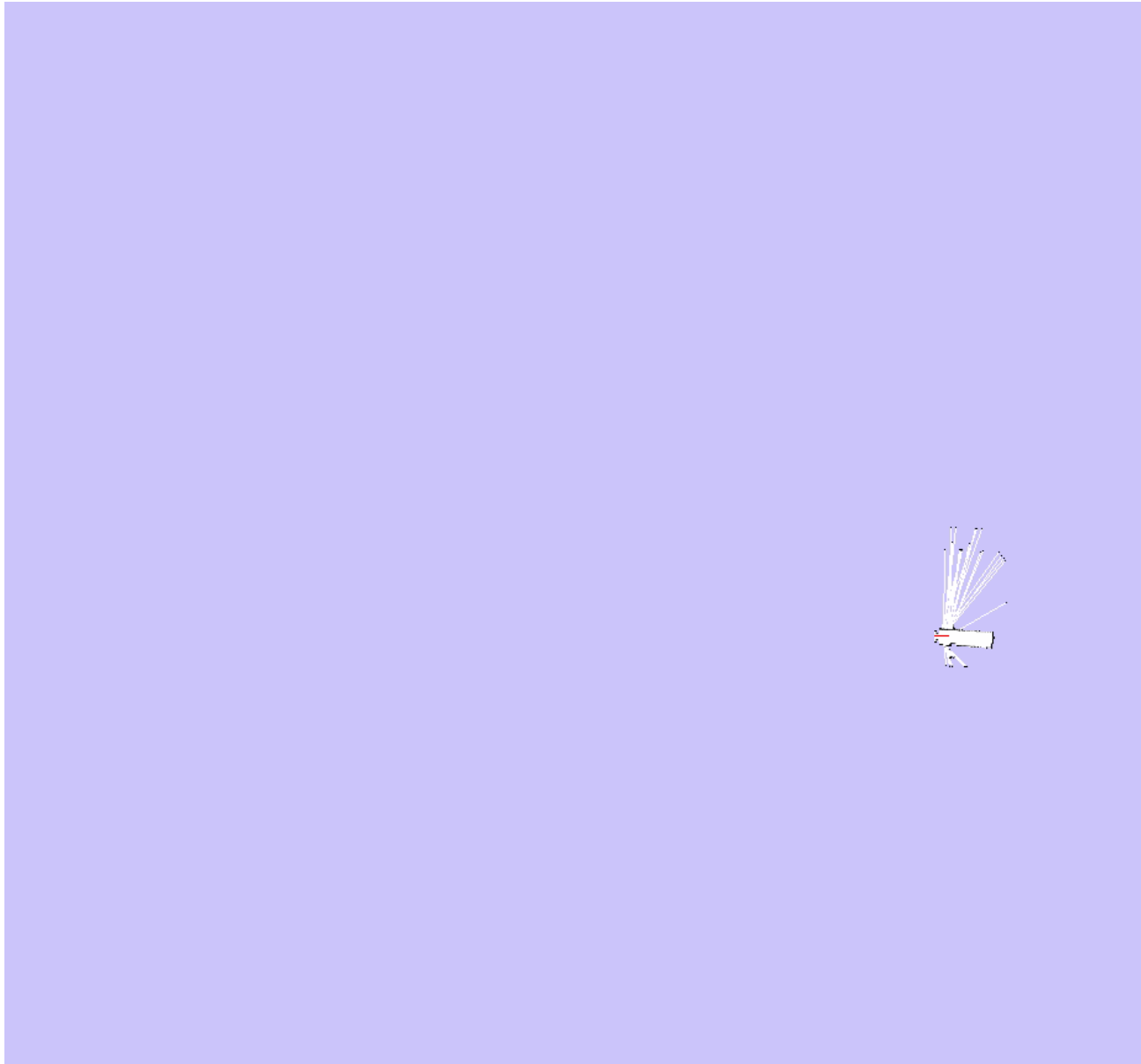


- The **“infinite-corridor-dataset”** at MIT

# MIT Killian Court



# MIT Killian Court - Video



# Conclusion

- The ideas of FastSLAM can also be applied in the context of grid maps
- Utilizing accurate sensor observation leads to good proposals and highly efficient filters
- It is similar to scan-matching on a per-particle base
- The number of necessary particles and resampling steps can seriously be reduced
- Improved versions of grid-based FastSLAM can handle larger environments than naïve implementations in “real time” since they need one order of magnitude fewer samples

# More Details on FastSLAM

- M. Montemerlo, S. Thrun, D. Koller, and B. Wegbreit. FastSLAM: A factored solution to simultaneous localization and mapping, *AAAI02 (The classic FastSLAM paper with landmarks)*
- D. Haehnel, W. Burgard, D. Fox, and S. Thrun. An efficient FastSLAM algorithm for generating maps of large-scale cyclic environments from raw laser range measurements, *IROS03 (FastSLAM on grid-maps using scan-matched input)*
- G. Grisetti, C. Stachniss, and W. Burgard. Improving grid-based SLAM with Rao-Blackwellized particle filters by adaptive proposals and selective resampling, *ICRA05 (Proposal using laser observation, adaptive resampling)*
- A. Eliazar and R. Parr. DP-SLAM: Fast, robust simultaneous localization and mapping without predetermined landmarks, *IJCAI03 (An approach to handle big particle sets)*