Introduction to Mobile Robotics

Iterative Closest Point Algorithm

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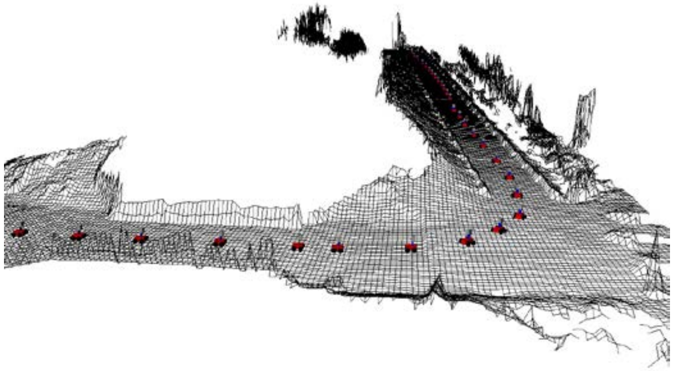
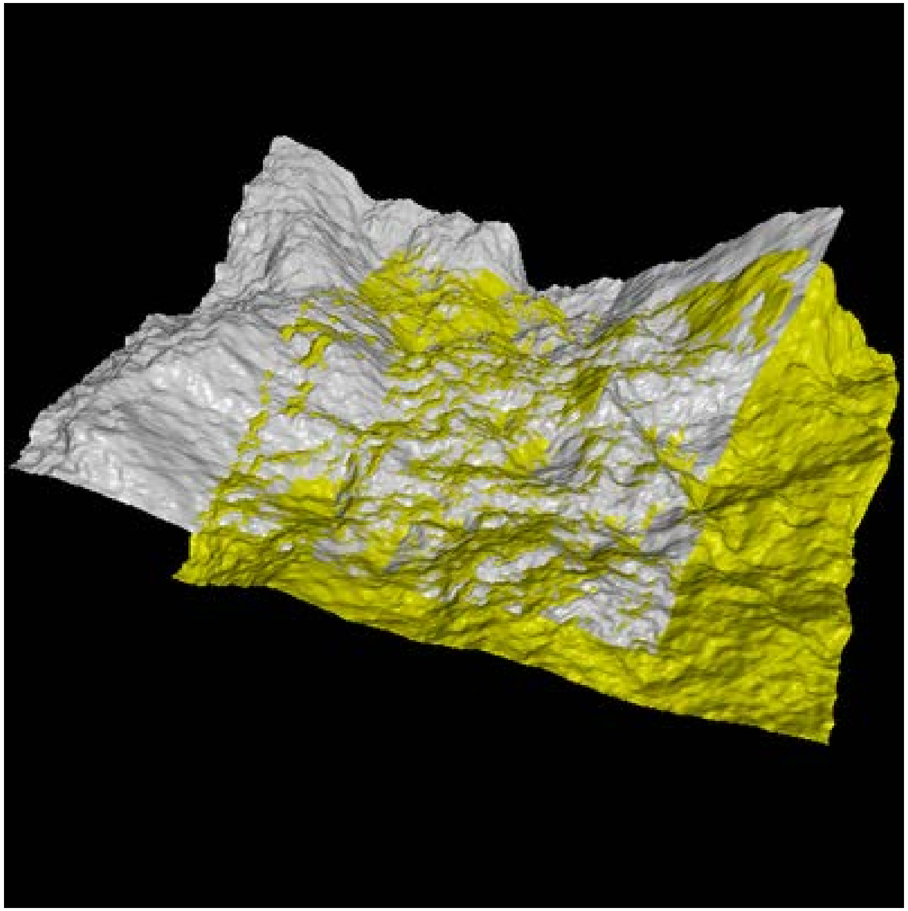
Motivation





Goal: Find local transformation to align points

The Problem

- Given two corresponding point sets:

$$X = \{x_1, ..., x_{N_x}\}$$

$$P = \{p_1, ..., p_{N_p}\}$$

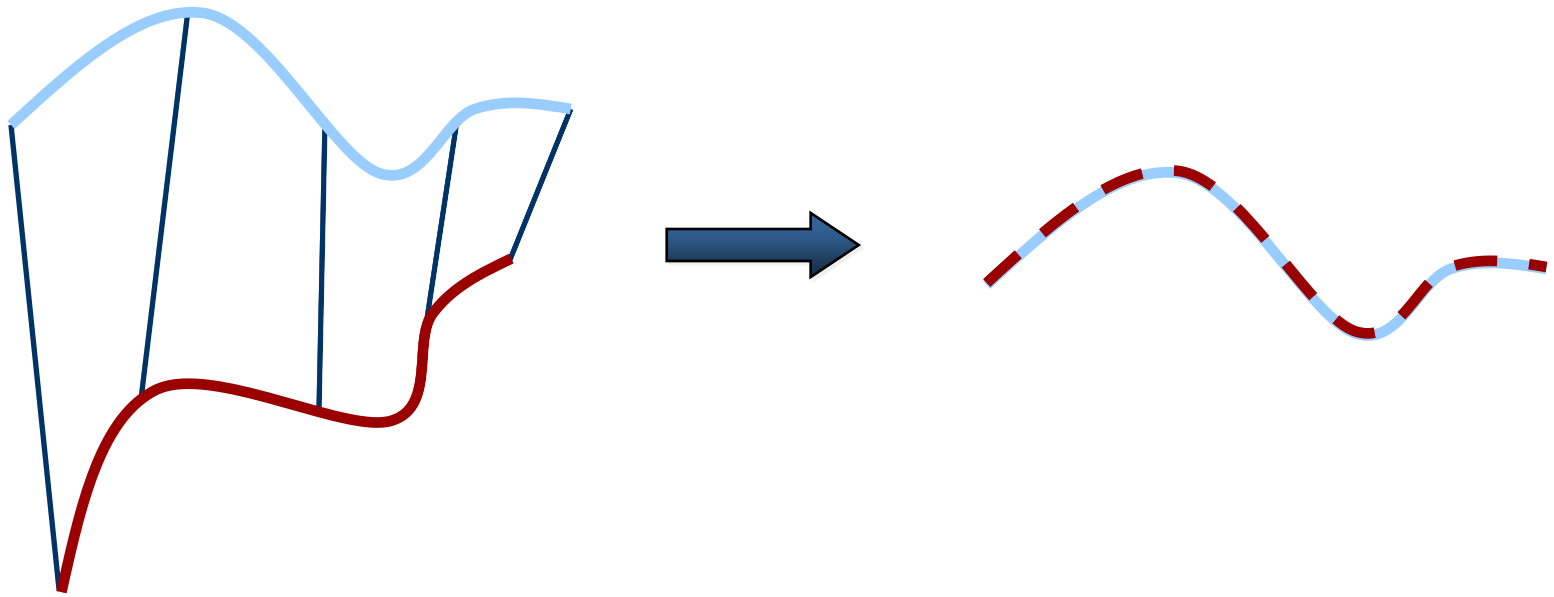
- Wanted: Translation *t* and rotation *R* that minimize the sum of the squared errors:

$$E(R,t) = \frac{1}{N_p} \sum_{i=1}^{N_p} ||x_i - Rp_i - t||^2$$

Here, x_i and p_i are corresponding points

Key Idea

- If the correct correspondences are known, the correct relative rotation/translation can be calculated in **closed form**



Center of Mass

$$\mu_x = \frac{1}{N_x} \sum_{i=1}^{N_x} x_i \quad \text{and} \quad \mu_p = \frac{1}{N_p} \sum_{i=1}^{N_p} p_i$$

are the centers of mass of the two point sets

Idea:

- Subtract the corresponding center of mass from every point in the two point sets before calculating the transformation
- The resulting point sets are:

$$X' = \{x_i - \mu_x\} = \{x'_i\} \quad \text{and}$$
$$P' = \{p_i - \mu_p\} = \{p'_i\}$$

Singular Value Decomposition

Let $W = \sum_{i=1}^{N_p} x_i' p_i'^T$

denote the singular value decomposition (SVD) of W by:

$$W = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V^T$$

where $U, V \in \mathbb{R}^{3\times 3}$ are unitary, and

$\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the singular values of W

SVD

Theorem (without proof):

If rank(W) = 3, the optimal solution of E(R, t) is unique and is given by:

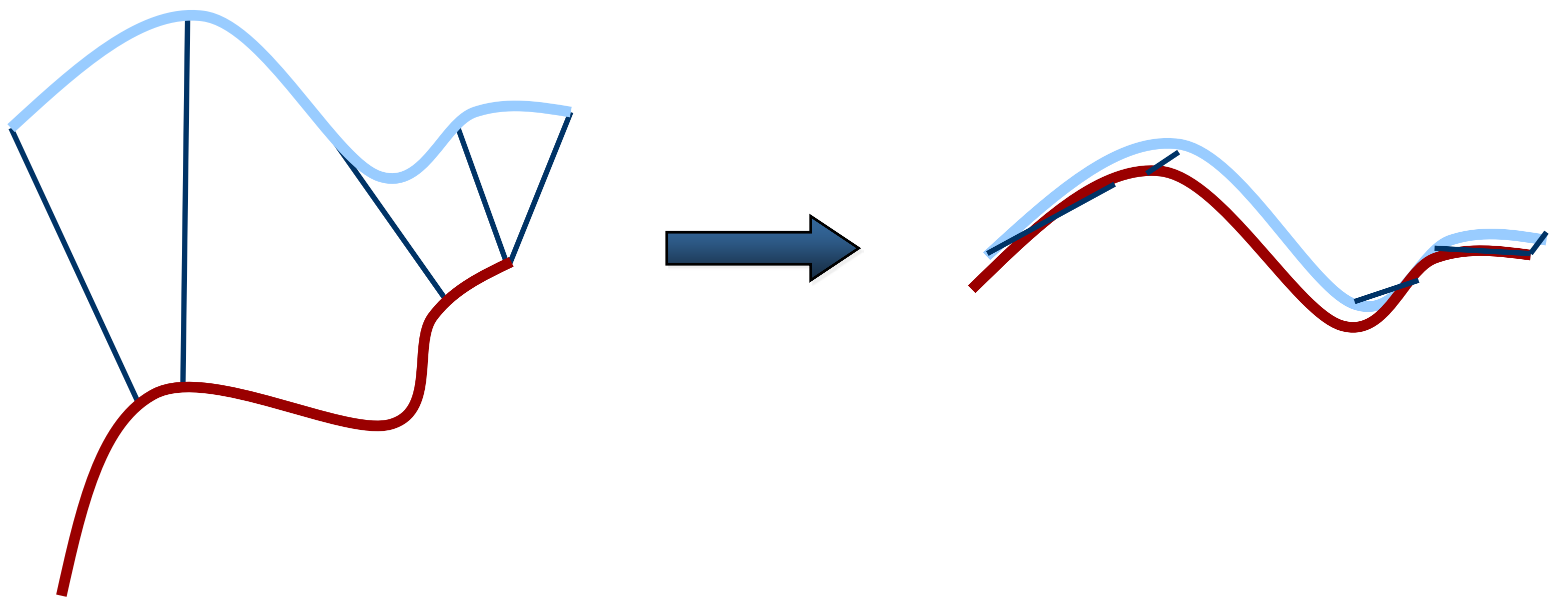
$$R = UV^T$$
$$t = \mu_x - R\mu_p$$

The minimal value of error function at (R, t) is:

$$E(R,t) = \sum_{i=1}^{N_p} (||x_i'||^2 + ||y_i'||^2) - 2(\sigma_1 + \sigma_2 + \sigma_3)$$

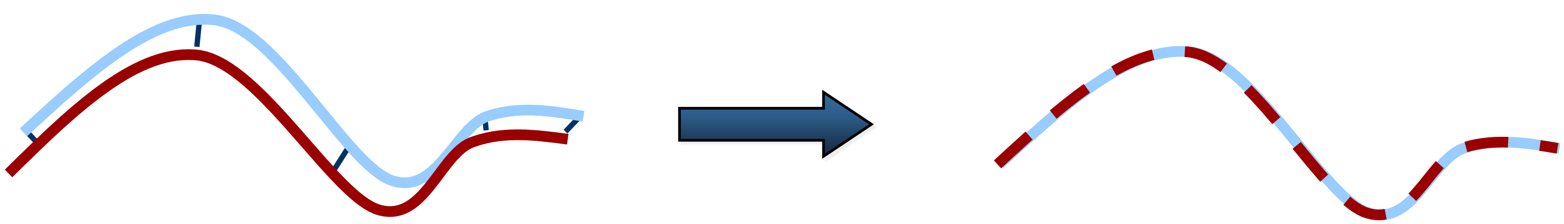
ICP with Unknown Data Association

- If the correct correspondences are **not known**, it is generally impossible to determine the optimal relative rotation and translation in one step



Iterative Closest Point (ICP) Algorithm

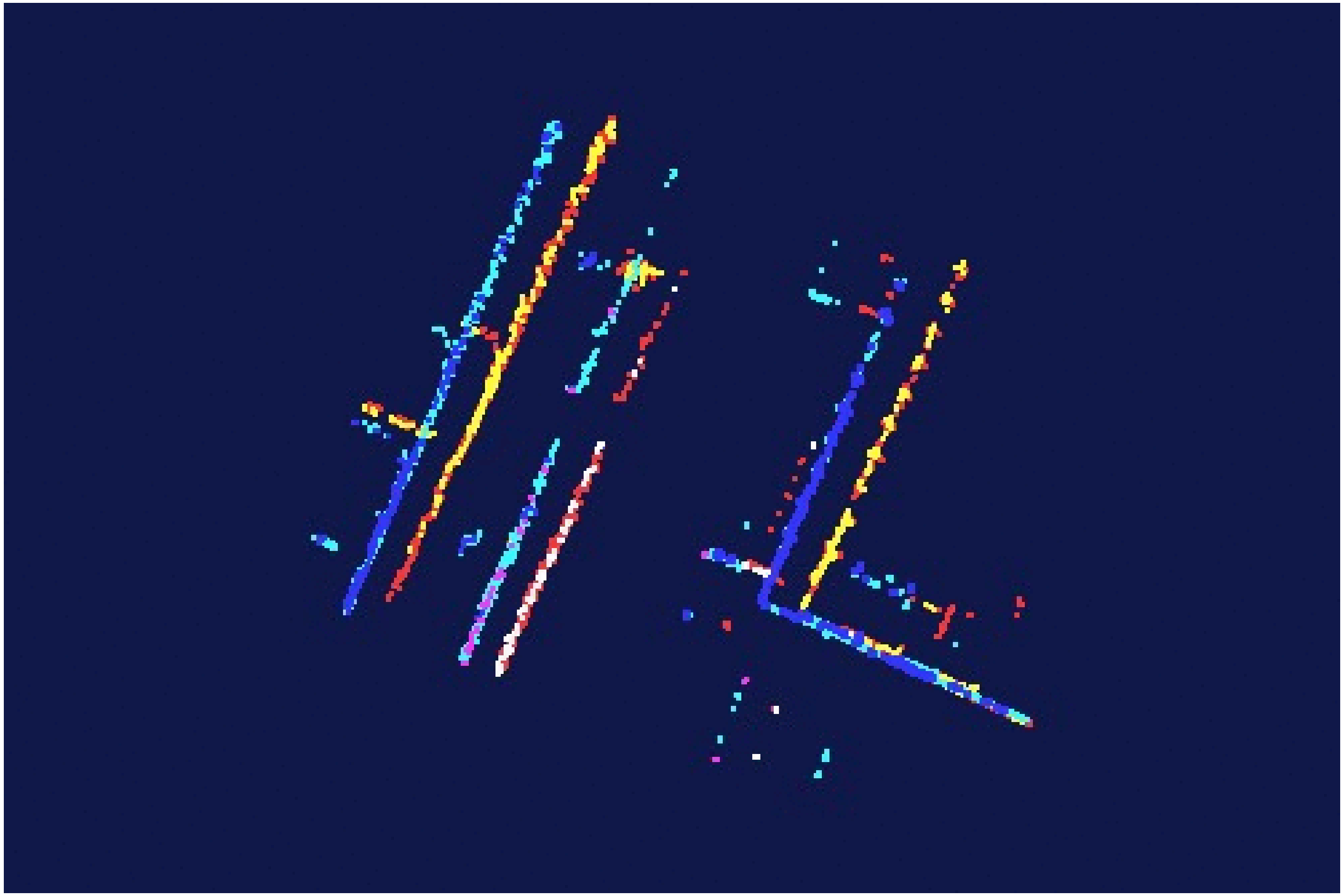
- Idea: Iterate to find alignment
- Iterative Closest Points
  [Besl & McKay 92]
- Converges if starting positions are "close enough"



Basic ICP Algorithm

- Determine corresponding points
- Compute rotation *R*, translation *t* via SVD
- Apply *R* and *t* to the points of the set to be registered
- Compute the error E(*R*,*t*)
- If error decreased and error > threshold
  - Repeat these steps
  - Stop and output final alignment, otherwise

ICP Example



ICP Variants

Variants on the following stages of ICP have been proposed:

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs

Performance of Variants

- Various aspects of performance:
  - Speed
  - Stability (local minima)
  - Tolerance wrt. noise and outliers
  - Basin of convergence (maximum initial misalignment)

ICP Variants

1. Point subsets (from one or both point sets)
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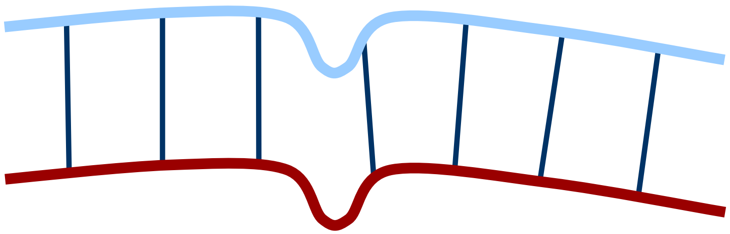
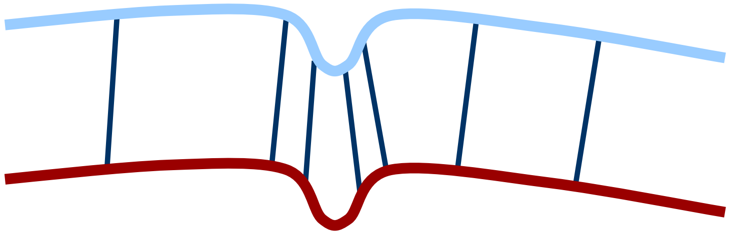
Selecting Source Points

- Use all points
- Uniform sub-sampling
- Random sampling
- Feature based sampling
- Normal-space sampling
  (Ensure that samples have normals distributed as uniformly as possible)

Normal-Space Sampling



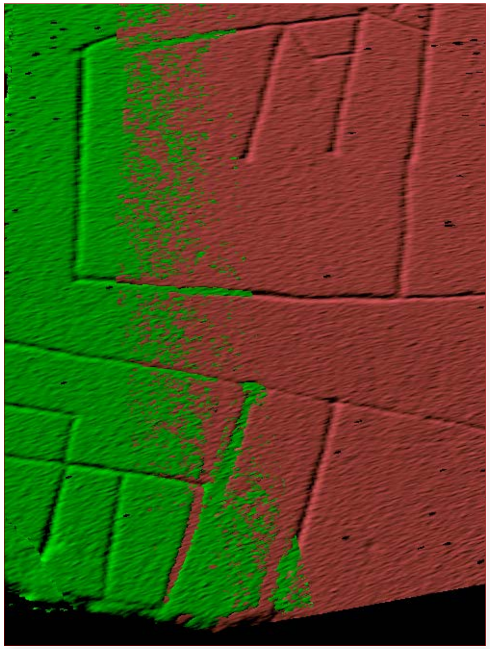
uniform sampling



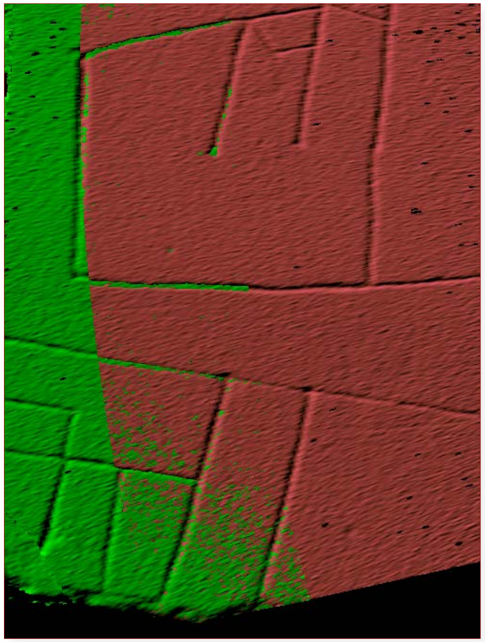
normal-space sampling

Comparison

- Normal-space sampling better for mostly smooth areas with sparse features
  [Rusinkiewicz et al., 01]



Random sampling



Normal-space sampling

Comparison

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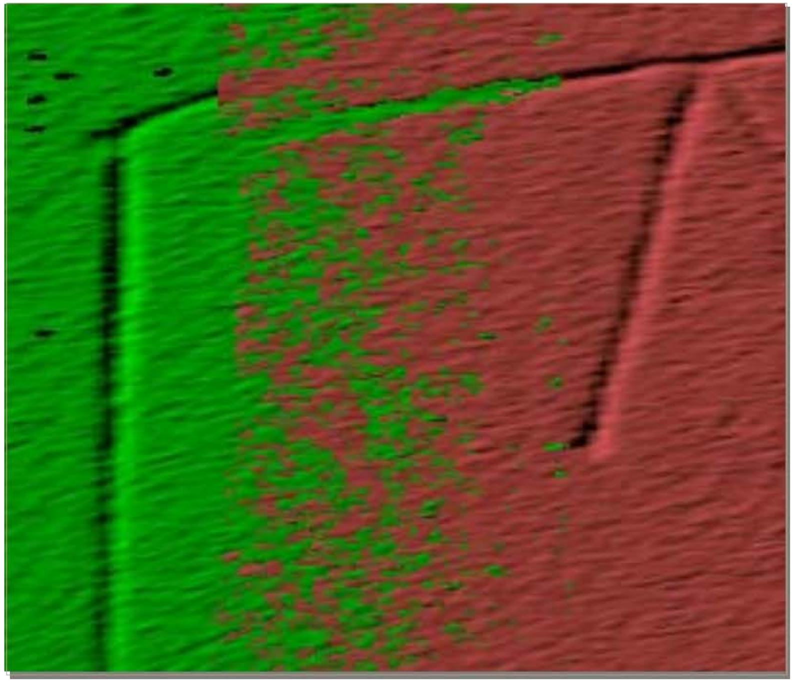
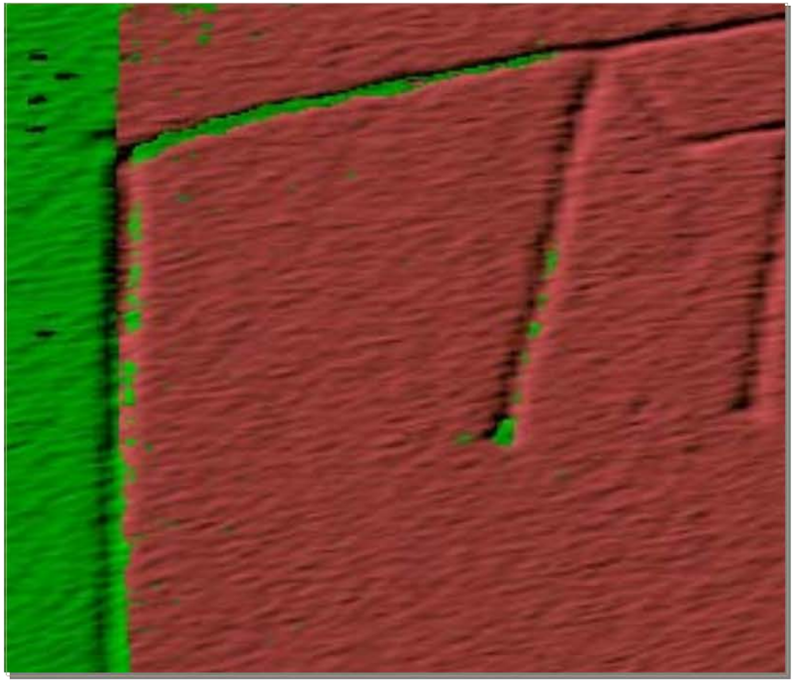
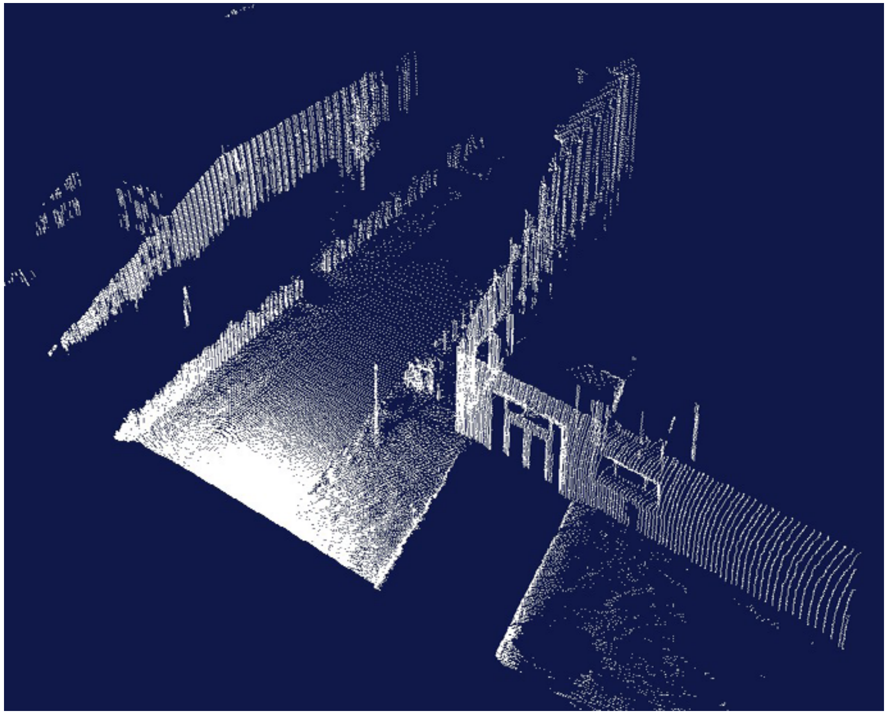




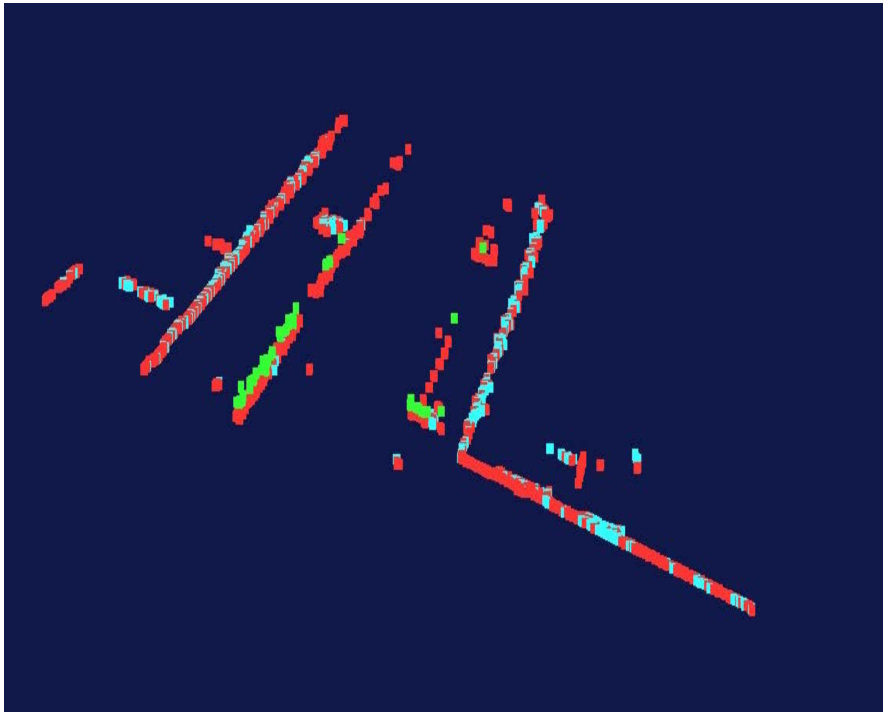
Random sampling Normal-space sampling

Feature-Based Sampling

- Try to find “important” points
- Decreases the number of correspondences to find
- Higher efficiency and higher accuracy
- Requires preprocessing

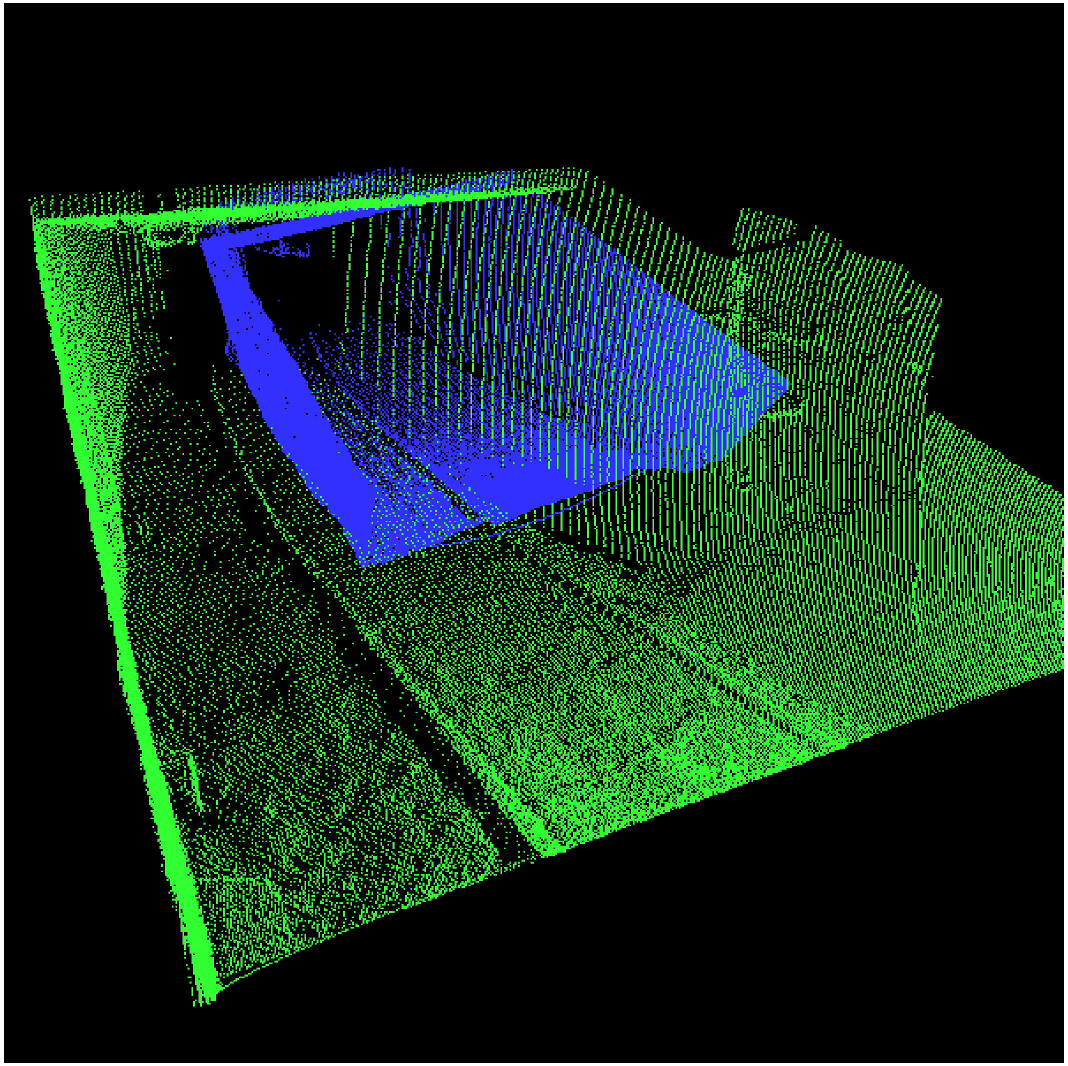


3D Scan (~200.000 Points)



Extracted Features (~5.000 Points)

ICP Application (With Uniform Sampling)



[Nuechter et al., 04]

ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
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4. Rejecting certain (outlier) point pairs

Weighting

- Select a set of points for each set
- Match the selected points of the two sets
- **Weight the corresponding pairs**
- E.g., assign lower weights for points with higher point-point distances
- Determine transformation that minimizes the error function

ICP Variants

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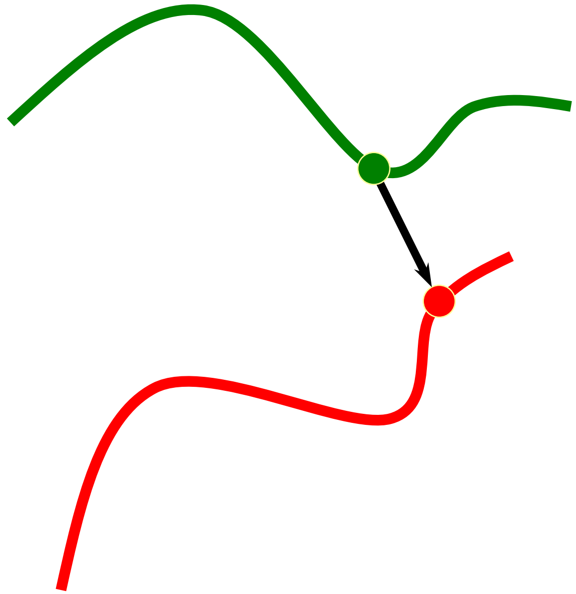
Data Association

- Has greatest effect on convergence and speed
- Matching methods:
  - Closest point
  - Normal shooting
  - Closest compatible point
  - Projection-based

Closest-Point Matching

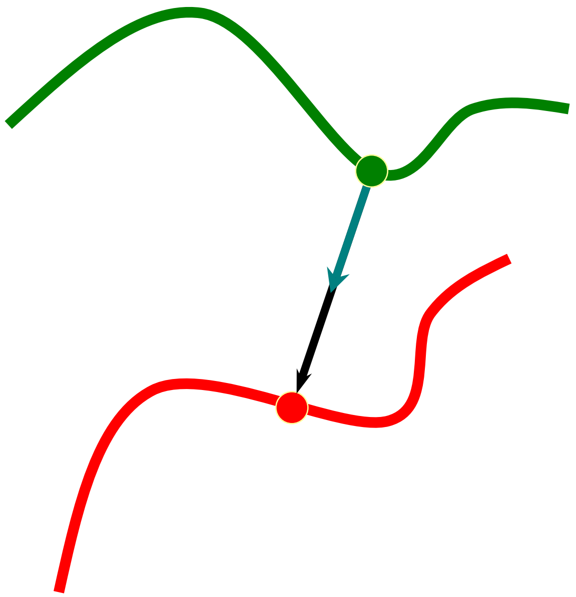
- Find closest point in other the point set (using kd-trees)



Generally stable, but slow convergence and requires preprocessing

Normal Shooting

- Project along normal, intersect other point set



Slightly better convergence results than closest point for smooth structures, worse for noisy or complex structures

Closest Compatible Point

- Improves the two previous variants by considering the **compatibility** of the points
- Only match compatible points
- Compatibility can be based on
  - Normals
  - Colors
  - Curvature
  - Higher-order derivatives
  - Other local features

Point-to-Plane Error Metric

- Minimize the sum of the squared distances between a point and the tangent plane at its correspondence point [Chen & Medioni 91]

destination

s_1
source
point

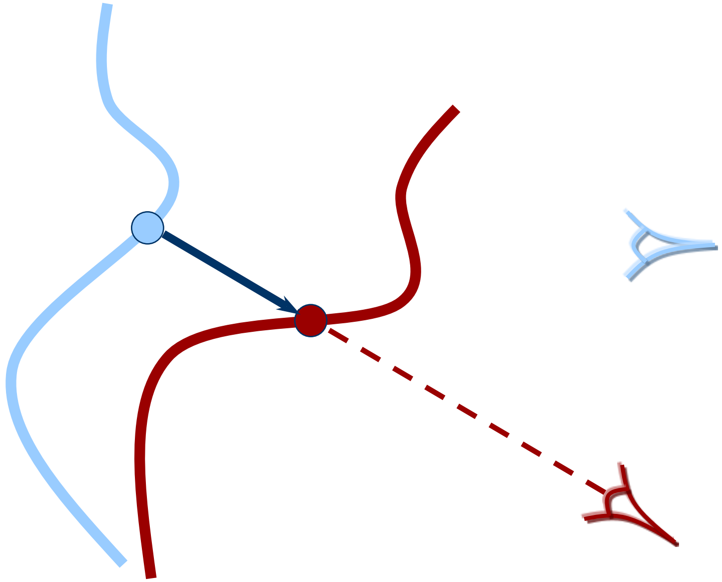
image from [Low 04]

Point-to-Plane Error Metric

- Solved using standard nonlinear least squares methods (e.g., Levenberg-Marquardt method [Press92]).
- Each iteration generally slower than the point-to-point version, however, often significantly better convergence rates [Rusinkiewicz01]
- Using point-to-plane distance instead of point-to-point lets flat regions slide along each other [Chen & Medioni 91]

Projection

- Finding the closest point is the most expensive stage of the ICP algorithm
- Idea: Simplified nearest neighbor search
- For range images, one can project the points according to the view-point [Blais 95]



Projection-Based Matching

- Constant time
- Does not require pre-computing a special data structure
- Requires point-to-plane error metric
- Slightly worse alignments per iteration

ICP Variants

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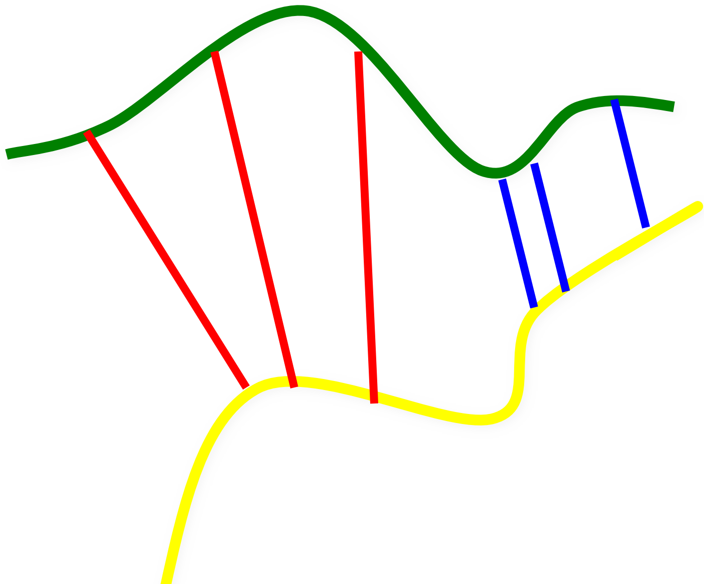
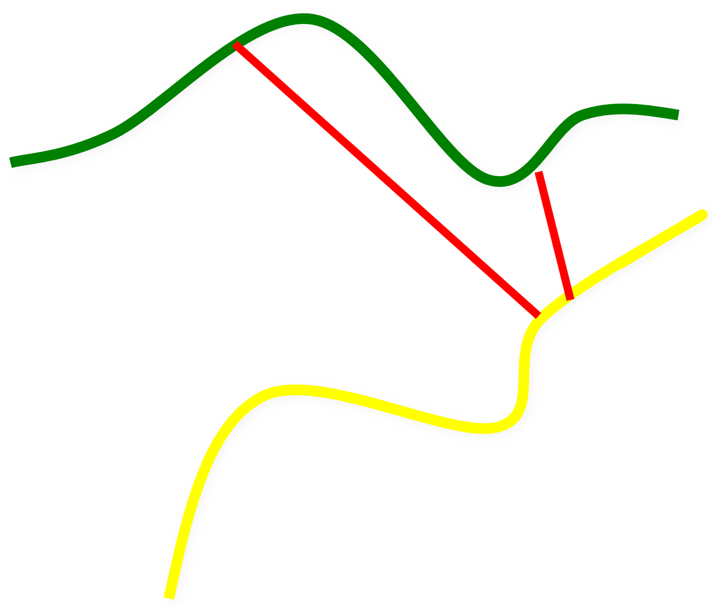
Rejecting (Outlier) Point Pairs

- Corresponding points with point to point distance higher than a given threshold



Rejecting (Outlier) Point Pairs

- Corresponding points with point to point distance higher than a given threshold
- Rejection of pairs that are not consistent with their neighboring pairs [Dorai 98]



Rejecting (Outlier) Point Pairs

- Corresponding points with point to point distance higher than a given threshold
- Rejection of pairs that are not consistent with their neighboring pairs [Dorai 98]
- Sort all correspondences with respect to their error and delete the worst *t*%, Trimmed ICP (TrICP) [Chetverikov et al. 02]
  - *t* is used to estimate the overlap
  - Problem: Knowledge about the overlap is necessary or has to be estimated

Summary: ICP Algorithm

- Potentially sample Points
- Determine corresponding points
- Potentially weight / reject pairs
- Compute rotation R, translation t (e.g. SVD)
- Apply R and t to all points of the set to be registered
- Compute the error $E(R,t)$
- If error decreased and error > threshold
  - Repeat to determine correspondences etc.
  - Stop and output final alignment, otherwise

ICP Summary

- ICP is a powerful algorithm for calculating the displacement between scans
- The major problem is to determine the correct data associations
- Convergence speed depends on point matched points
- Given the correct data associations, the transformation can be computed efficiently using SVD
- ICP does not always converge