Introduction to Mobile Robotics

SLAM – Landmark-based FastSLAM

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Partial slide courtesy of Mike Montemerlo

The SLAM Problem

- SLAM stands for simultaneous localization and mapping
- The task of building a map while estimating the pose of the robot relative to this map

- Why is SLAM hard?
  Chicken-or-egg problem:
  - A map is needed to localize the robot
  - A pose estimate is needed to build a map

The SLAM Problem

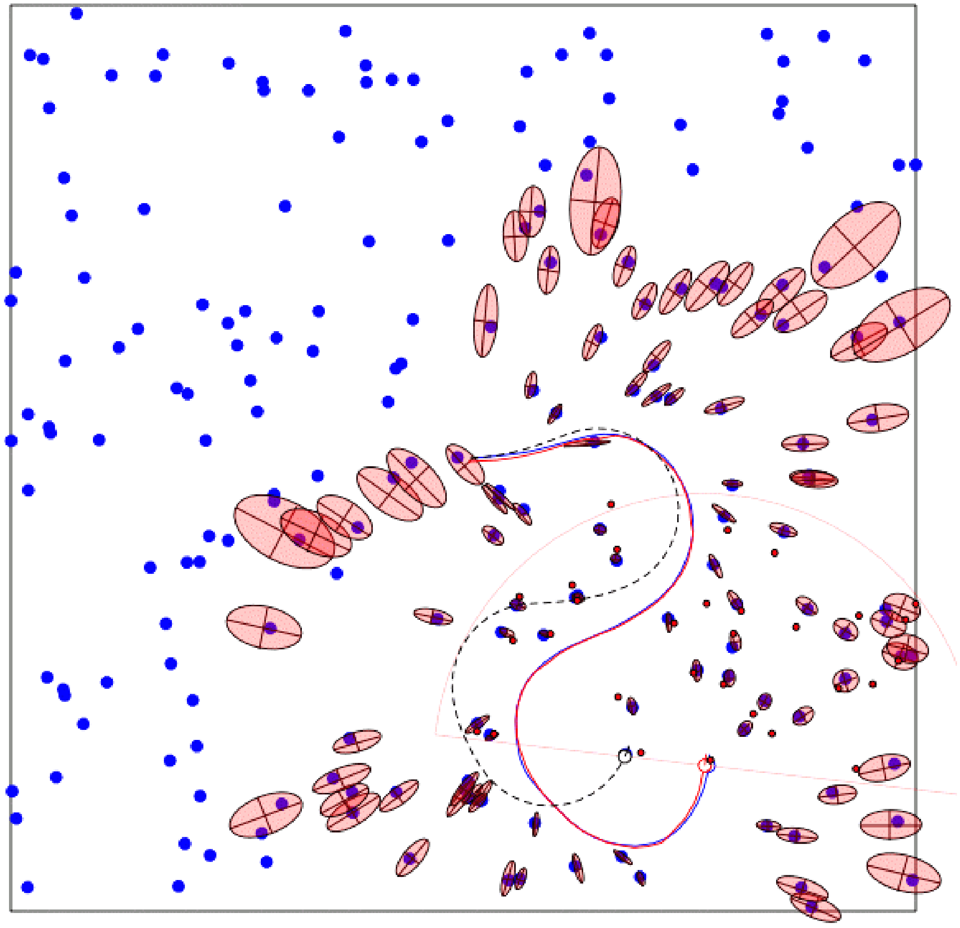
A robot moving through an unknown, static environment

Given:

- The robot's controls
- Observations of nearby features

Estimate:

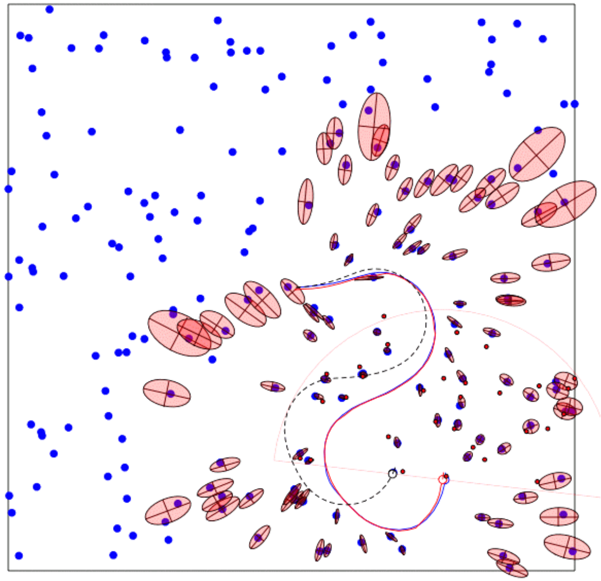
- Map of features
- Path of the robot



Map Representations

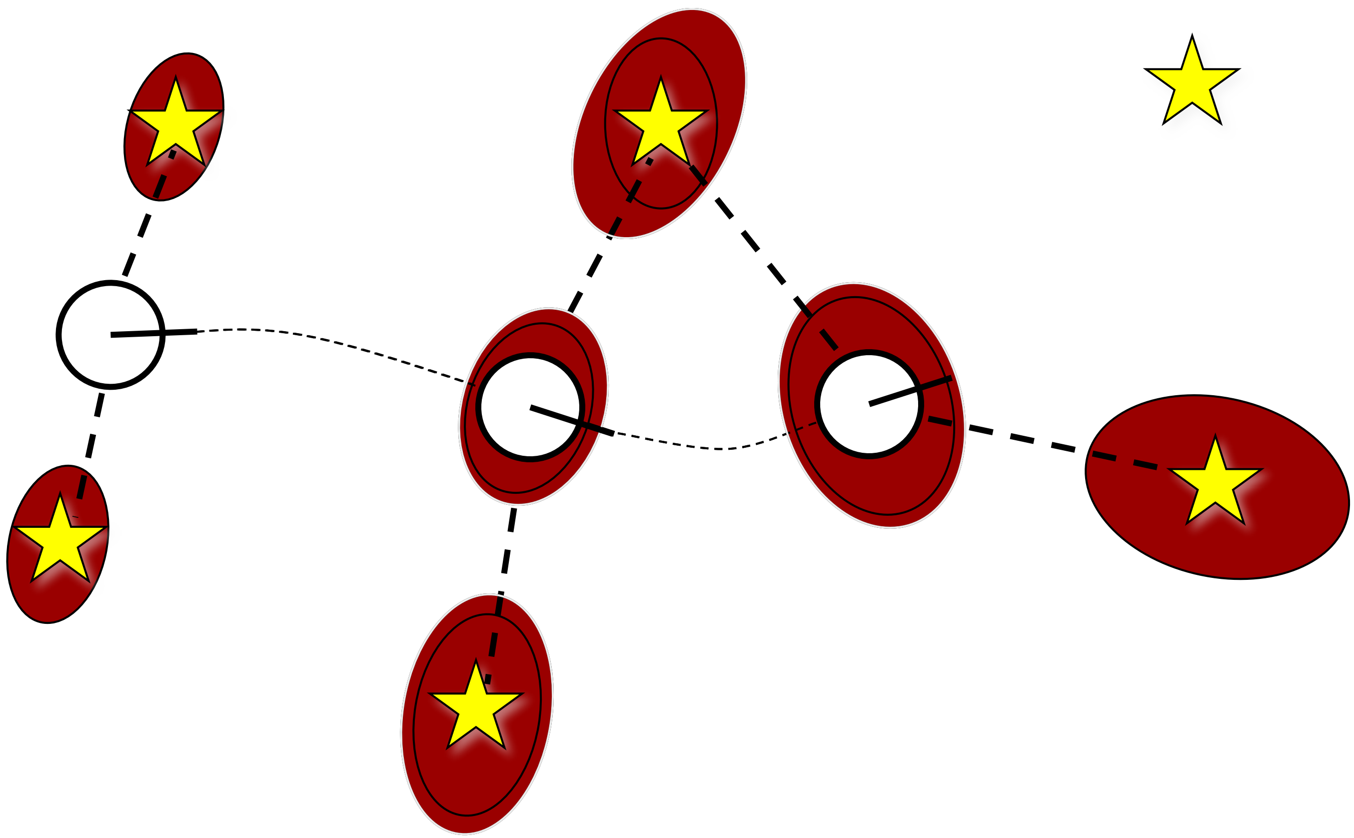
Typical models are:

- Feature maps
- Grid maps (occupancy or reflection probability maps)



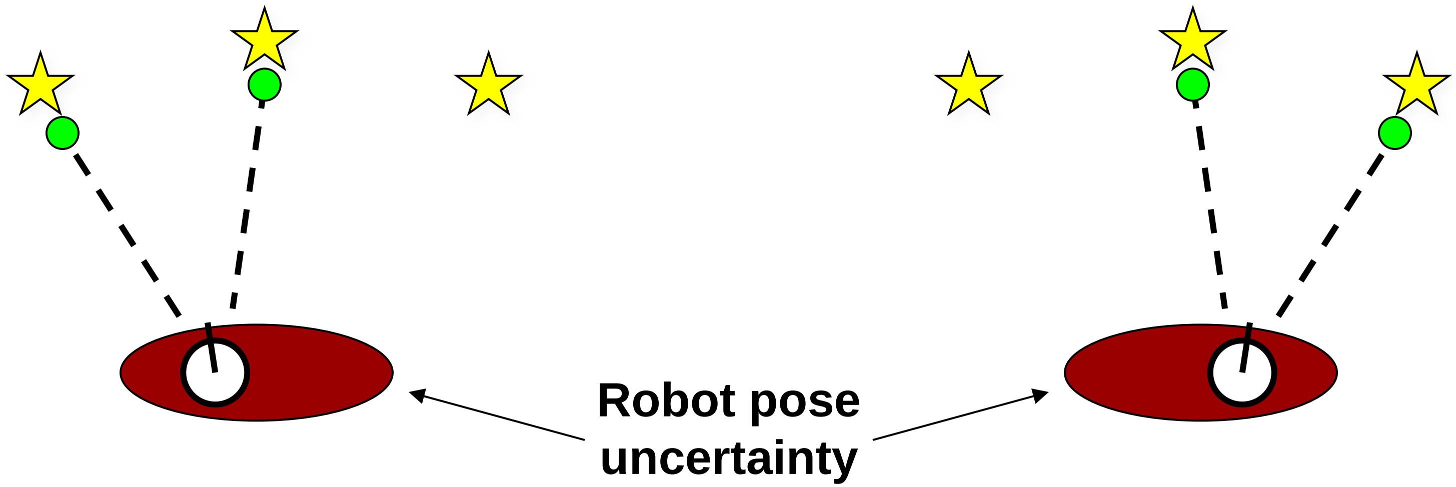
Why is SLAM a Hard Problem?

SLAM: robot path and map are both **unknown!**



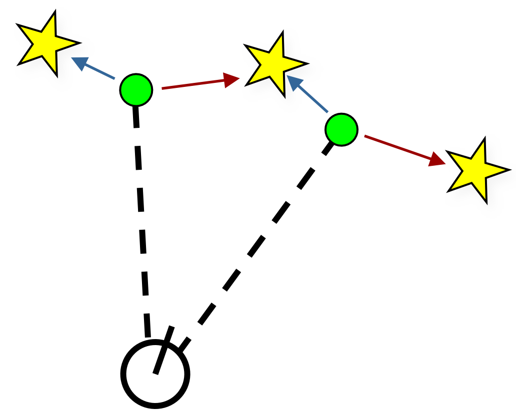
Robot path error correlates errors in the map

Why is SLAM a Hard Problem?



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations

Data Association Problem



- A data association is an assignment of observations to landmarks
- In general there are more than $\binom{n}{m}$ (n observations, m landmarks) possible associations
- Also called "assignment problem"

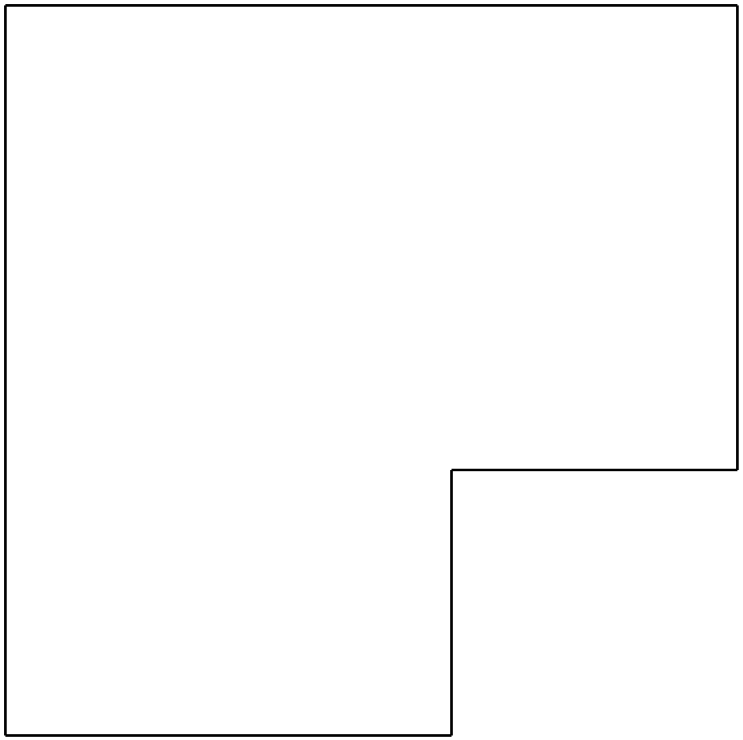
Particle Filters

- Represent belief by random samples
- Estimation of non-Gaussian, nonlinear processes

- Sampling Importance Resampling (SIR) principle
  - Draw the new generation of particles
  - Assign an importance weight to each particle
  - Resample

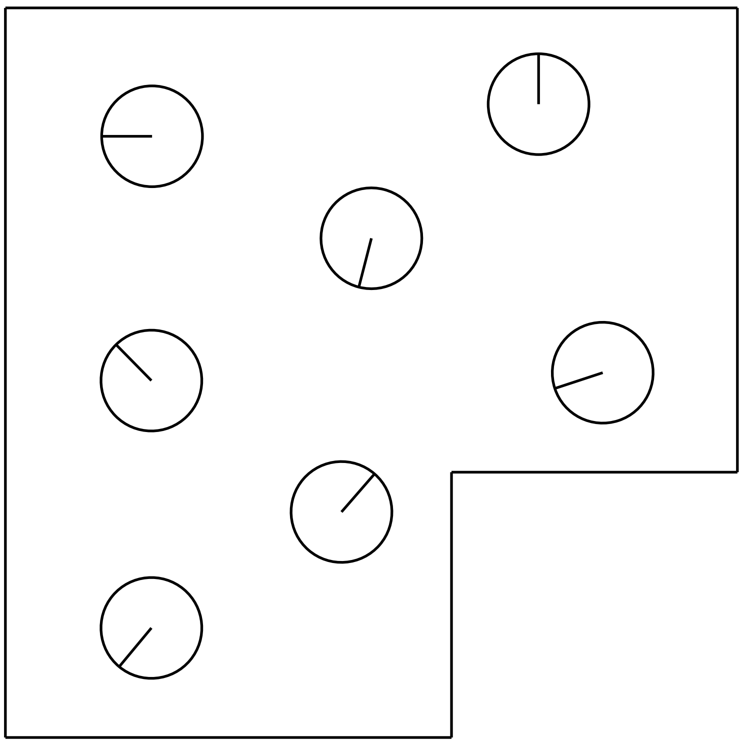
- Typical application scenarios are tracking, localization, ...

Recap: Particle Filter Localization



1. initialize particles
2. apply motion model
3. weight particles (sensor model)
4. resample according to weight

Recap: Particle Filter Localization



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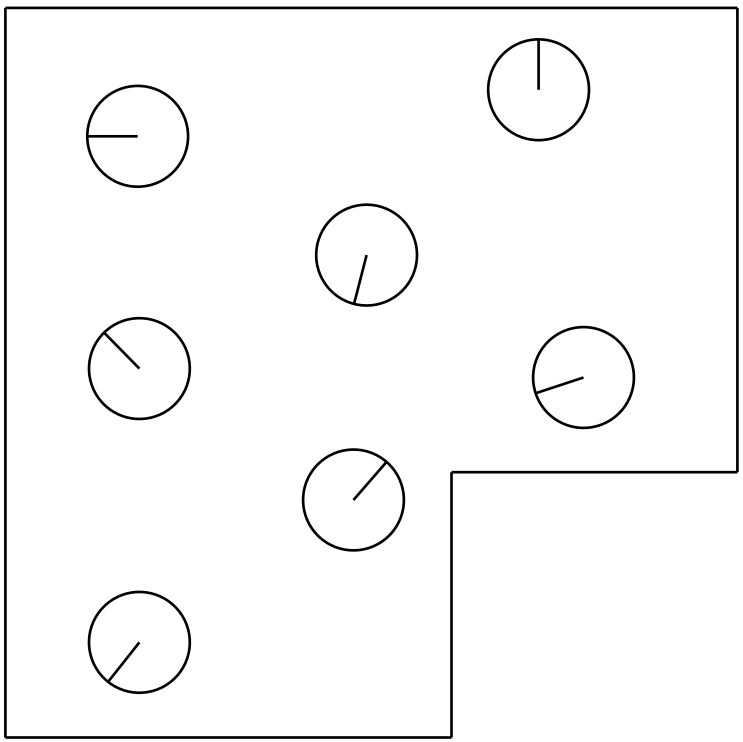

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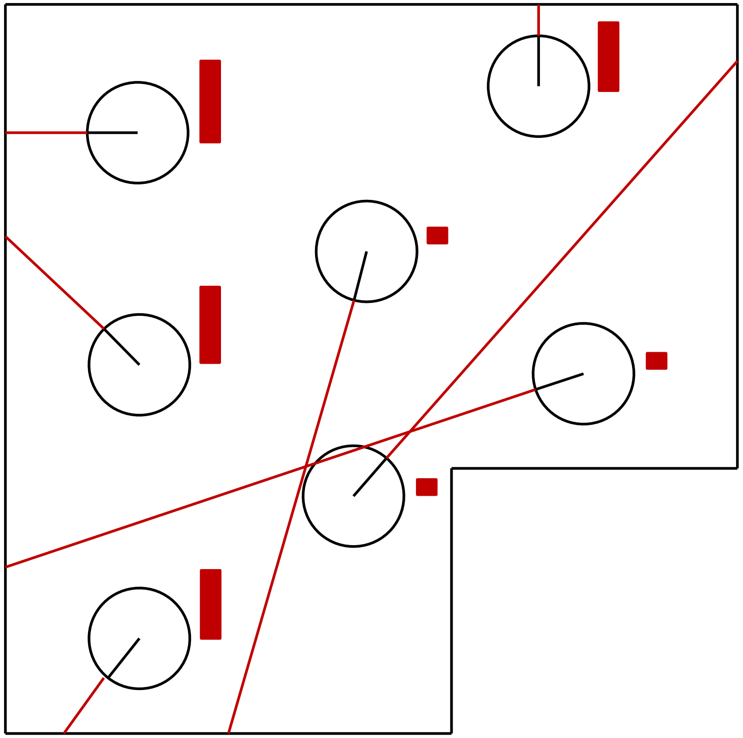
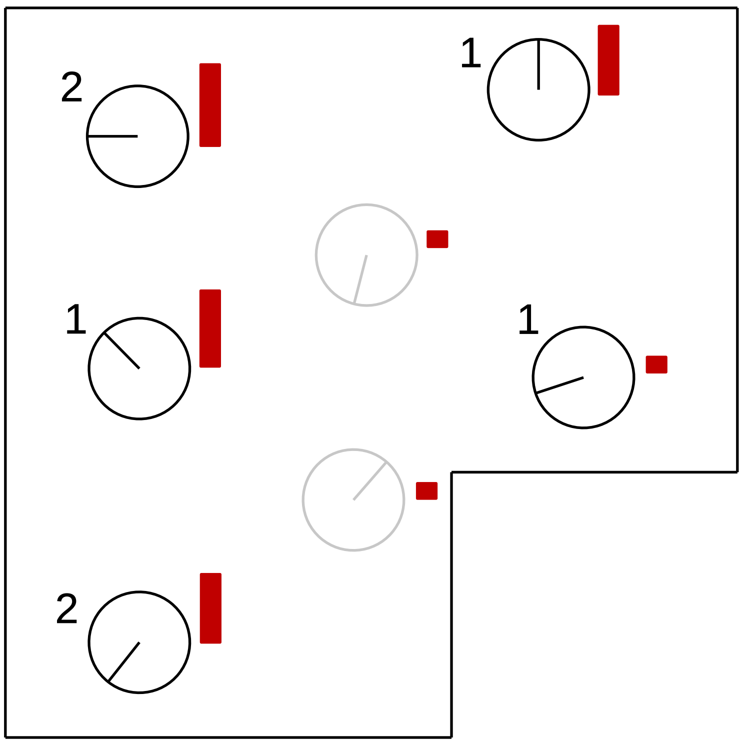
Recap: Particle Filter Localization



Actual measurement:

1. initialize particles
2. apply motion model
3. weight particles (sensor model)
4. resample according to weight

Recap: Particle Filter Localization



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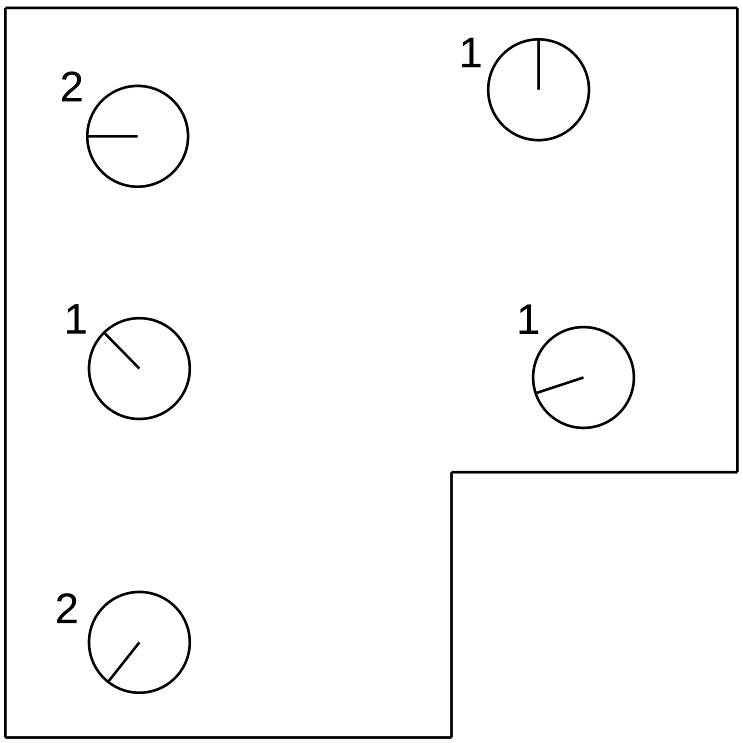
Recap: Particle Filter Localization



1. initialize particles
2. apply motion model
3. weight particles (sensor model)
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Localization vs. SLAM

- A particle filter can be used to solve both problems
- Localization: state space $<x, y, \theta>$
- SLAM: state space $<x, y, \theta, map>$
  - for landmark maps = $<l_1, l_2, \ldots, l_m>$
  - for grid maps = $<c_{11}, c_{12}, \ldots, c_{1n}, c_{21}, \ldots, c_{nm}>$

- **Problem:** The number of particles needed to represent a posterior grows exponentially with the dimension of the state space!

Dependencies

- Is there a dependency between certain dimensions of the state space?
- If so, can we use the dependency to solve the problem more efficiently?
- In the SLAM context
  - The map depends on the poses of the robot.
  - We know how to build a map given the position of the sensor is known.

Rao-Blackwellization

- Factorization to exploit dependencies between variables:

$$p(a, b) \;=\; p(a) \cdot p(b \mid a)$$

- If $p(b \mid a)$ can be computed in closed form, represent only $p(a)$ with samples and compute $p(b \mid a)$ for every sample

- It comes from the Rao-Blackwell theorem

Factored Posterior (Landmarks)

poses map observations & movements

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$

Factorization first introduced by Murphy in 1999

Factored Posterior (Landmarks)

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Factored Posterior (Landmarks)

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$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$

$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(l_{1:m} \mid x_{1:t}, z_{1:t})$$

SLAM posterior

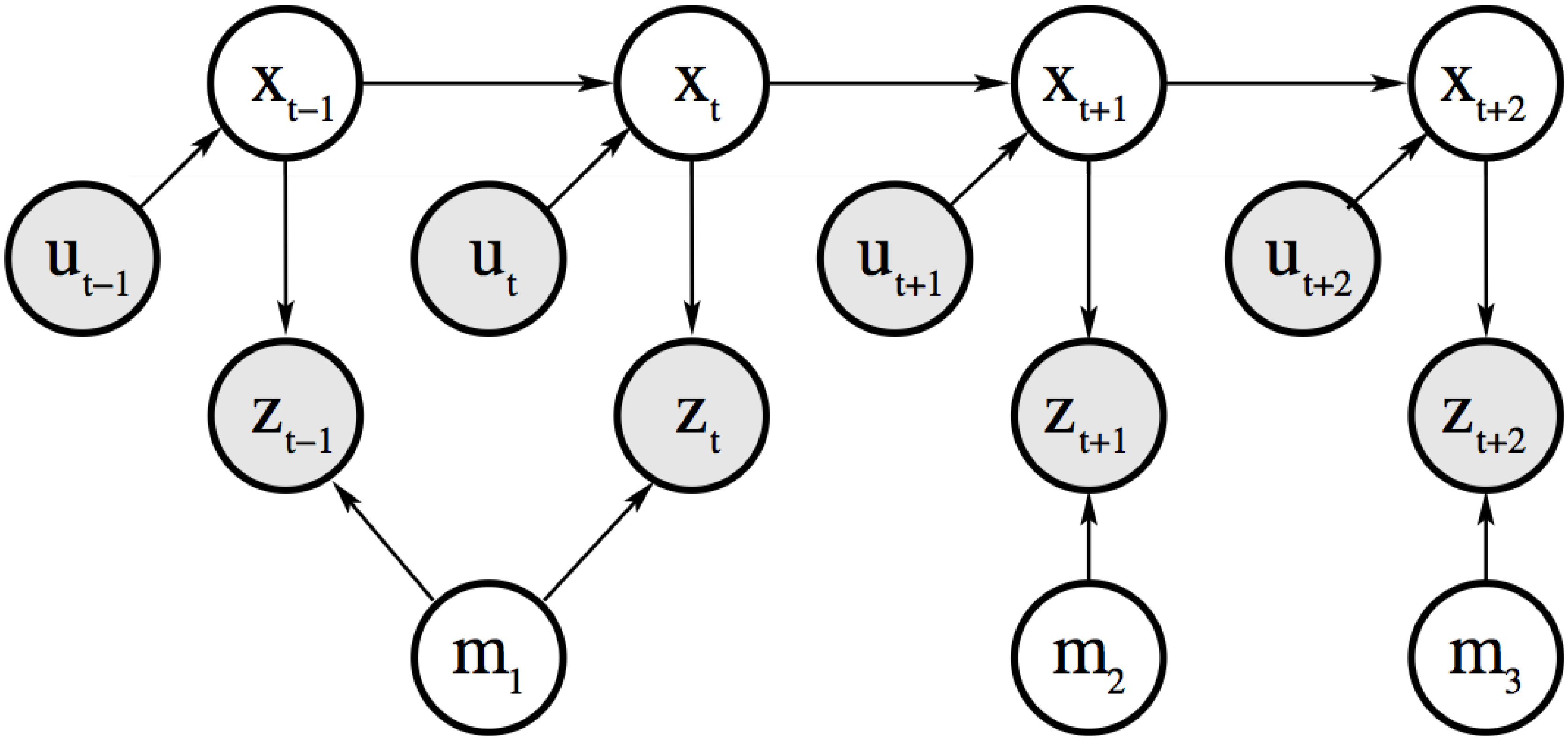
Robot path posterior

landmark positions

Does this help to solve the problem?

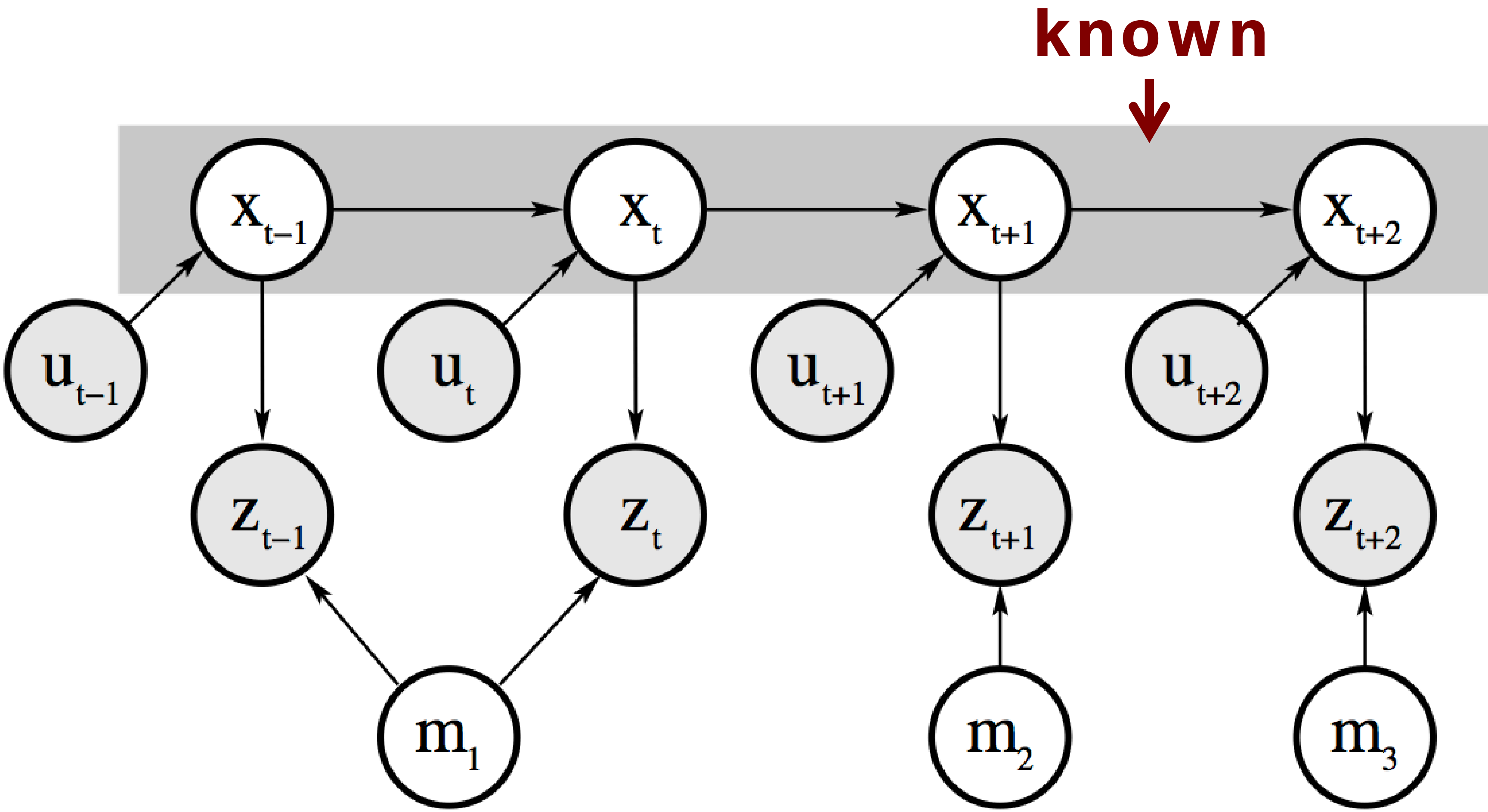
Factorization first introduced by Murphy in 1999

Revisit the Graphical Model



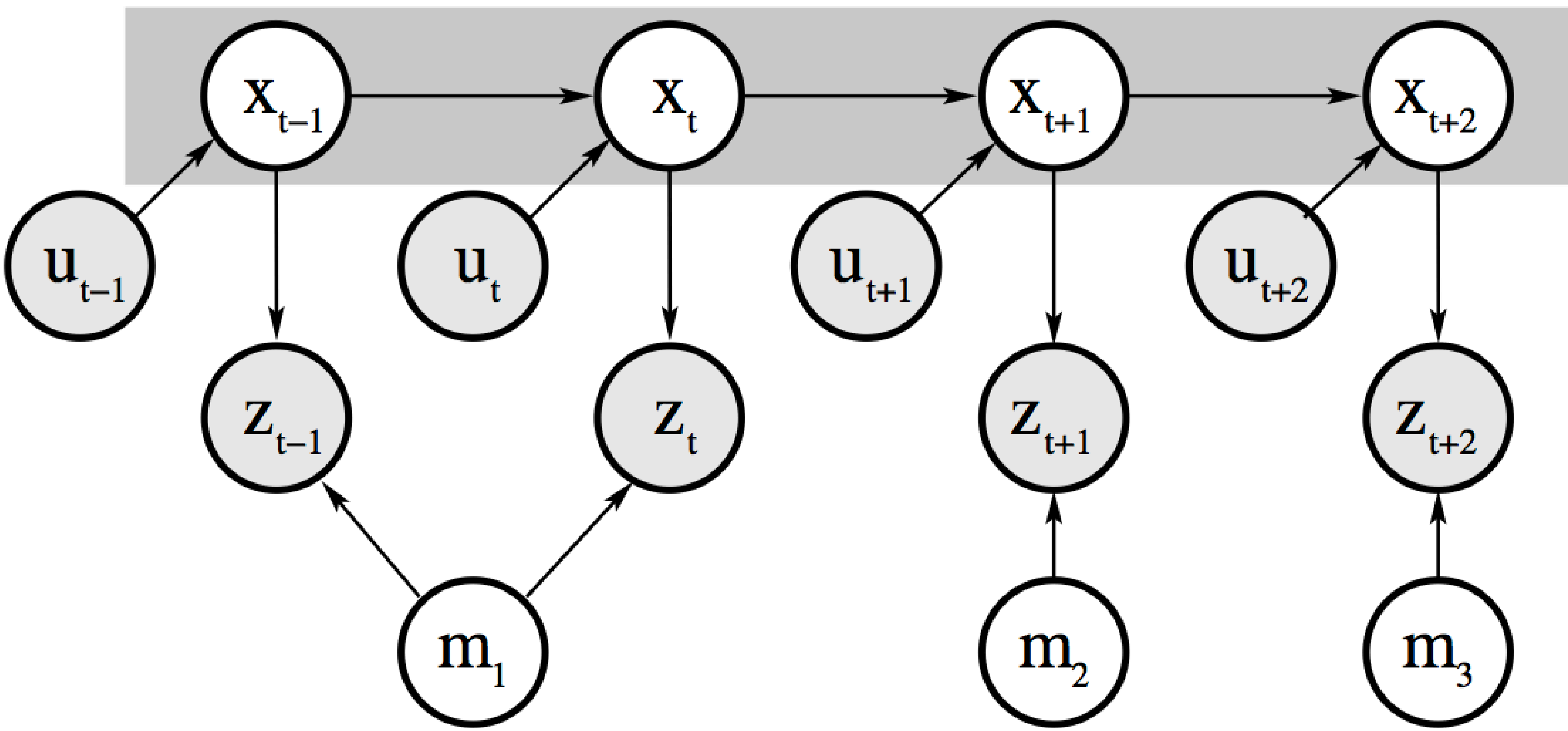
Courtesy: Thrun, Burgard, Fox

Revisit the Graphical Model



Courtesy: Thrun, Burgard, Fox

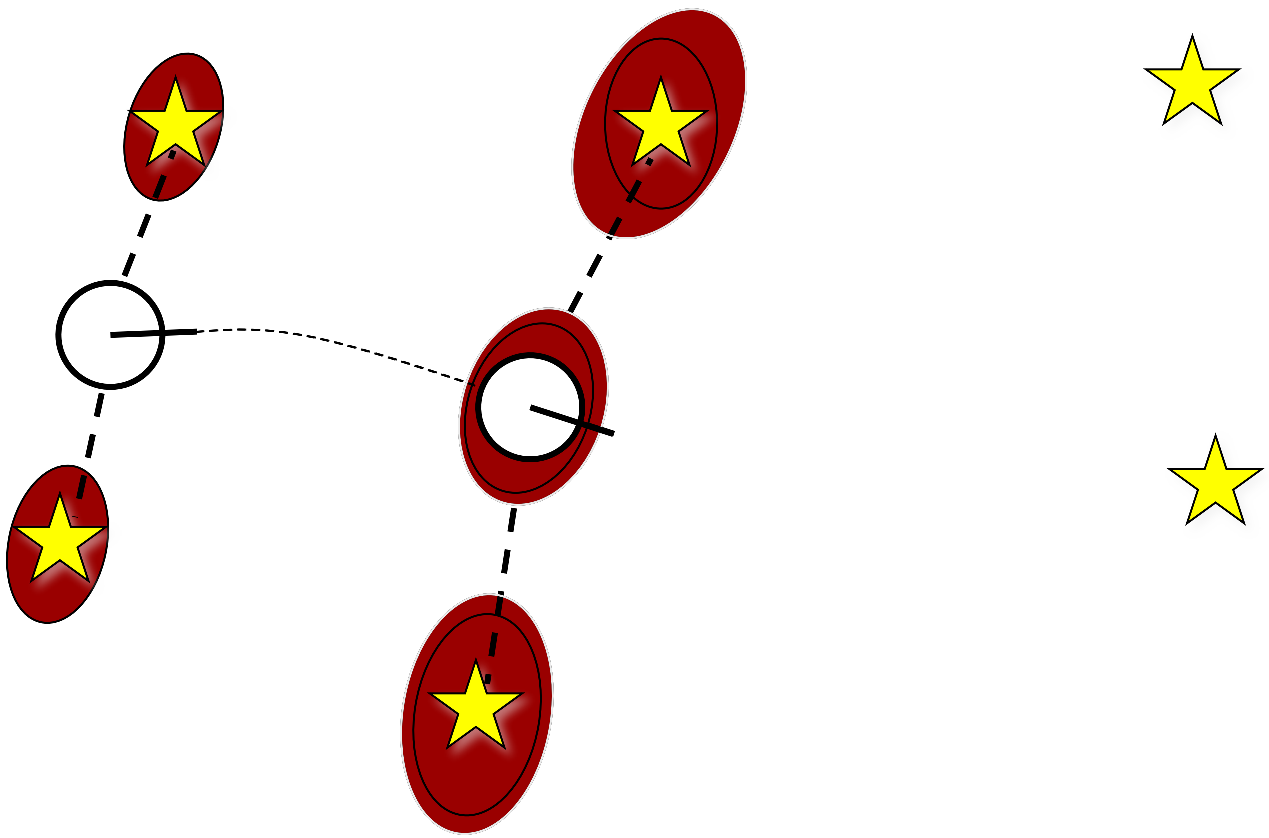
Landmarks are Conditionally Independent Given the Poses



Landmark variables are all disconnected (i.e. independent) given the robot's path

Remember: Landmarks Correlated

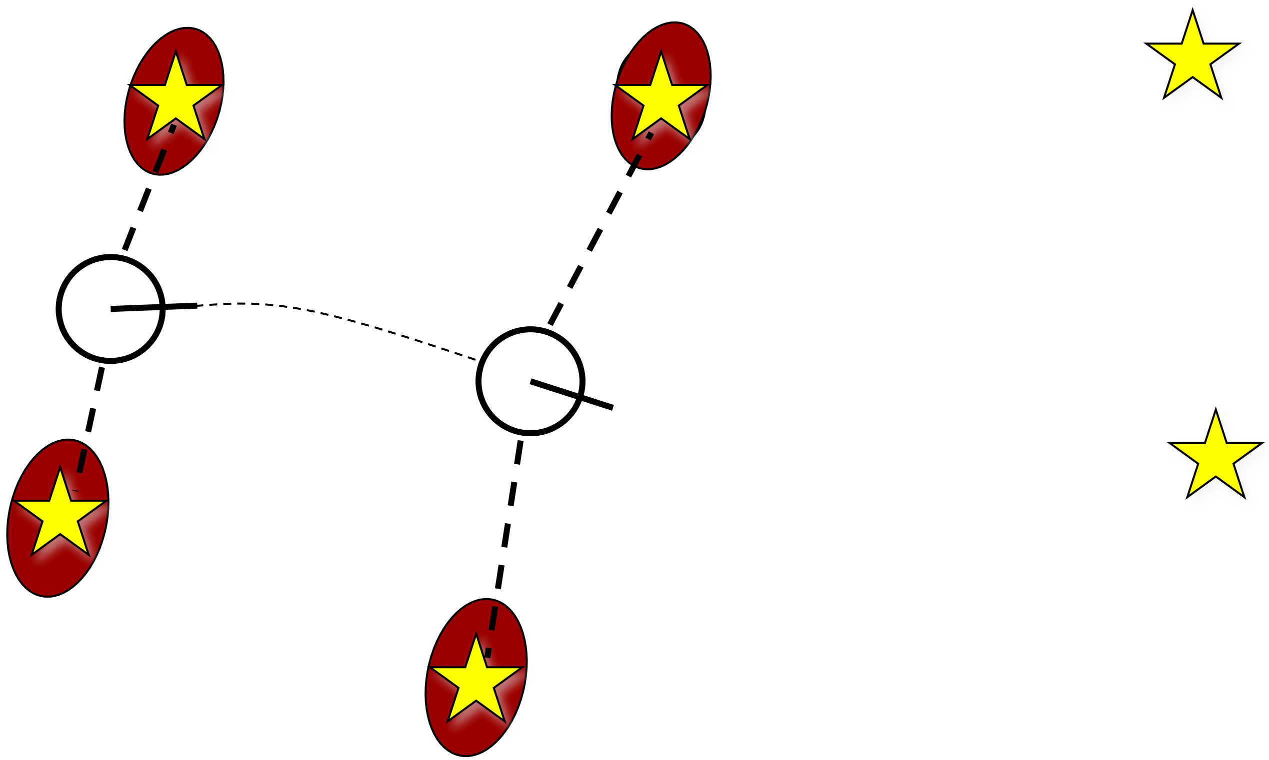
SLAM: robot path and map are both **unknown!**



Robot path error correlates errors in the map

After Factorization

For estimating landmarks: robot path **known!**



Landmarks are not correlated

Factored Posterior

$$
\begin{aligned}
& p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) \\
& \quad = \; p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(l_{1:m} \mid x_{1:t}, z_{1:t}) \\
& \quad = \; \underbrace{p(x_{1:t} \mid z_{1:t}, u_{0:t-1})}_{\text{Robot path posterior (localization problem)}} \cdot \prod_{i=1}^{M} \underbrace{p(l_i \mid x_{1:t}, z_{1:t})}_{\text{Conditionally independent landmark positions}}
\end{aligned}
$$

Robot path posterior (localization problem)

Conditionally independent landmark positions

Rao-Blackwellization for SLAM

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$
$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot \prod_{i=1}^{M} p(l_i \mid x_{1:t}, z_{1:t})$$

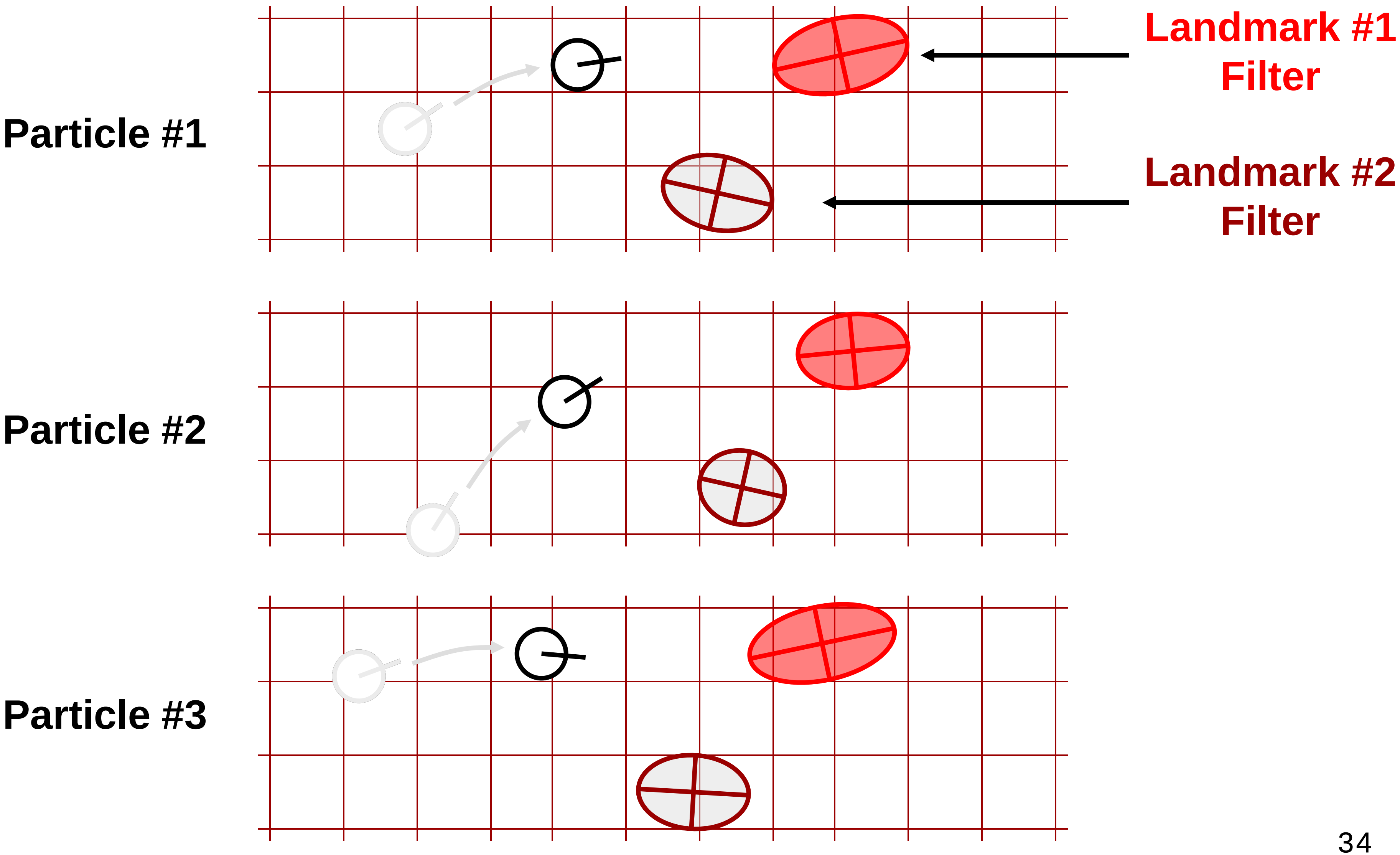
- Given that the second term can be computed efficiently, particle filtering becomes possible!

FastSLAM

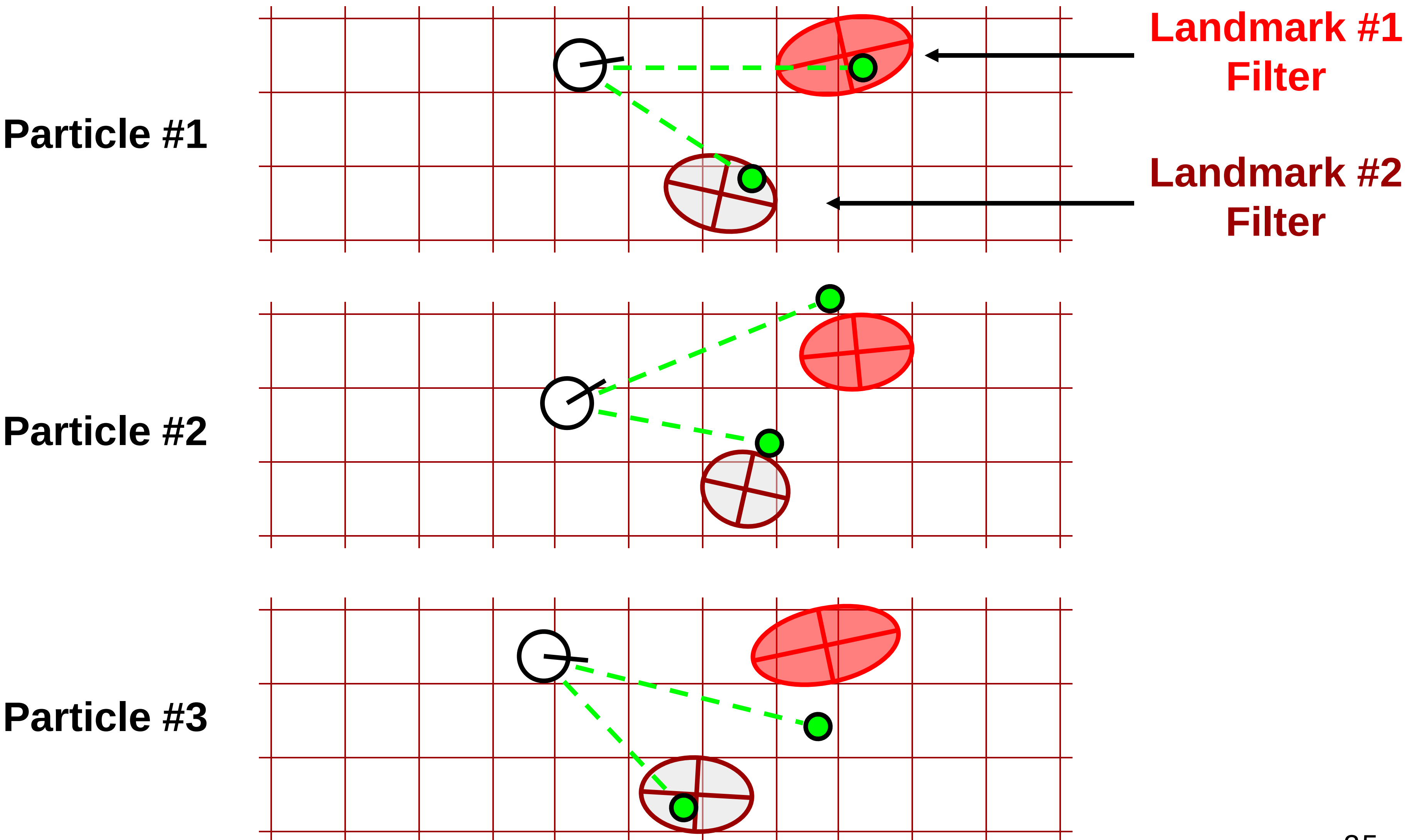
- Rao-Blackwellized particle filtering based on landmarks [Montemerlo et al., 2002]
- Each landmark is represented by a 2x2 Extended Kalman Filter (EKF)
- Each particle therefore has to maintain *M* EKFs

Particle #1	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**
Particle #2	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**
⋮					
Particle N	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**

FastSLAM – Action Update



FastSLAM – Sensor Update



FastSLAM – Sensor Update

Particle #1

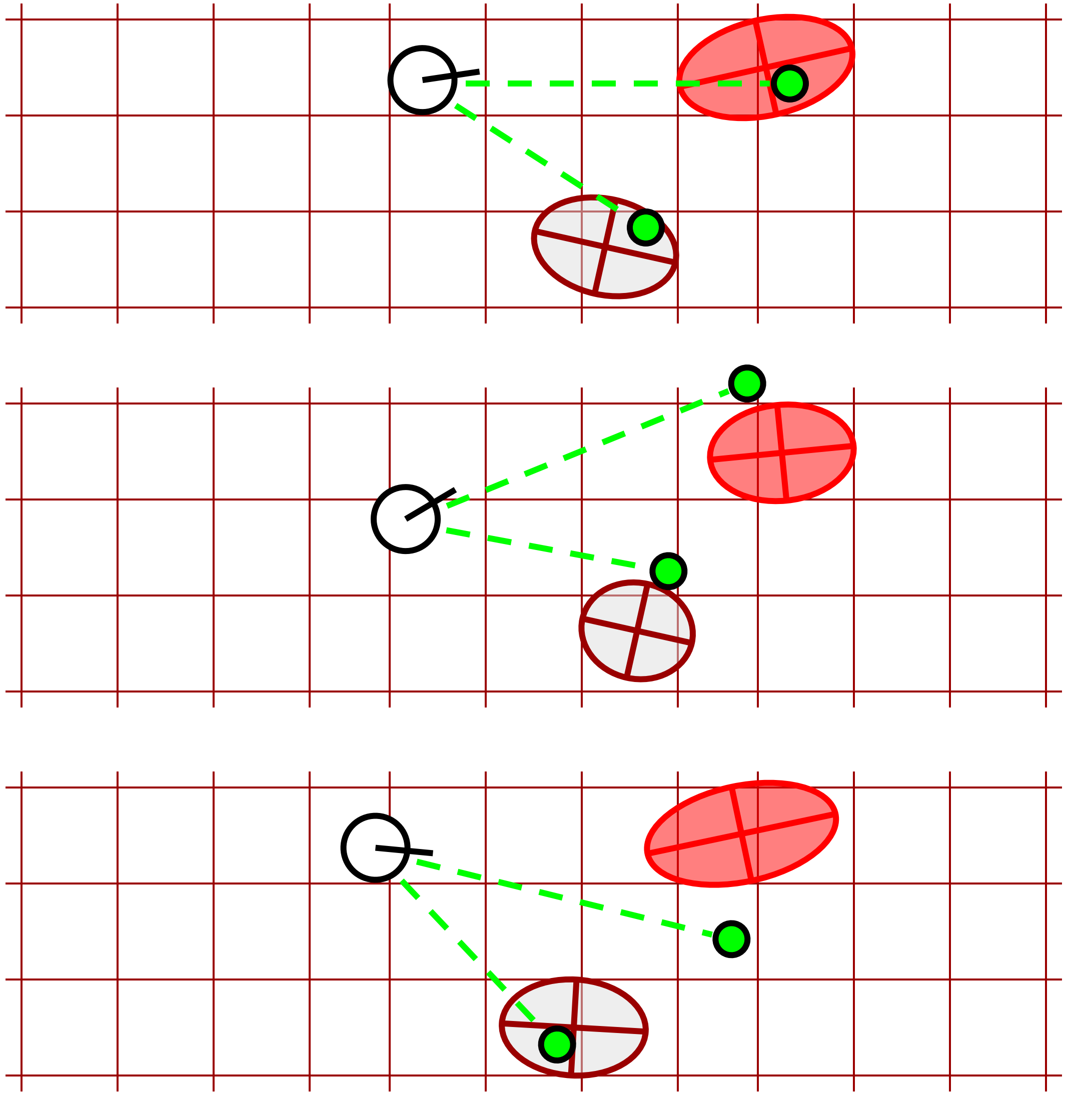
Weight = 0.8

Particle #2

Weight = 0.4

Particle #3

Weight = 0.1

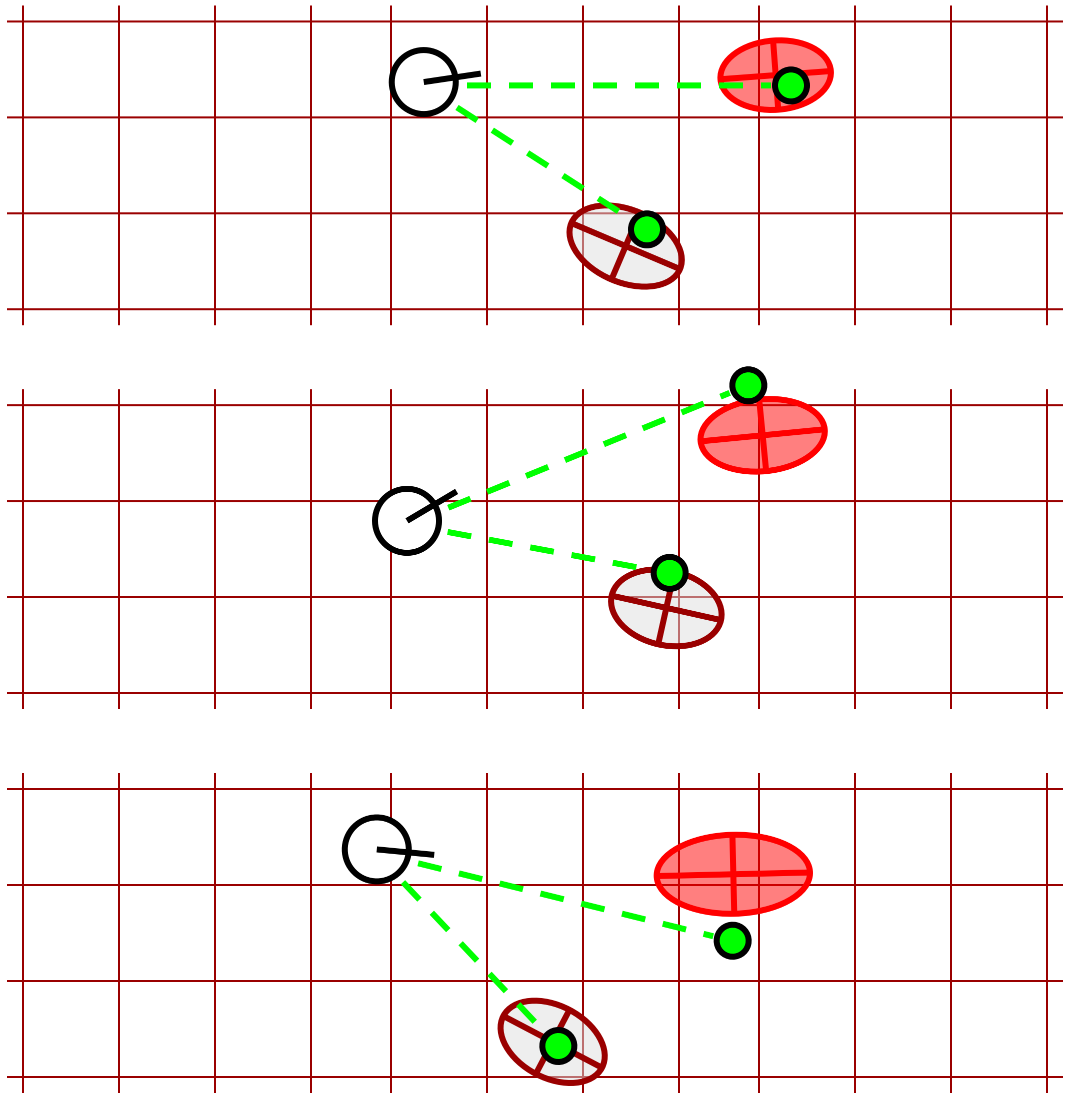


FastSLAM – Sensor Update

Particle #1

Particle #2

Particle #3

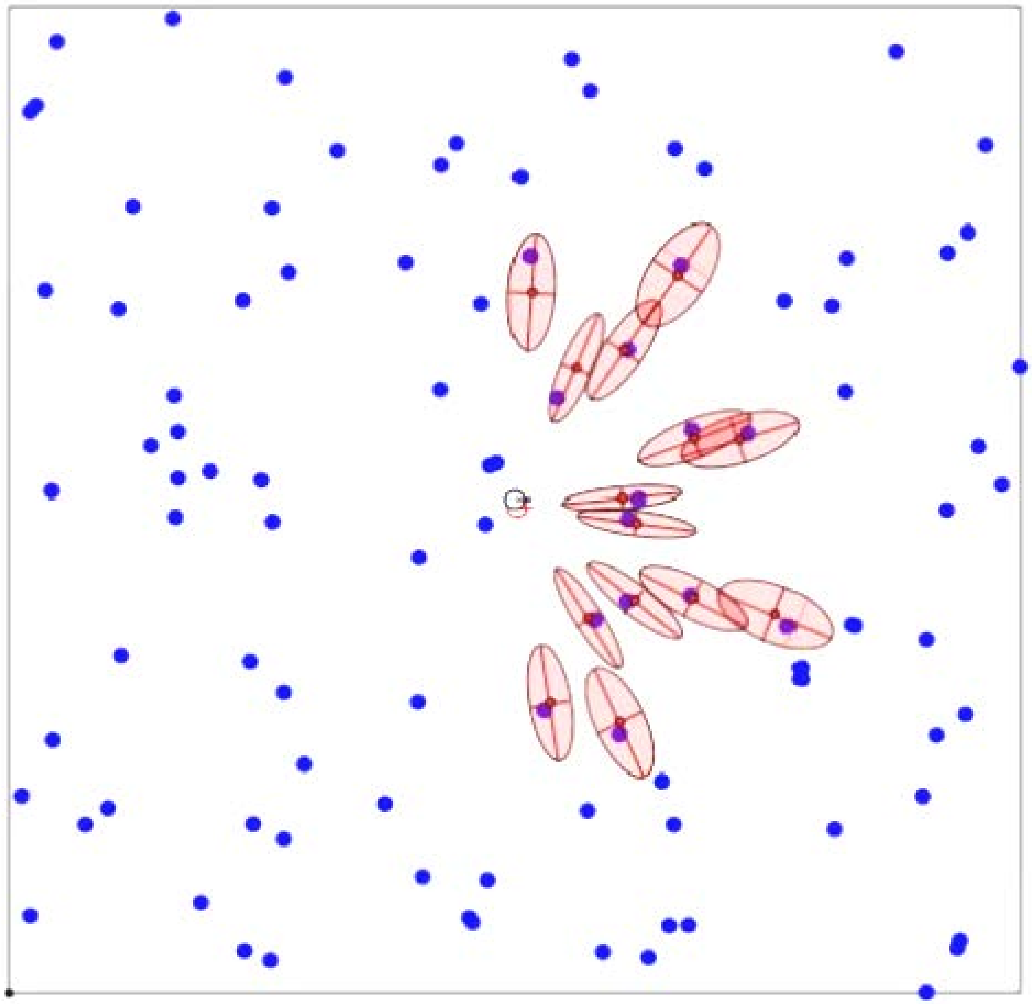


Update map of particle #1

Update map of particle #2

Update map of particle #3

FastSLAM - Video



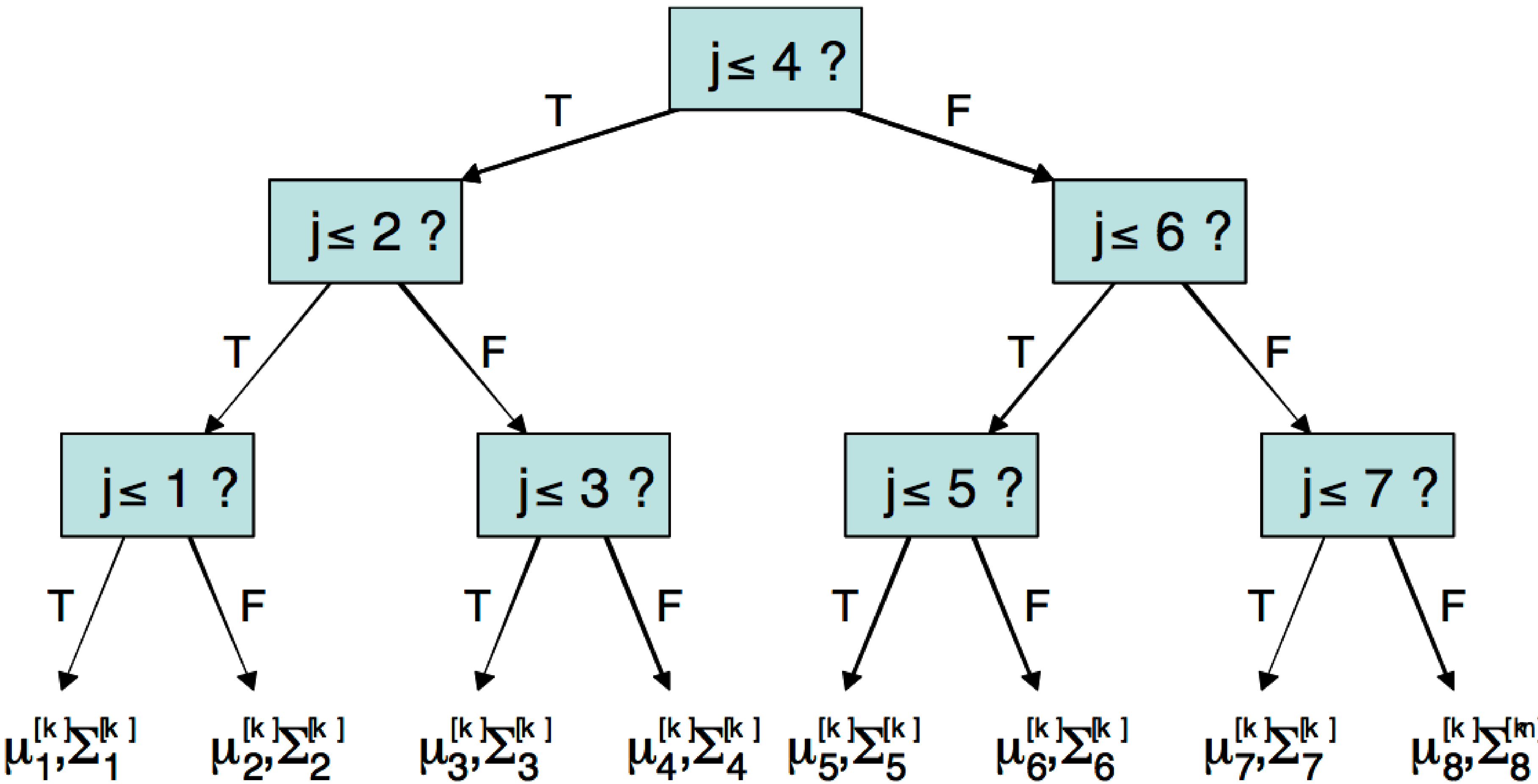
FastSLAM Complexity – Naive

- Update robot particles based on the control $\mathcal{O}(N)$
- Incorporate an observation into the Kalman filters (given the data association) $\mathcal{O}(N)$
- Resample particle set $\mathcal{O}(NM)$

N = Number of particles
M = Number of map features

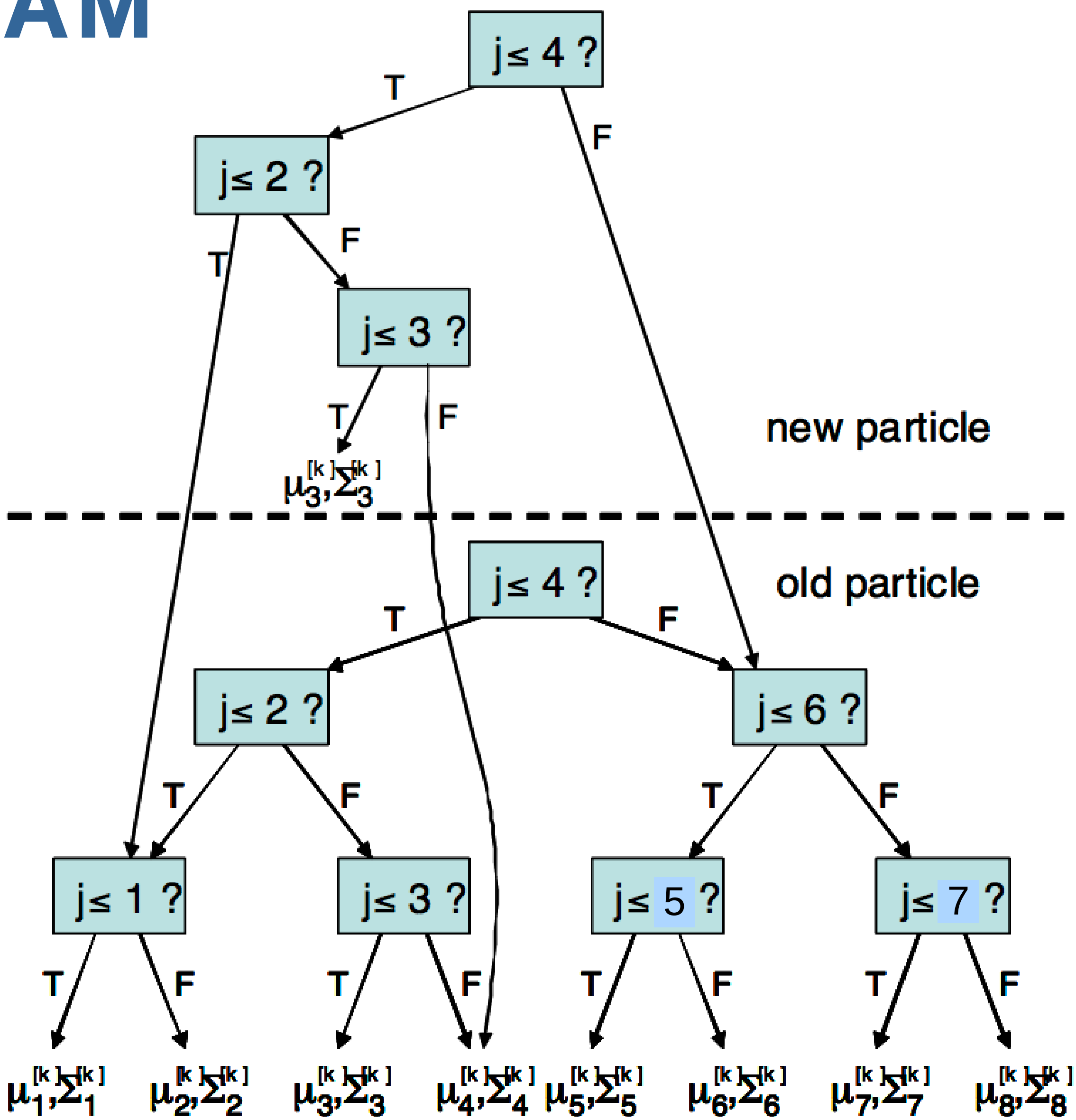
$\mathcal{O}(NM)$

A Better Data Structure for FastSLAM



Courtesy: M. Montemerlo

A Better Data Structure for FastSLAM



FastSLAM Complexity

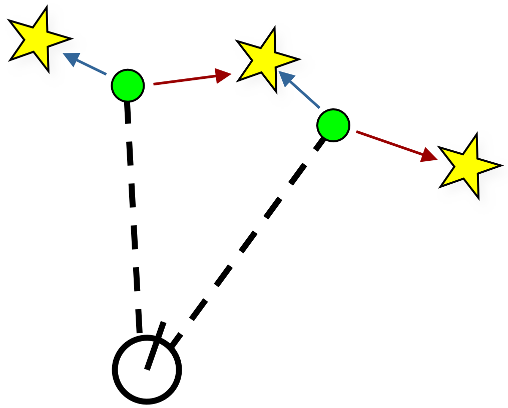
- Update robot particles based on the control $\mathcal{O}(N)$
- Incorporate an observation into the Kalman filters (given the data association) $\mathcal{O}(N \log M)$
- Resample particle set $\mathcal{O}(N)$

N = Number of particles
M = Number of map features

$$\mathcal{O}(N \log M)$$

Data Association Problem

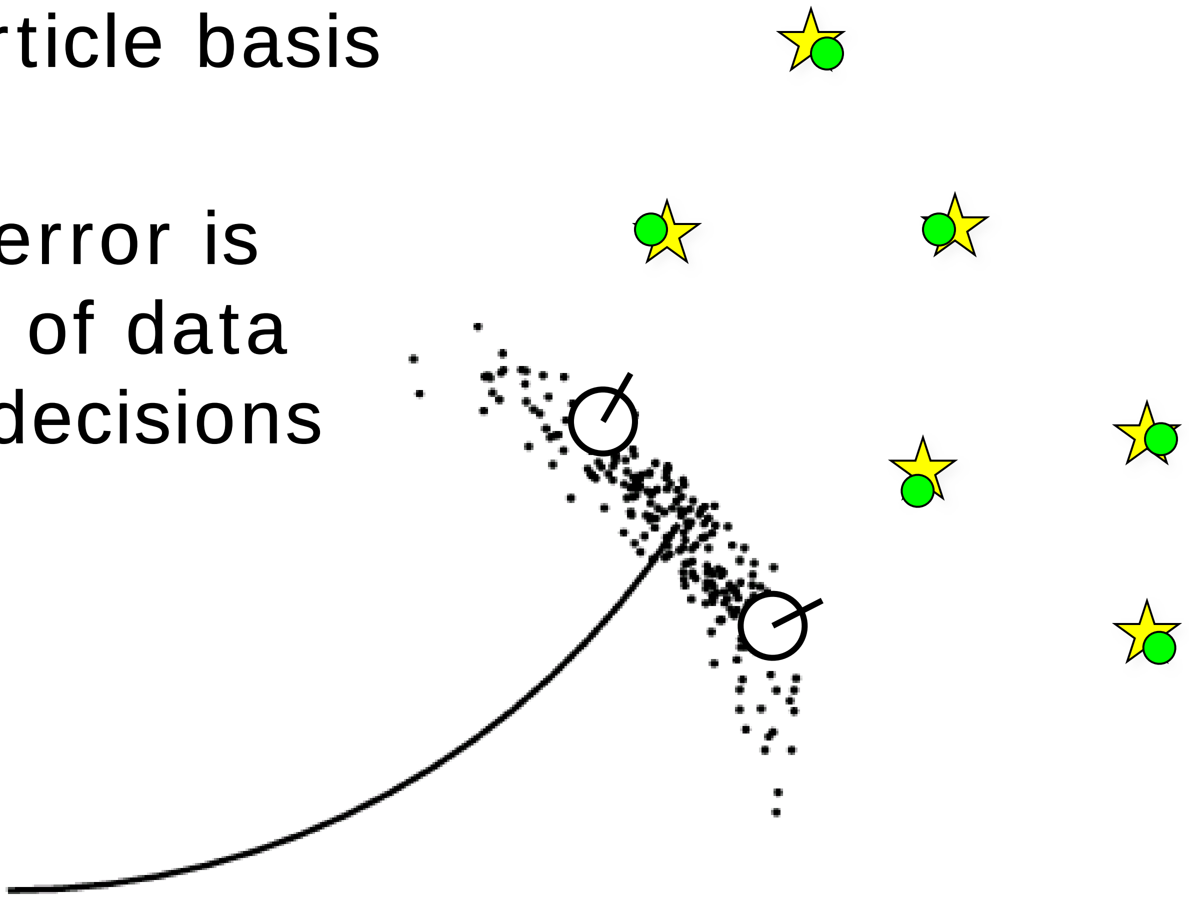
- Which observation belongs to which landmark?



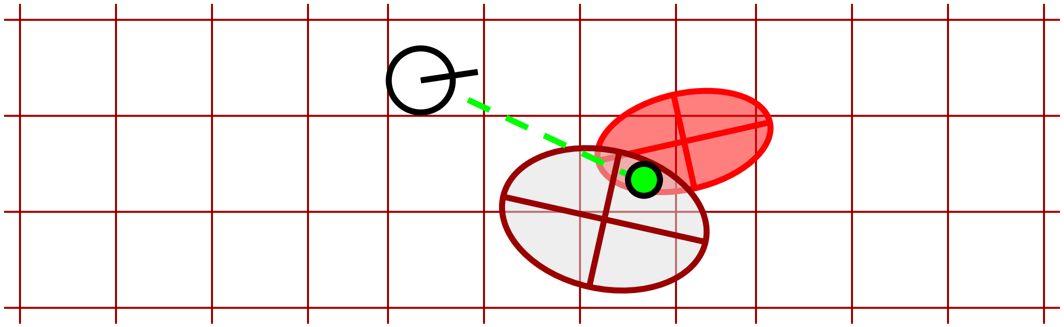
- A robust SLAM solution must consider possible data associations
- Potential data associations depend also on the pose of the robot

Multi-Hypothesis Data Association

- Data association is done on a per-particle basis
- Robot pose error is factored out of data association decisions



Per-Particle Data Association



Was the observation generated by the red or the brown landmark?

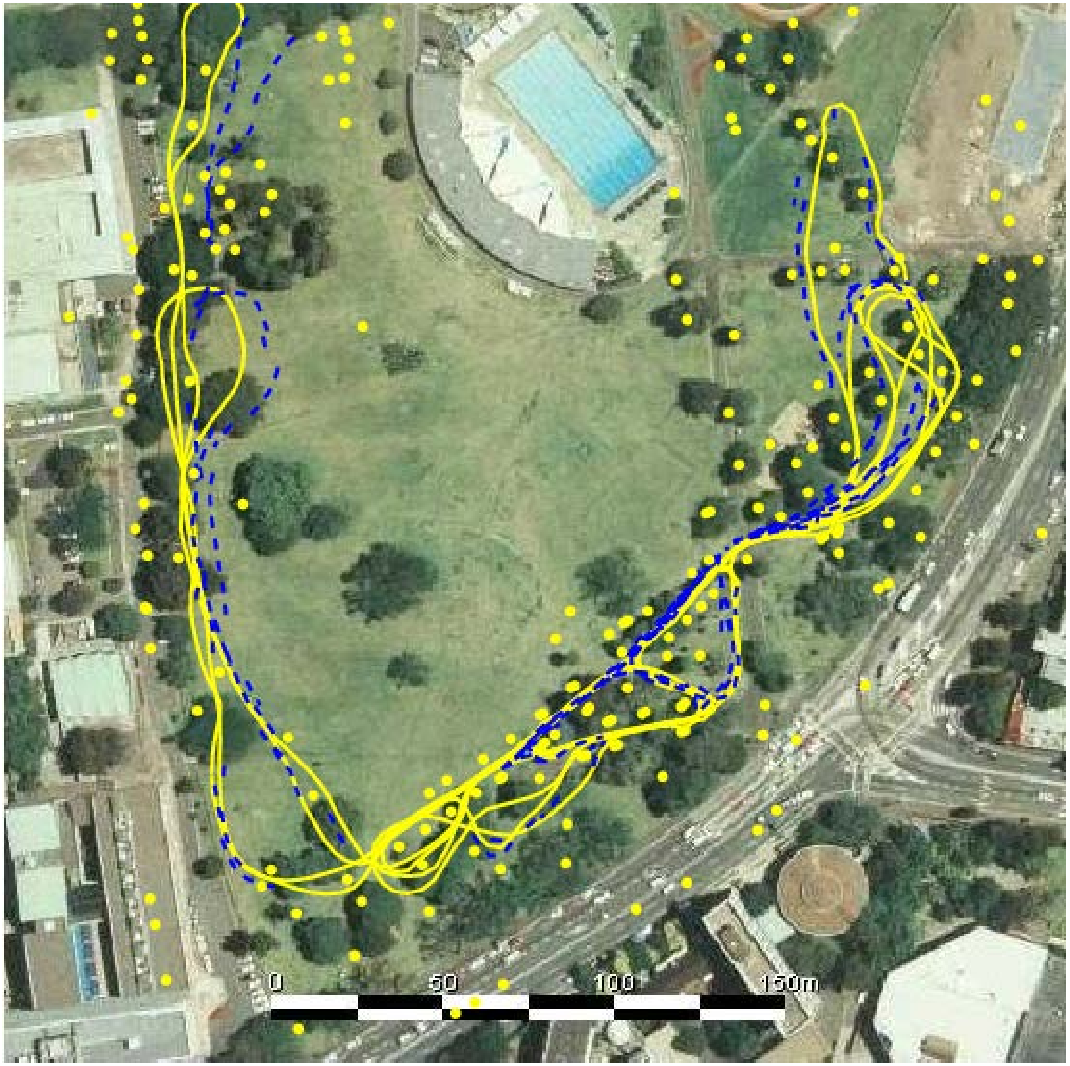
P(observation|red) = 0.3 P(observation|brown) = 0.7

- Two options for per-particle data association
  - Pick the most probable match
  - Pick a random association weighted by the observation likelihoods
- If the probability is too low, generate a new landmark

Results – Victoria Park

- 4 km traverse
- < 5 m RMS position error
- 100 particles

Blue = GPS
Yellow = FastSLAM

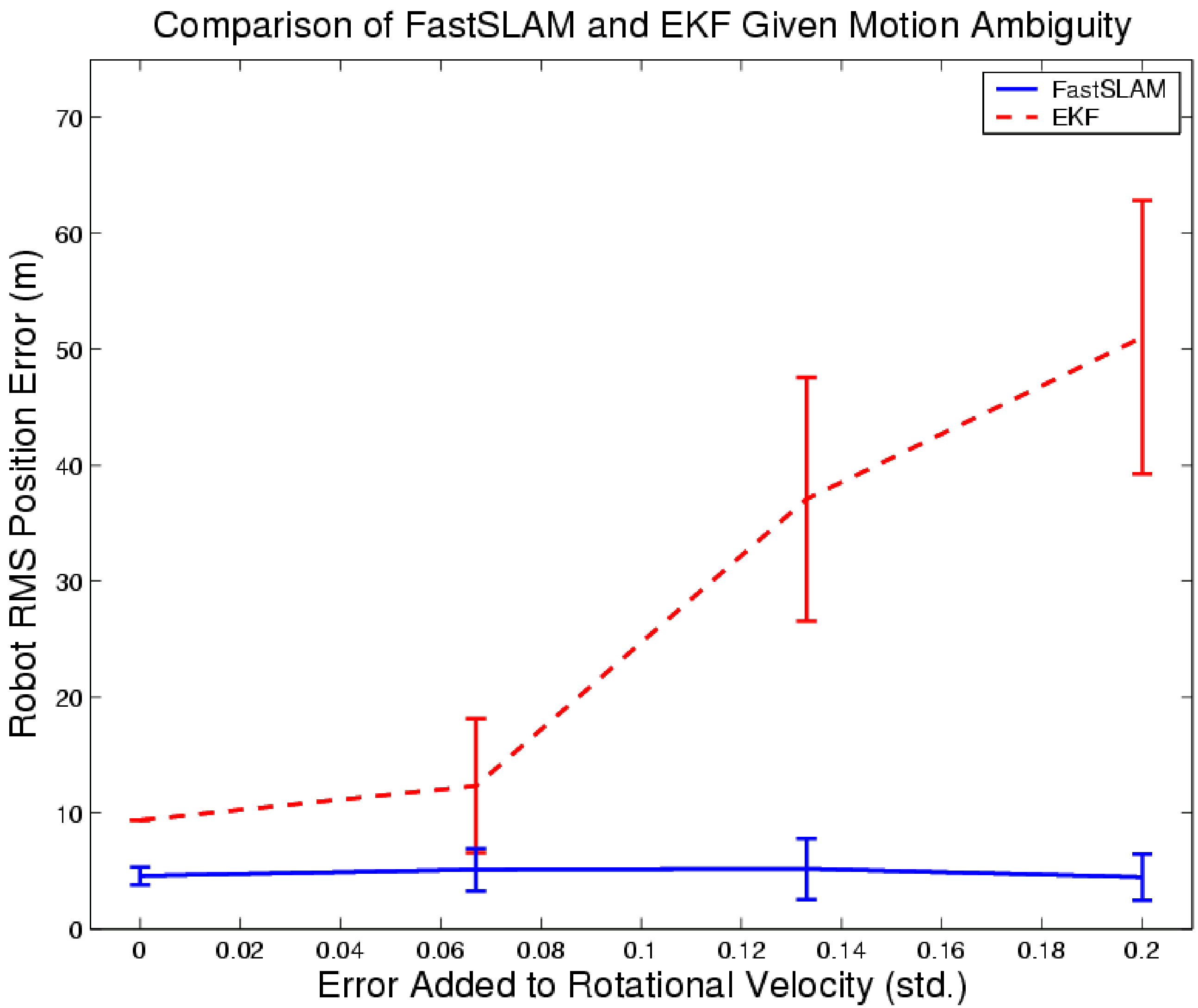


Dataset courtesy of University of Sydney

Results – Victoria Park (Video)

Dataset courtesy of University of Sydney

Results – Data Association



FastSLAM Summary

- FastSLAM factors the SLAM posterior into low-dimensional estimation problems
  - Scales to problems with over 1 million features
- FastSLAM factors robot pose uncertainty out of the data association problem
  - Robust to significant ambiguity in data association
  - Allows data association decisions to be delayed until unambiguous evidence is collected
- Advantages compared to the classical EKF approach (especially with non-linearities)
- Complexity of $O(N \log M)$