Introduction to Mobile Robotics

Wheeled Locomotion

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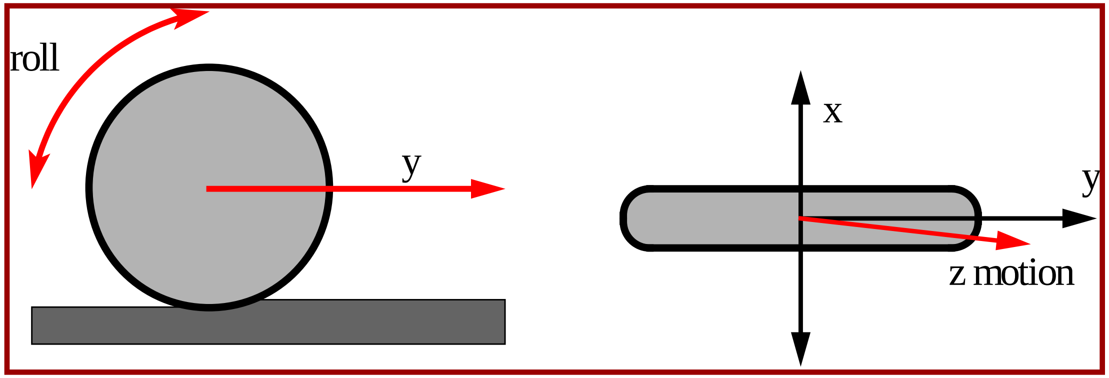
Locomotion of Wheeled Robots

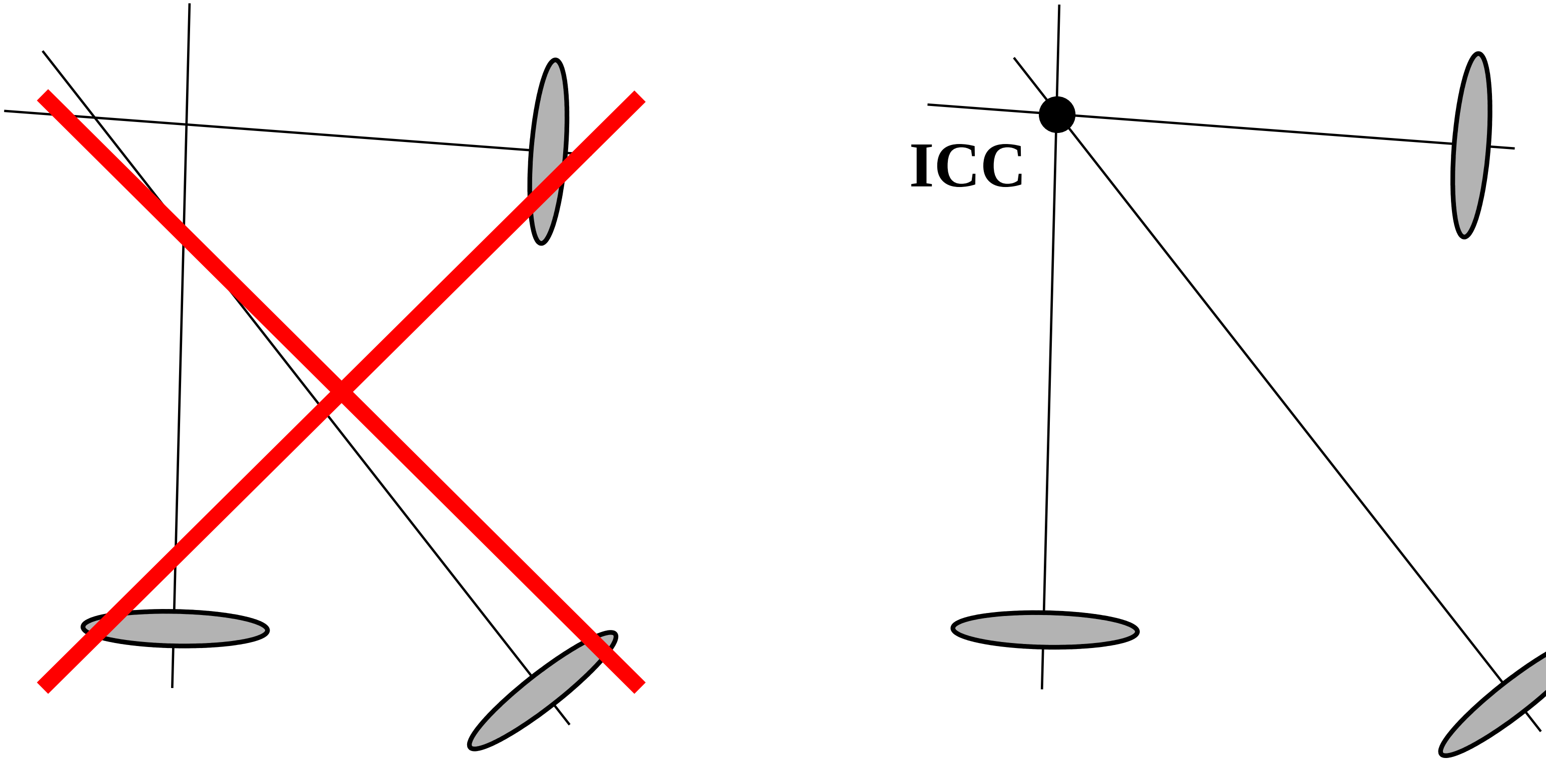
Locomotion (Oxford Dict.): Power of motion from place to place

- Differential drive (lawn mover, cleaning robots)
- Ackerman drive (cars)
- Synchronous drive
- XR4000
- Mecanum wheels



Wheels rotate around the x axis and possibly z axis

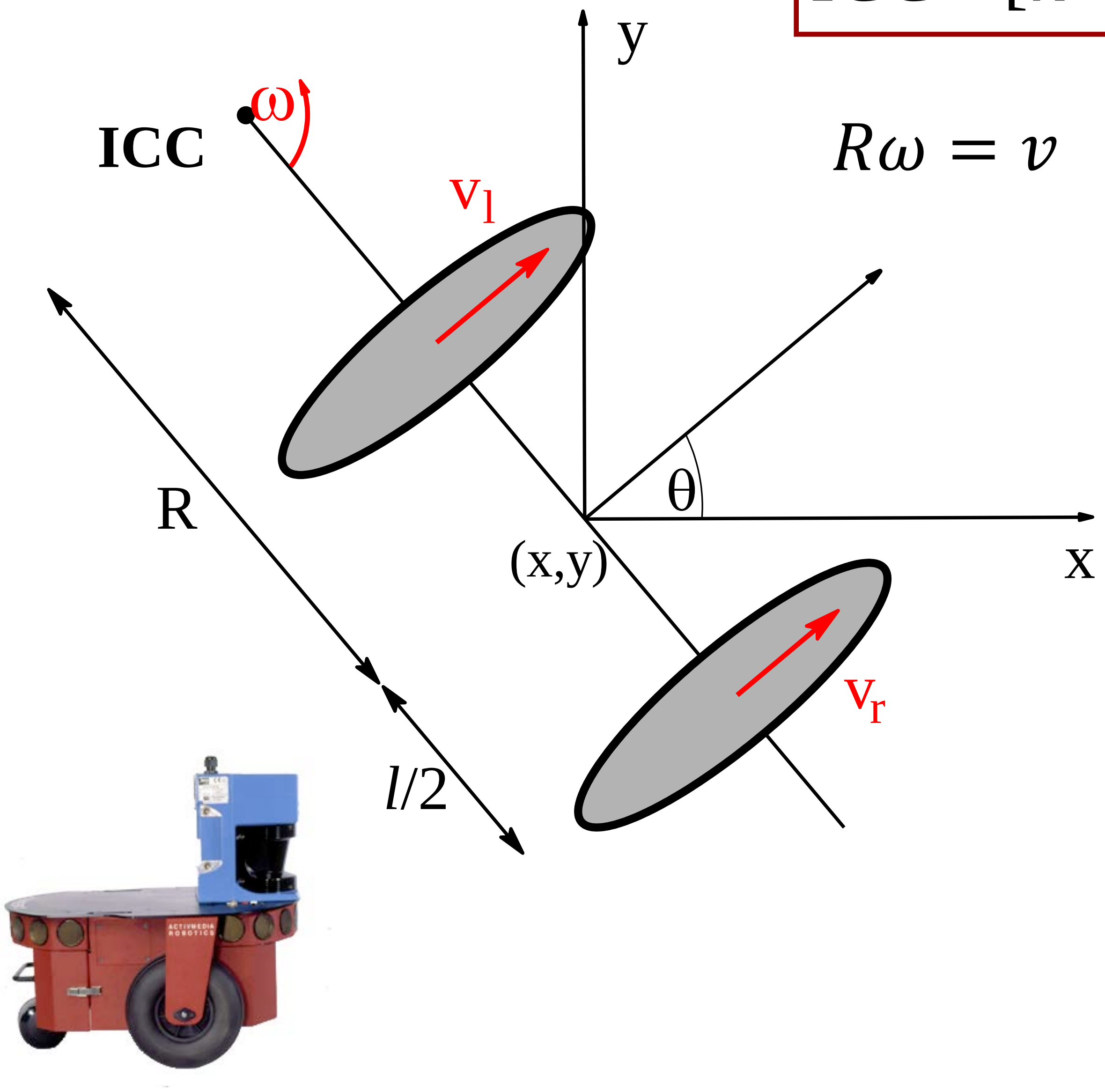
Instantaneous Center of Curvature



- For rolling motion: axis need to meet in one point

Differential Drive

$$ICC = [x - R\sin\theta, y + R\cos\theta]$$



$$R\omega = v$$

$$\omega(R + l/2) = v_r$$

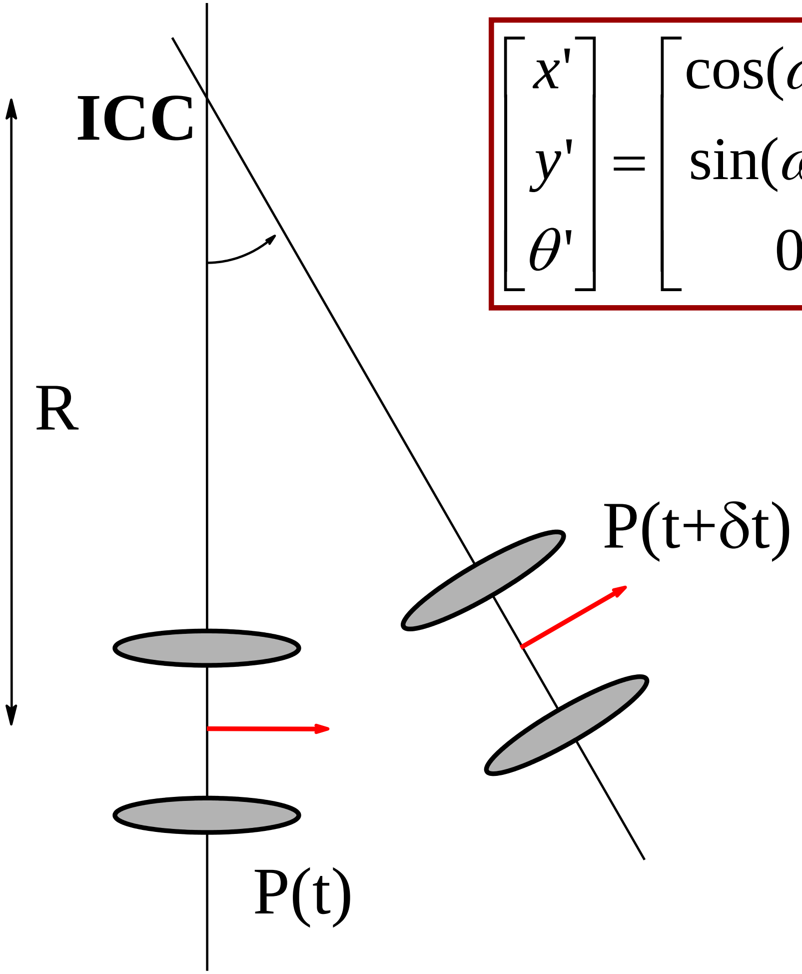
$$\omega(R - l/2) = v_l$$

$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

$$v = \frac{v_r + v_l}{2}$$

Differential Drive: Forward Kinematics



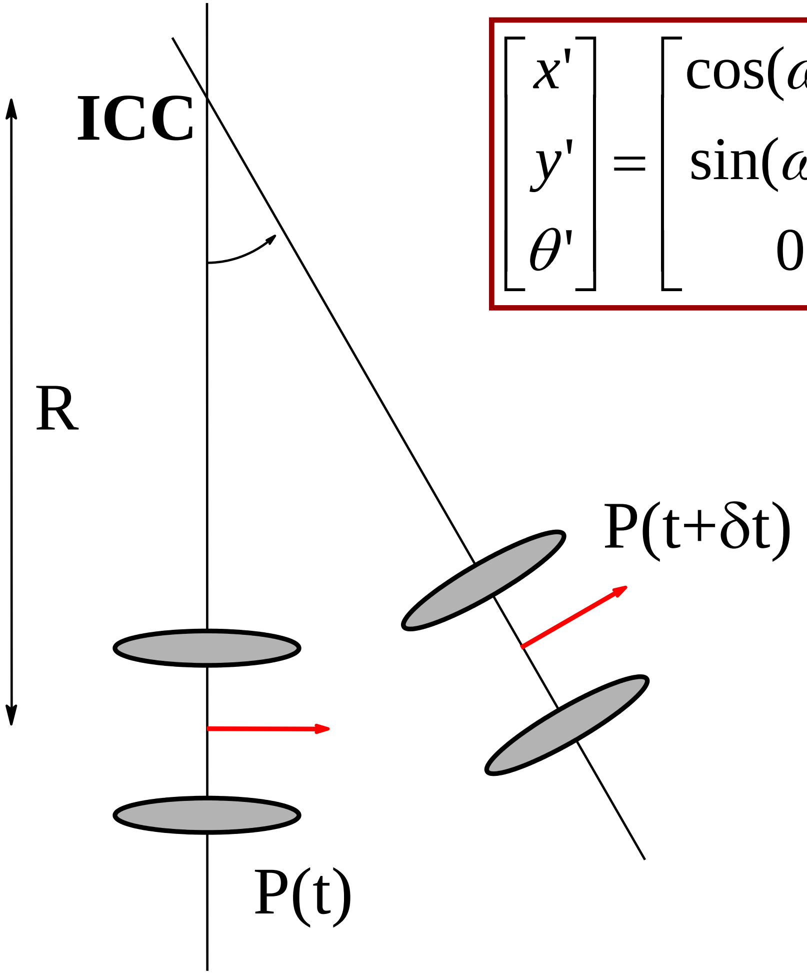
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \mathrm{ICC}_x \\ y - \mathrm{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \mathrm{ICC}_x \\ \mathrm{ICC}_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t v(t')\ \cos[\theta(t')]dt'$$

$$y(t) = \int_0^t v(t')\ \sin[\theta(t')]dt'$$

$$\theta(t) = \int_0^t \omega(t')\ dt'$$

Differential Drive: Forward Kinematics



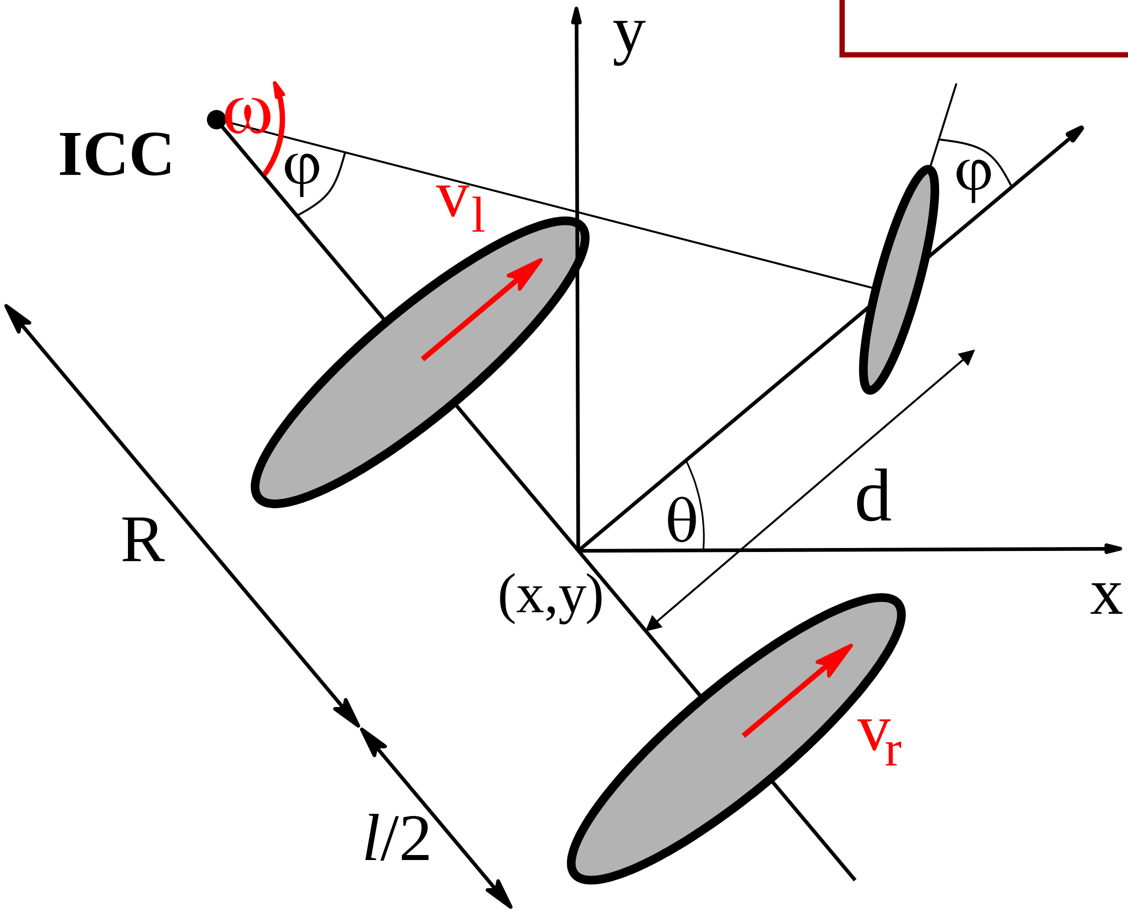
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \mathrm{ICC}_x \\ y - \mathrm{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \mathrm{ICC}_x \\ \mathrm{ICC}_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \frac{1}{2}\int_0^t [v_r(t') + v_l(t')]\ \cos[\theta(t')]dt'$$

$$y(t) = \frac{1}{2}\int_0^t [v_r(t') + v_l(t')]\ \sin[\theta(t')]dt'$$

$$\theta(t) = \frac{1}{l}\int_0^t [v_r(t') - v_l(t')]\, dt'$$

Ackermann Drive



$$ICC = [x - R\sin\theta, y + R\cos\theta]$$

$$R = \frac{d}{\tan\varphi}$$

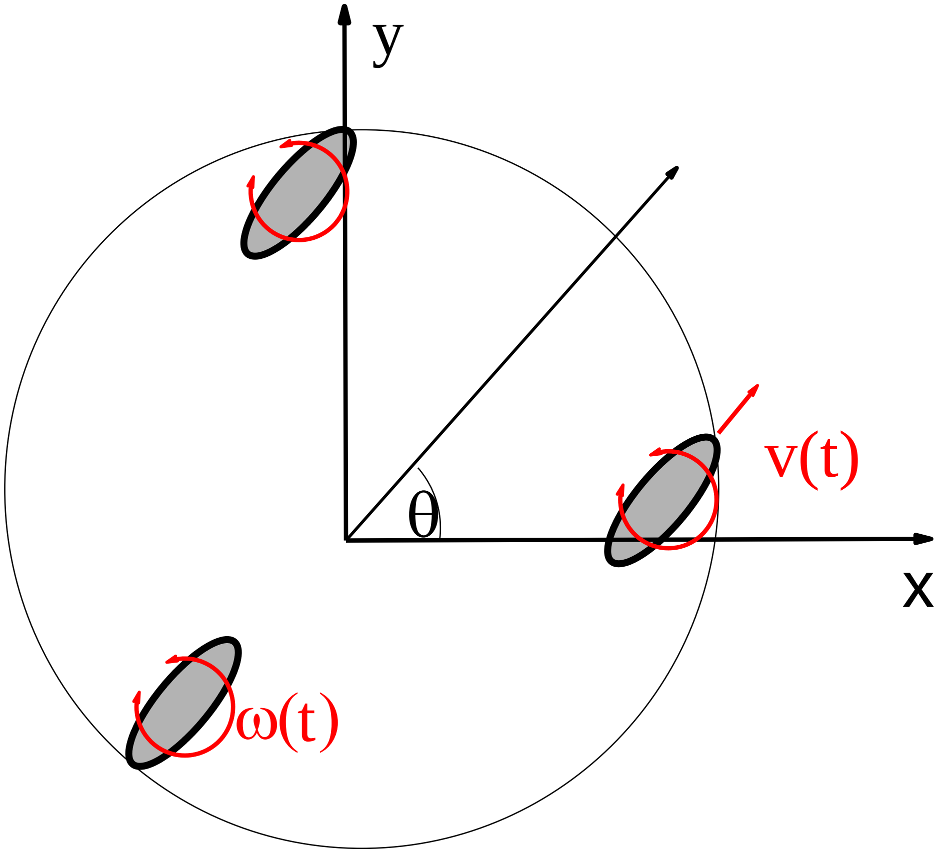
$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

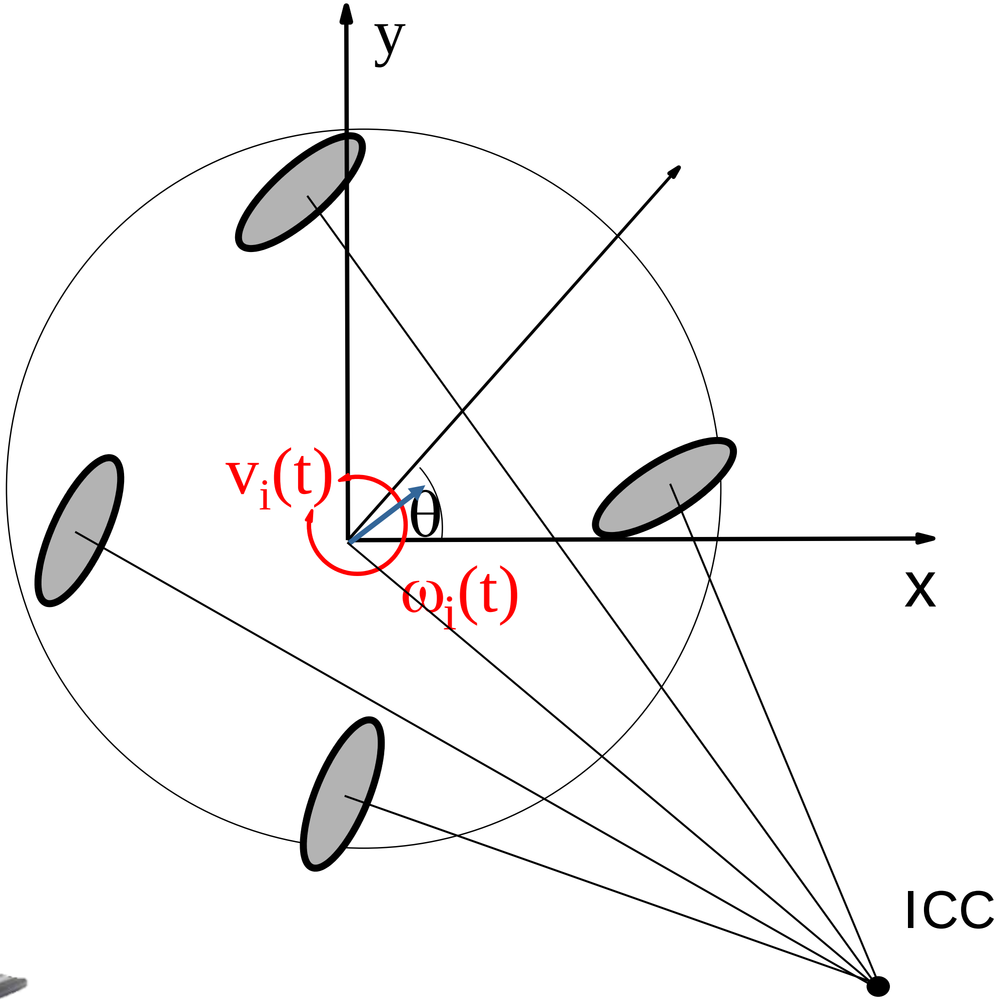
$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

Synchronous Drive



XR4000 Drive



Mecanum Wheels

$$v_y = (v_0 + v_1 + v_2 + v_3)/4$$

$$v_x = (v_0 - v_1 + v_2 - v_3)/4$$

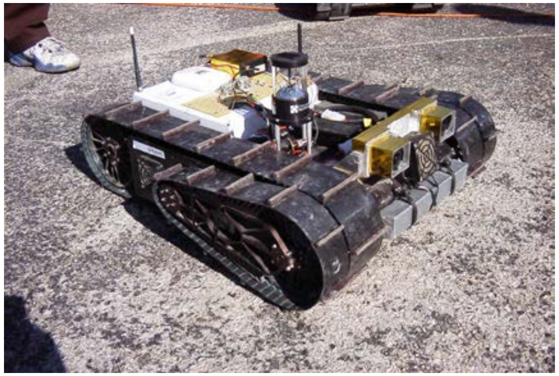
$$v_\theta = (v_0 + v_1 - v_2 - v_3)/4$$

$$v_{error} = (v_0 - v_1 - v_2 + v_3)/4$$

Example: KUKA youBot

Example: KUKA OmniRob

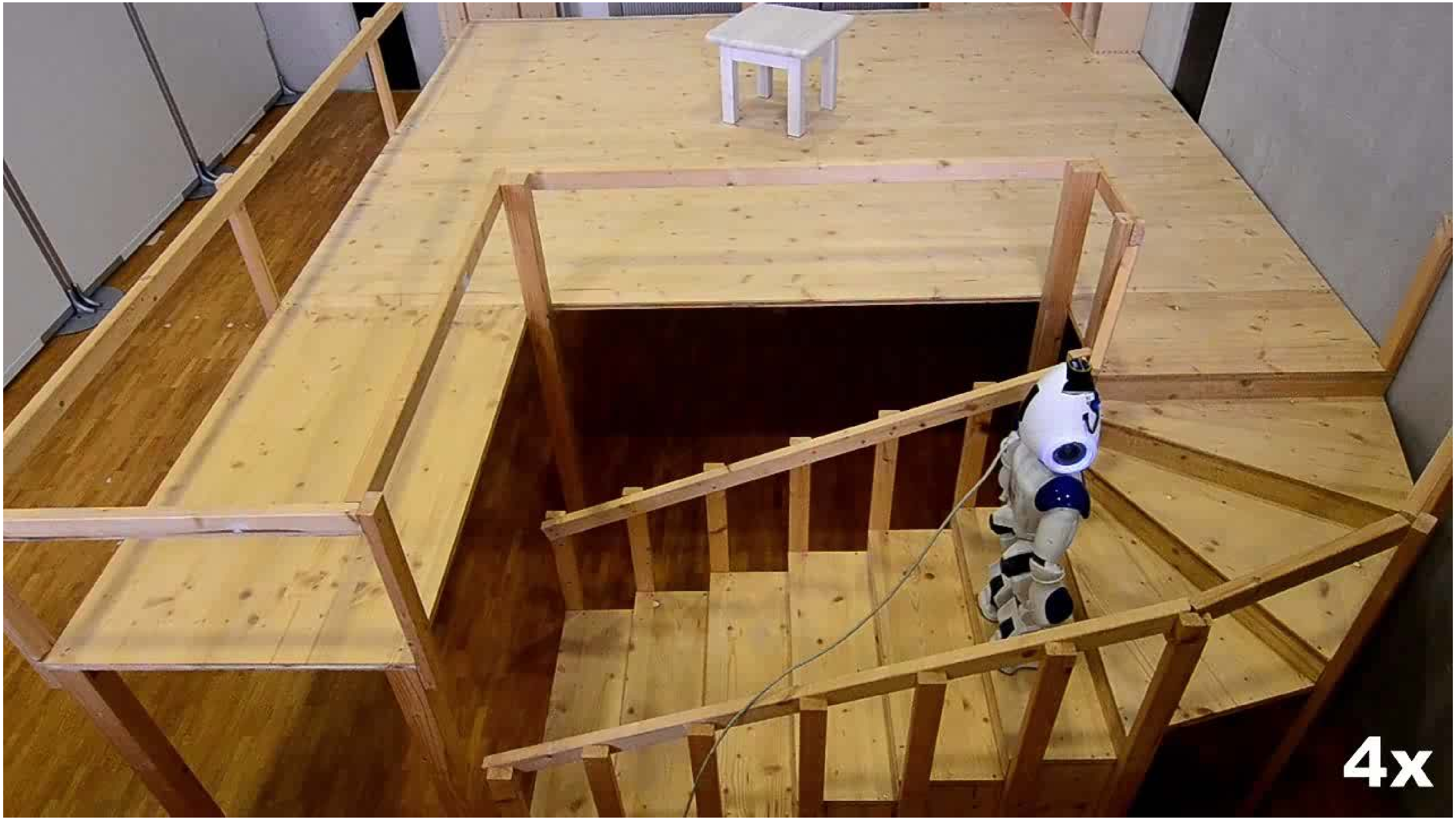
Tracked Vehicles



Other Robots: OmniTread

[courtesy by Johann Borenstein]

Other Robots: Humanoids



Holonomic and Non-Holonomic Constraints

- Holonomic: reduced configuration space
  - E.g., a train on tracks: not all positions and orientations on the plane are possible
- Non-holonomic: reduced control space
  - Limits of the possible incremental movements within the configuration space of the robot
  - E.g., a robot on a plane is not able to move sideways

Drives with Non-Holonomic Constraints

Limited to circular trajectories:

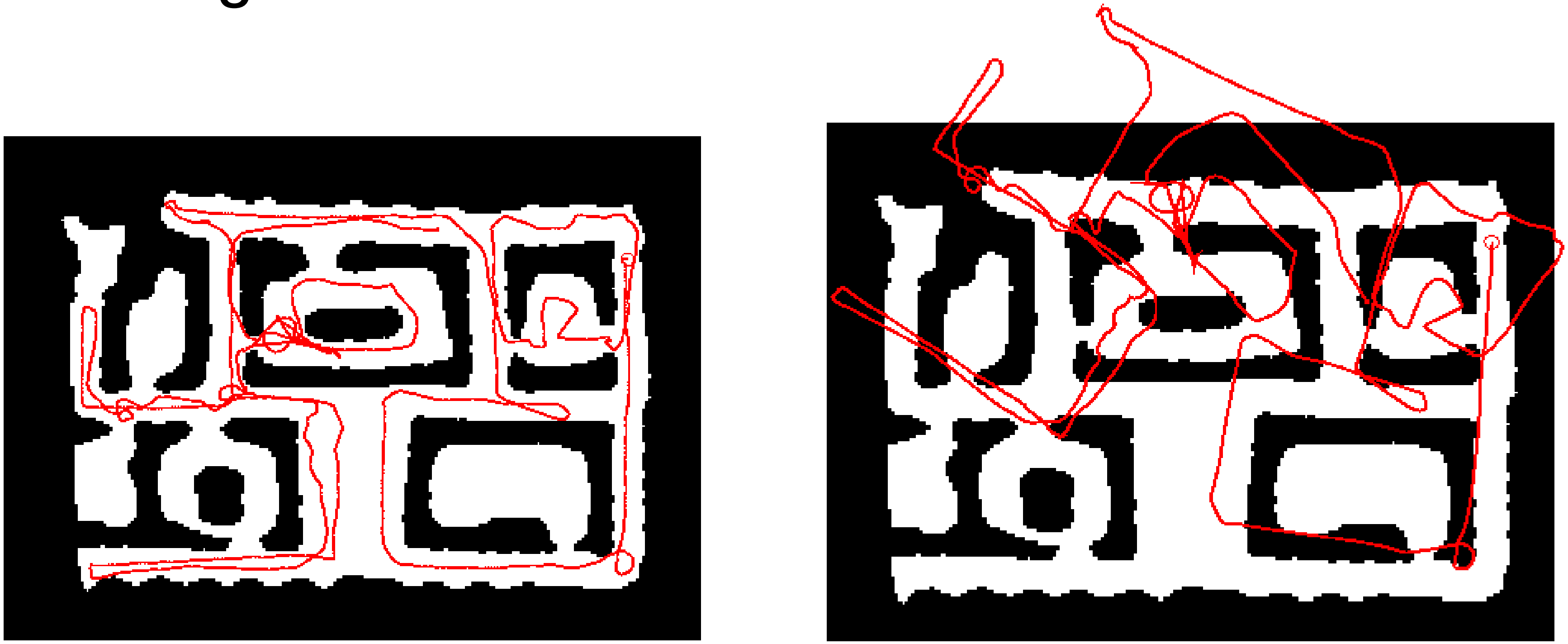
- Differential drive
- Ackermann drive
- Synchro-drive

Drives without Non-Holonomic Constraints

- Mecanum wheels

Dead Reckoning and Odometry

- Estimating the motion based on the issued controls/wheel encoder readings
- Integrated over time



Summary

- Introduced different types of drives for wheeled robots
- Math to describe the motion of the basic drives given the speed of the wheels
- Non-holonomic constraints
- Odometry and dead reckoning