# Sheet 9 solutions

July 8, 2019

## Exercise 1: Bearing-only SLAM

*Bearing-only SLAM refers to the SLAM problem when the sensors can only measure the bearing of a landmark but not its range. One problem in bearing only SLAM with EKFs concerns the initialization of landmark location estimates, even if the correspondences are known. Discuss why, and devise a technique for initializing the landmark location estimates (means and covariances) that can be applied in bearing only SLAM.*

A single bearing measurement in 2D gives mostly information about the angle $\alpha$ of the landmark in the robot frame. The distance is only known to lie somewhere between zero and the maximum range reading $d_{\max}$ of the detector.

For the EKF framework we will require the possible position of the landmark to be described by a 2D Gaussian ellipse. This assumption is certainly not well met in reality with a single bearing measurement. However, it will be ok once a second measurement is made from a different location.

One can choose the mean position of the landmark to be at the center between the robot and the maximum range reading with $d = d_{\max}/2$:

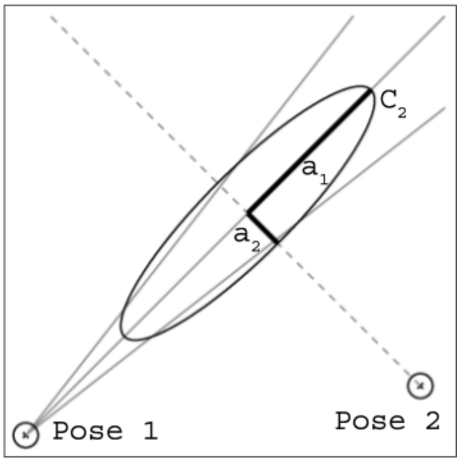
$$\begin{pmatrix} x_l \\ y_l \end{pmatrix} = d \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

The uncertainty of the landmark position is described by a 2D Gaussian ellipse with axes $a_1$ and $a_1$. The covariance matrix is then given in the landmark frame ($\vec{x}$ pointing to the robot) as

$$E = \begin{pmatrix} a_1^2 & 0 \\ 0 & a_2^2 \end{pmatrix}$$

The larger axis $a_1$ corresponds to the uncertainty of the landmark distance, which can be chosen e.g. as $a_1 = d$. The angular uncertainty is given by $a_2 = d \sin\sigma_\alpha \approx d\sigma_\alpha$.

The covariance in the robot frame is $C_2 = RER^T$, where $R(\alpha)$ is the usual 2D rotation matrix. This is illustrated in the following figure, which can be found along more details in: *A. Costa, G. Kantor, H. Choset,* "Bearing-only Landmark Initialization with Unknown Data Association", ICRA proceedings, 2004.




Once a new measurement from a different location is available, the landmark position distribution is described by the multiplication of two lengthy ellipses at different angles, resulting in a more circular distribution (corresponding to the overlap of the ellipses).

## Exercise 2: Data Association

*Features extracted from an observation can be interpreted as either matches with existing features in a map, previously unobserved features, or false alarms (noise). Consider two features $z_t^1$ and $z_t^2$ extracted from an observation $z_t$, and a map $m_t = \{l_1, l_2\}$ with two landmarks. Each observed feature $z_t^i$ is either assigned to an existing or a new landmark, or it is marked as a false alarm.*

(a) *Write down all possible assignments for the two observed features $z_t^1$ and $z_t^2$. Note that each feature can be associated to at most one landmark and vice versa.*

At maximum one observation $z_t^{1,2}$ is assigned to each landmark $l_{1,2}$. A new landmark $l_3$ is initialized if one observation cannot be assigned, and a second new landmark $l_4$ is initialized only if both observations cannot be assigned. The possible solutions are (*fa* is false alarm):

| Solution | $l_1$ | $l_2$ | $l_3$ | $l_4$ | *fa* | *fa* |
|---|---|---|---|---|---|---|
| 1 | $\mathbf{z_t^1}$ | $z_t^2$ | | | | |
| 2 | $\mathbf{z_t^1}$ | | $z_t^2$ | | | |
| 3 | $\mathbf{z_t^1}$ | | | | $z_t^2$ | |
| 4 | $z_t^2$ | $\mathbf{z_t^1}$ | | | | |
| 5 | | $\mathbf{z_t^1}$ | $z_t^2$ | | | |
| 6 | | $\mathbf{z_t^1}$ | | | $z_t^2$ | |
| 7 | $z_t^2$ | | $\mathbf{z_t^1}$ | | | |
| 8 | | $z_t^2$ | $\mathbf{z_t^1}$ | | | |
| 9 | | | $\mathbf{z_t^1}$ | $z_t^2$ | | |
| 10 | | | $\mathbf{z_t^1}$ | | $z_t^2$ | |
| 11 | | | $z_t^2$ | $\mathbf{z_t^1}$ | | |
| 12 | $z_t^2$ | | | | $\mathbf{z_t^1}$ | |
| 13 | | $z_t^2$ | | | $\mathbf{z_t^1}$ | |
| 14 | | | $z_t^2$ | | $\mathbf{z_t^1}$ | |
| 15 | | | | | $\mathbf{z_t^1}$ | $z_t^2$ |

In total 15 possible solutions can be found. The solutions 9 and 11 are equivalent if the order of new assignments is fixed, such that only 14 solutions remain.

(b) *Now consider an update of the map to obtain $m_{t+1}$. Here, every new feature is added to the map as a new landmark, and every existing landmark without a match is removed. Suppose no false alarm is detected. How many solutions for the assignments remain? Are there any two solutions that will result in the same map?*

If no false alarms is detected, seven solutions remain in the table above. Each of the solutions results in a map with two landmarks (corresponding to the two observed features). Some solutions result in the same landmarks indices (e.g. solution 5 and 8 will both result in $m_{t+1} = \{l_2, l_3\}$). However, the resulting maps can still differ, since the landmark positions are updated with different observed feature positions.

(c) *How many new assignments can be generated from this set of maps in total if at time $t+1$ a single feature $z^1_{t+1}$ is observed?*

Seven possible maps remain from the previous time step, each with two landmarks $l_{a,b}$. For each of these maps, exactly four new assignments can be made:

$$z^1_{t+1} \rightarrow l_a \qquad z^1_{t+1} \rightarrow l_b \qquad z^1_{t+1} \rightarrow l_{\text{new}} \qquad z^1_{t+1} \rightarrow fa$$

in total $7 \cdot 4 = 28$ possible assignments are found.