Introduction to Mobile Robotics
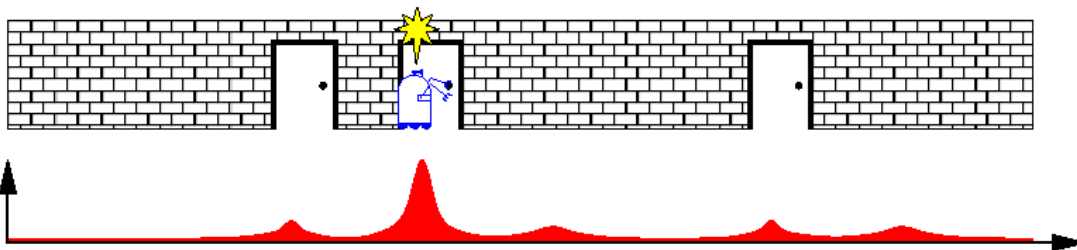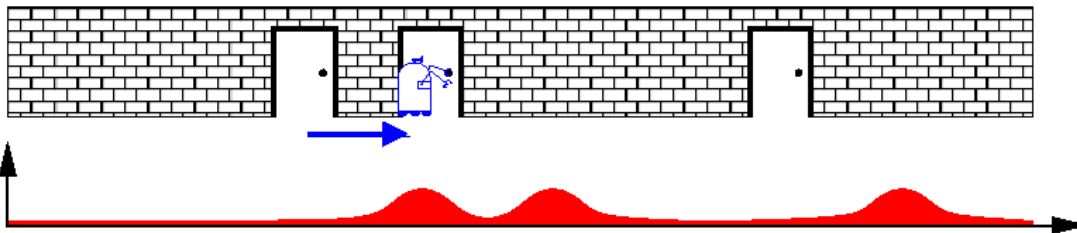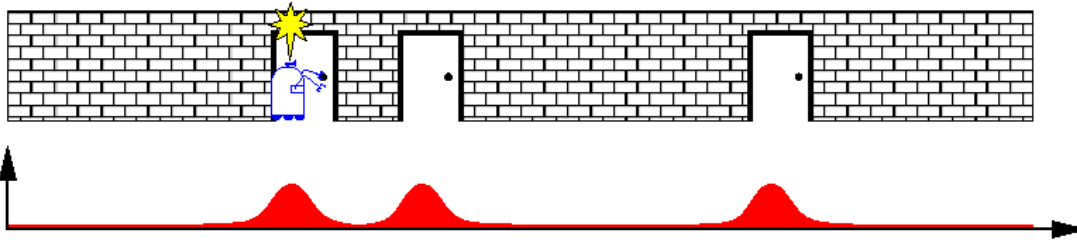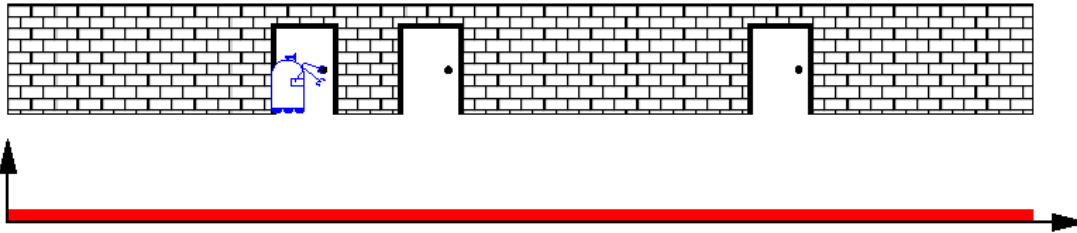
Bayes Filter – Discrete Filters

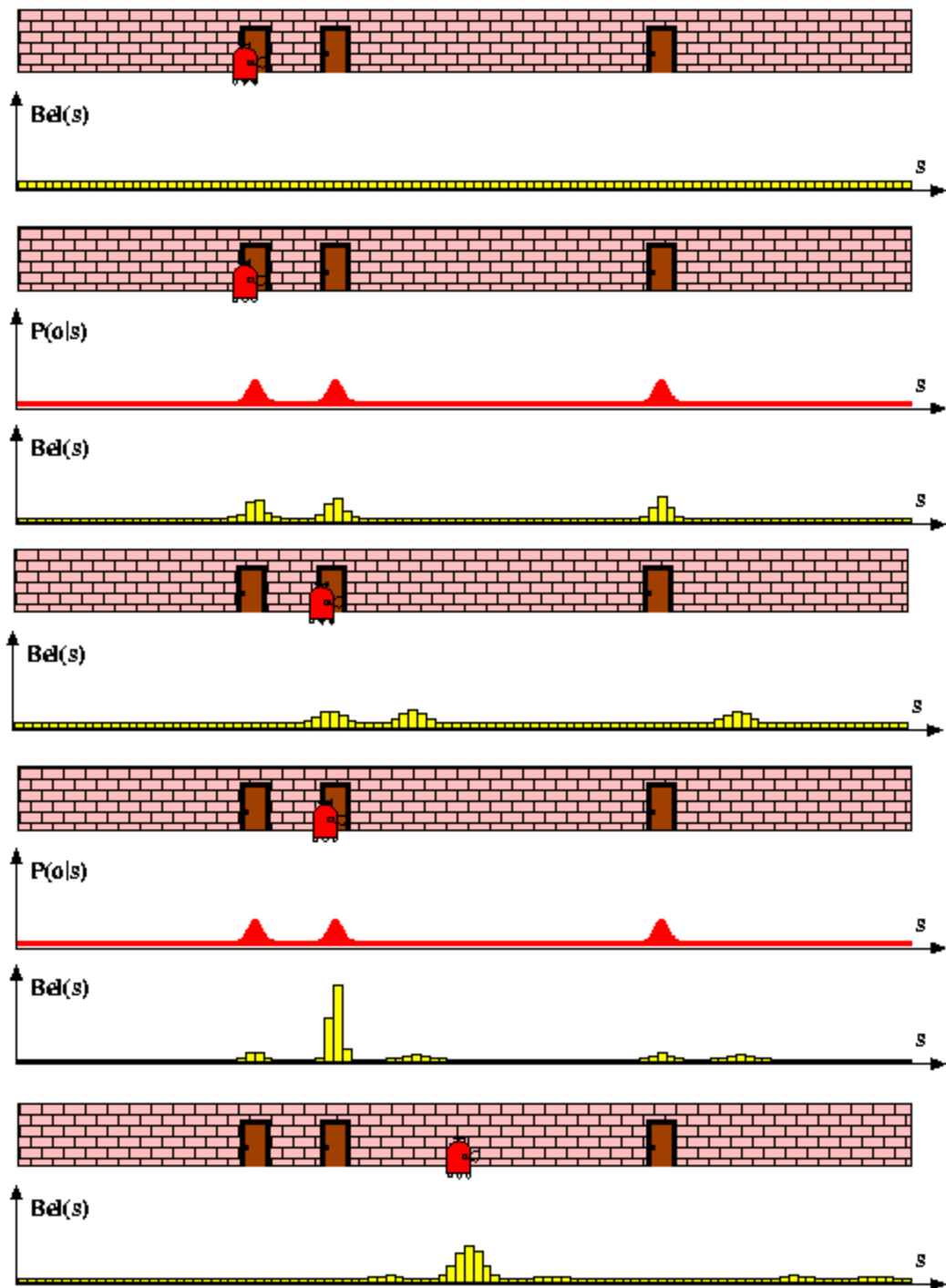Wolfram Burgard

$$Bel(x \mid z, u) \;=\; \alpha \, p(z \mid x) \int_{x'} p(x \mid u, x') \, Bel(x') dx'$$
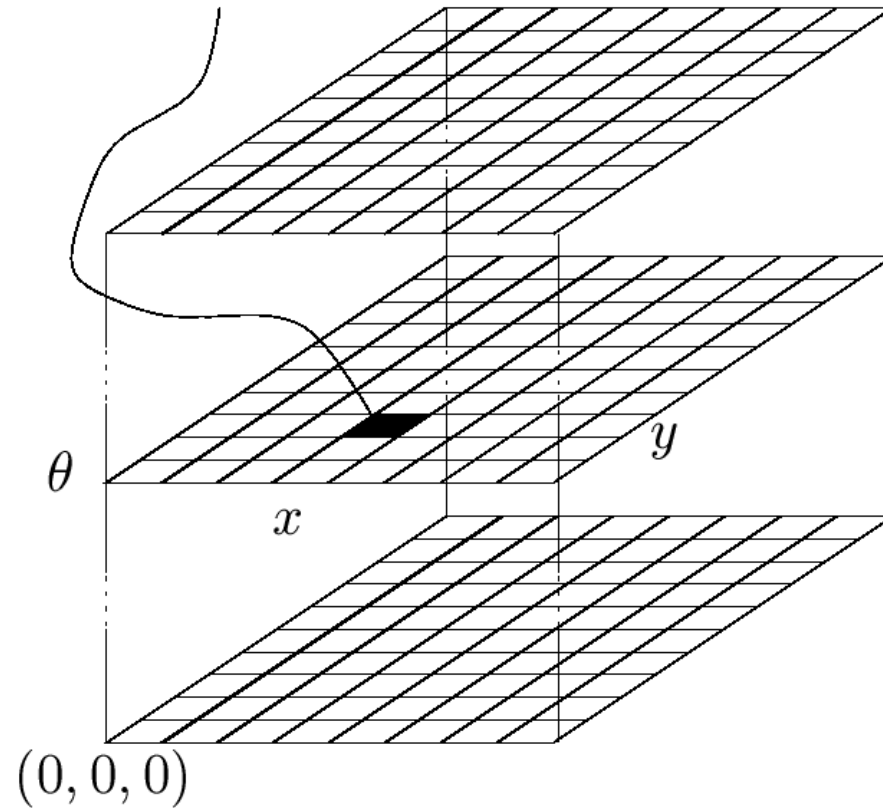
Piecewise Constant

Discrete Bayes Filter Algorithm

1. Algorithm **Discrete_Bayes_filter**(*Bel(x),d*):
2. *η=0*
3. If *d* is a perceptual data item *z* then
4.   For all *x* do
5. $Bel'(x) = P(z \mid x)Bel(x)$
6. $\eta = \eta + Bel'(x)$
7.   For all *x* do
8. $Bel'(x) = \eta^{-1}Bel'(x)$
9. Else if *d* is an action data item *u* then
10.   For all *x* do
11. $Bel'(x) = \sum_{x'} P(x \mid u, x') \, Bel(x')$
12. Return *Bel'(x)*

Piecewise Constant Representation

$$Bel(x_t = < x, y, \theta >)$$

Implementation (1)

- To update the belief upon sensory input and to carry out the normalization one has to iterate over all cells of the grid.

- Especially when the belief is peaked (which is generally the case during position tracking), one wants to avoid updating irrelevant aspects of the state space.

- One approach is not to update entire sub-spaces of the state space.

- This, however, requires to monitor whether the robot is de-localized or not.

- To achieve this, one can consider the likelihood of the observations given the active components of the state space.

Implementation (2)

- To efficiently update the belief upon robot motions, one typically assumes a bounded Gaussian model for the motion uncertainty.
- This reduces the update cost from $O(n^2)$ to $O(n)$, where n is the number of states.
- The update can also be realized by shifting the data in the grid according to the measured motion.
- In a second step, the grid is then convolved using a separable Gaussian Kernel.
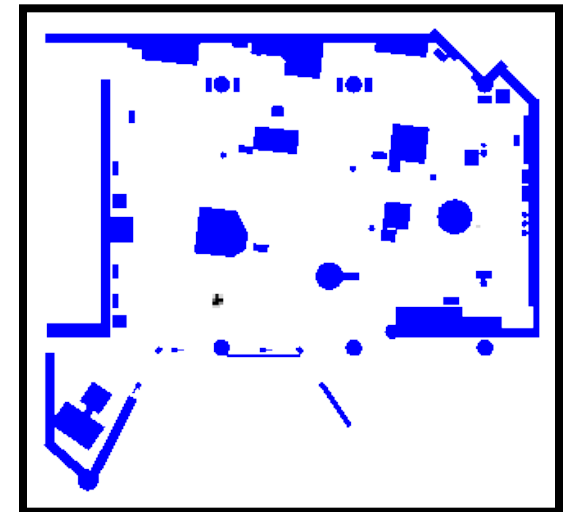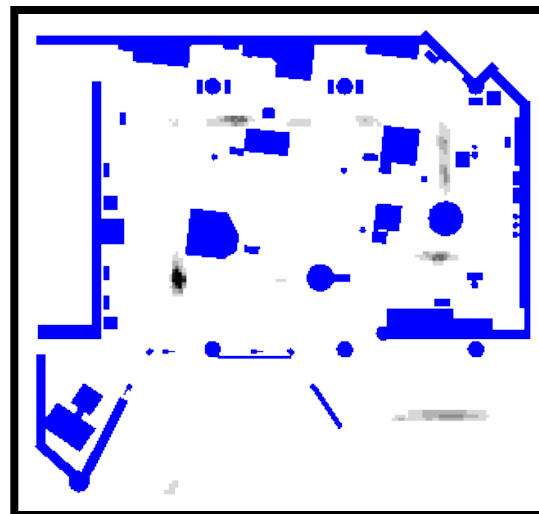- Two-dimensional example:

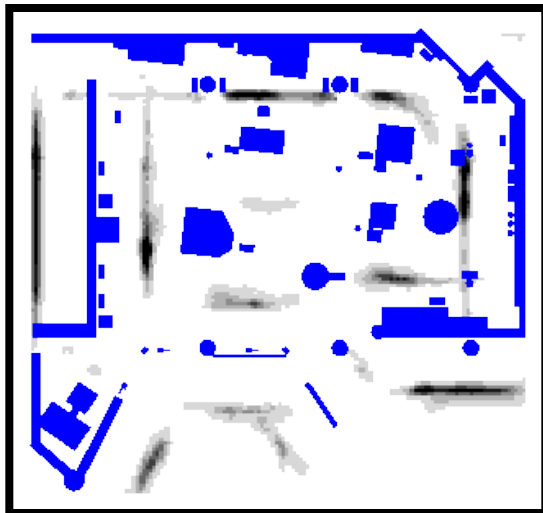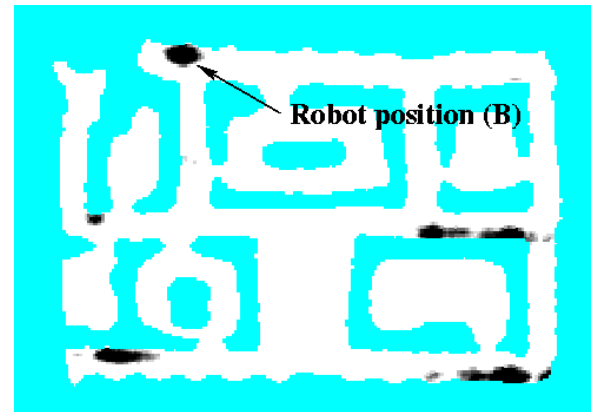| 1/16	1/8	1/16
1/8	1/4	1/8
1/16	1/8	1/16

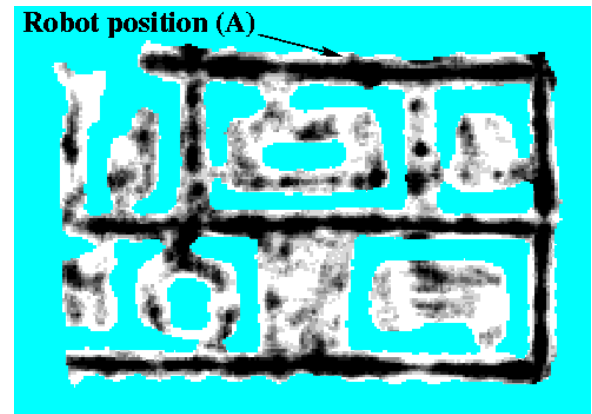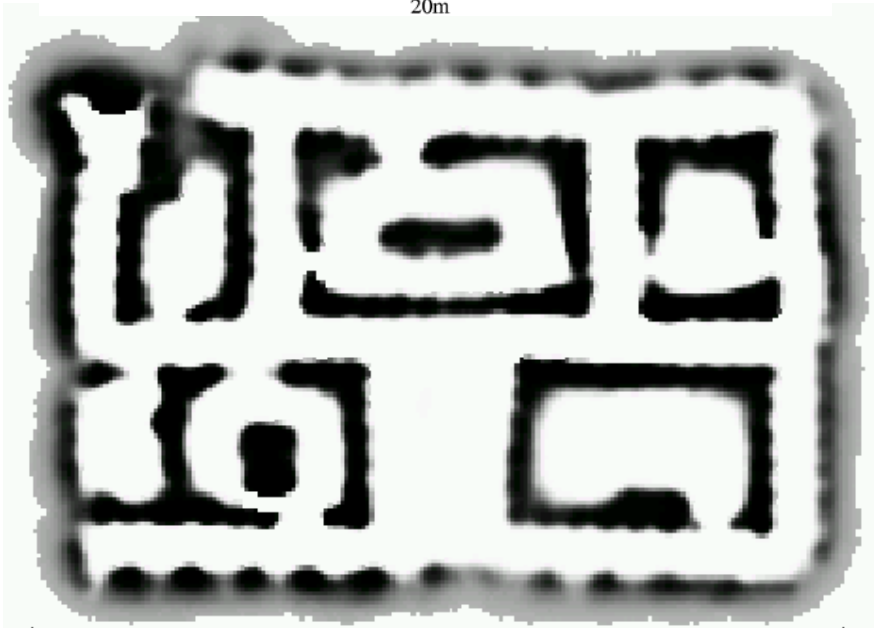$\cong$

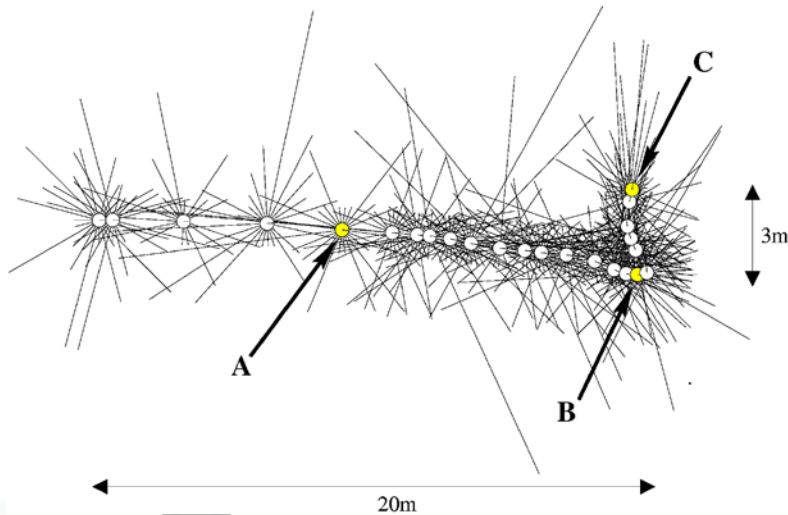| 1/4 |
|-----|
| 1/2 |
| 1/4 |

$+$

1/4	1/2	1/4

- Fewer arithmetic operations
- Easier to implement
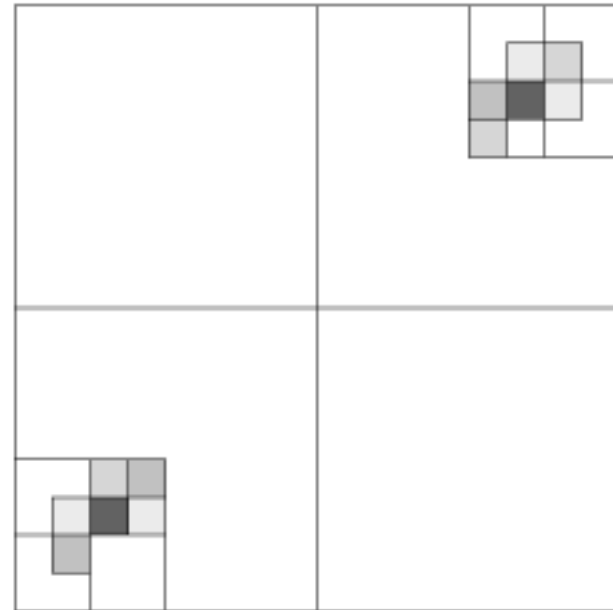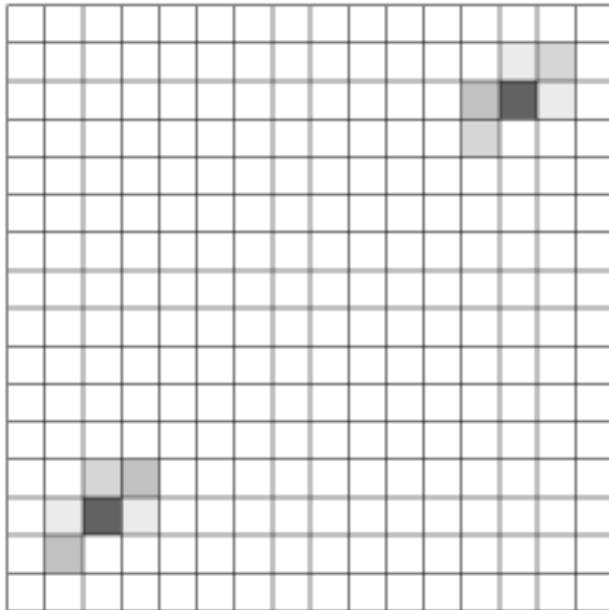
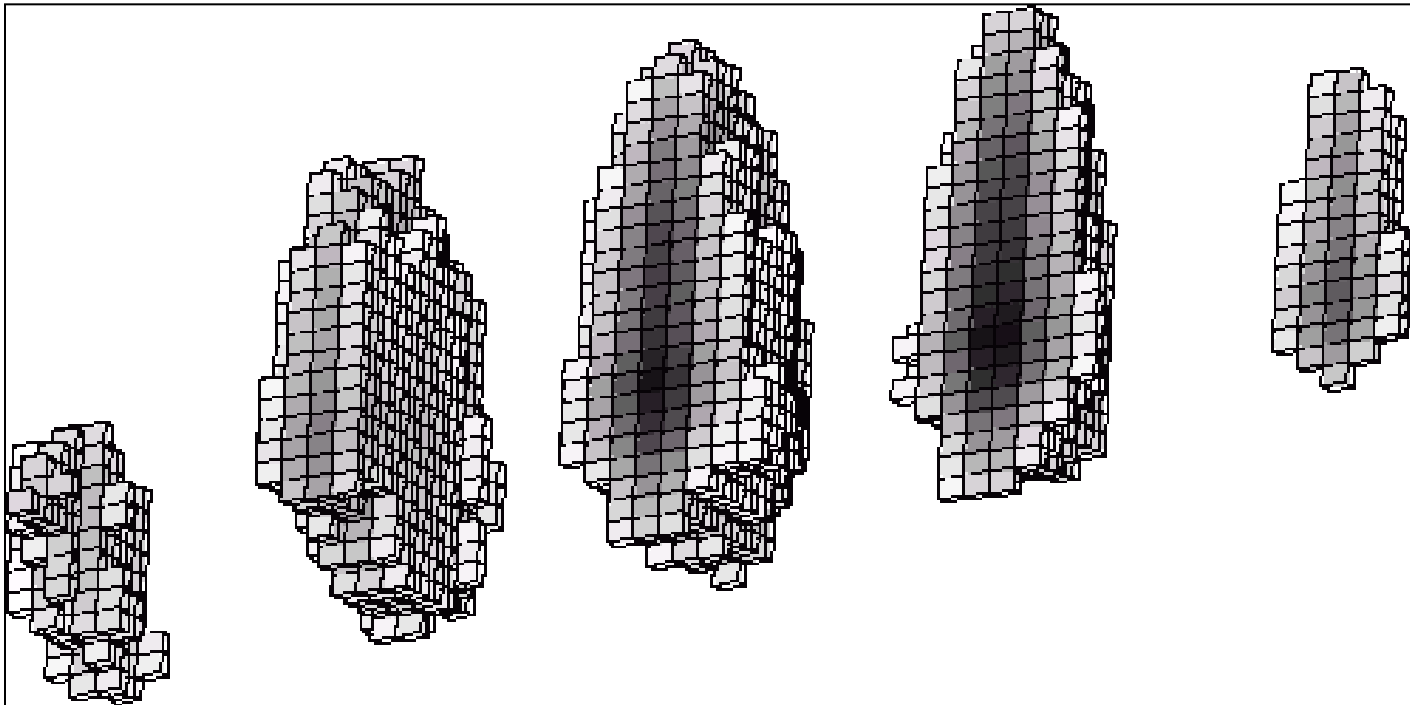# Grid-based Localization
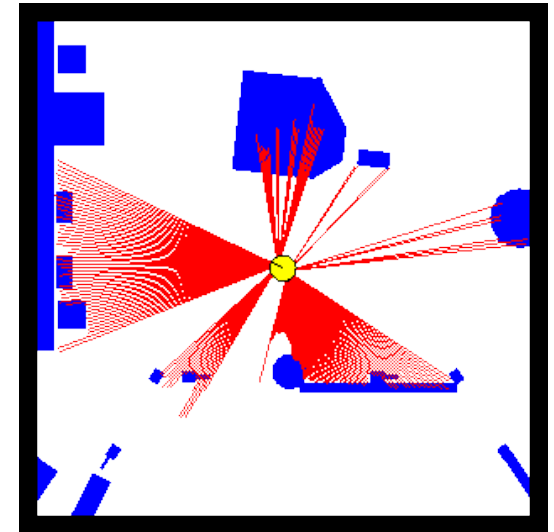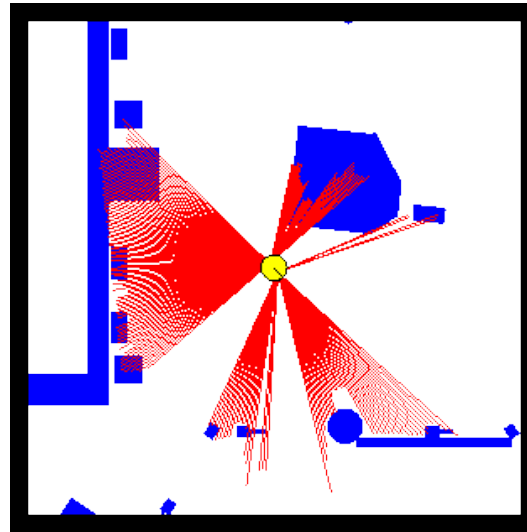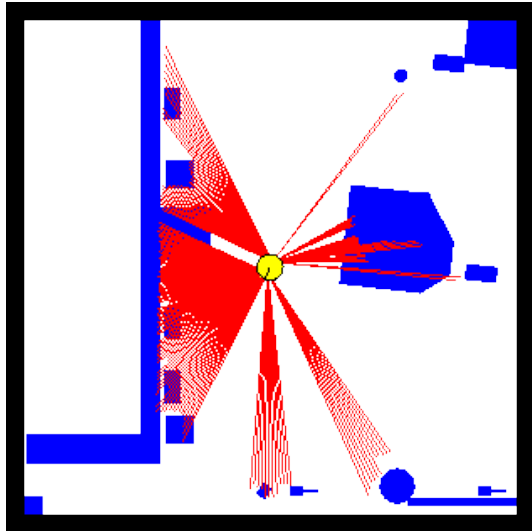
Sonars and Occupancy Grid Map
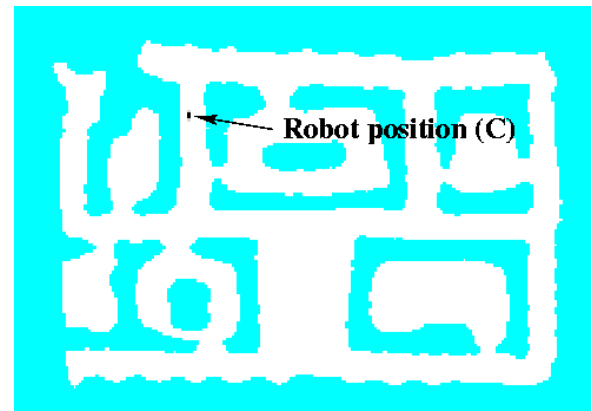
Tree-based Representation

Idea: Represent density using a variant of octrees

Tree-based Representations

- Efficient in space and time
- Multi-resolution

Summary

- Discrete filters are an alternative way for implementing Bayes Filters

- They are based on histograms for representing the density.

- They have huge memory and processing requirements

- Can easily recover from localization errors

- Their accuracy depends on the resolution of the grid.

- Special approximations need to be made to make this approach having dynamic memory and computational requirements.