Introduction to Mobile Robotics

Summary

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Probabilistic Robotics

Probabilistic Robotics

Key idea: Explicit representation of uncertainty

(using the calculus of probability theory)

- Perception = state estimation
- Action = utility optimization

Bayes Formula

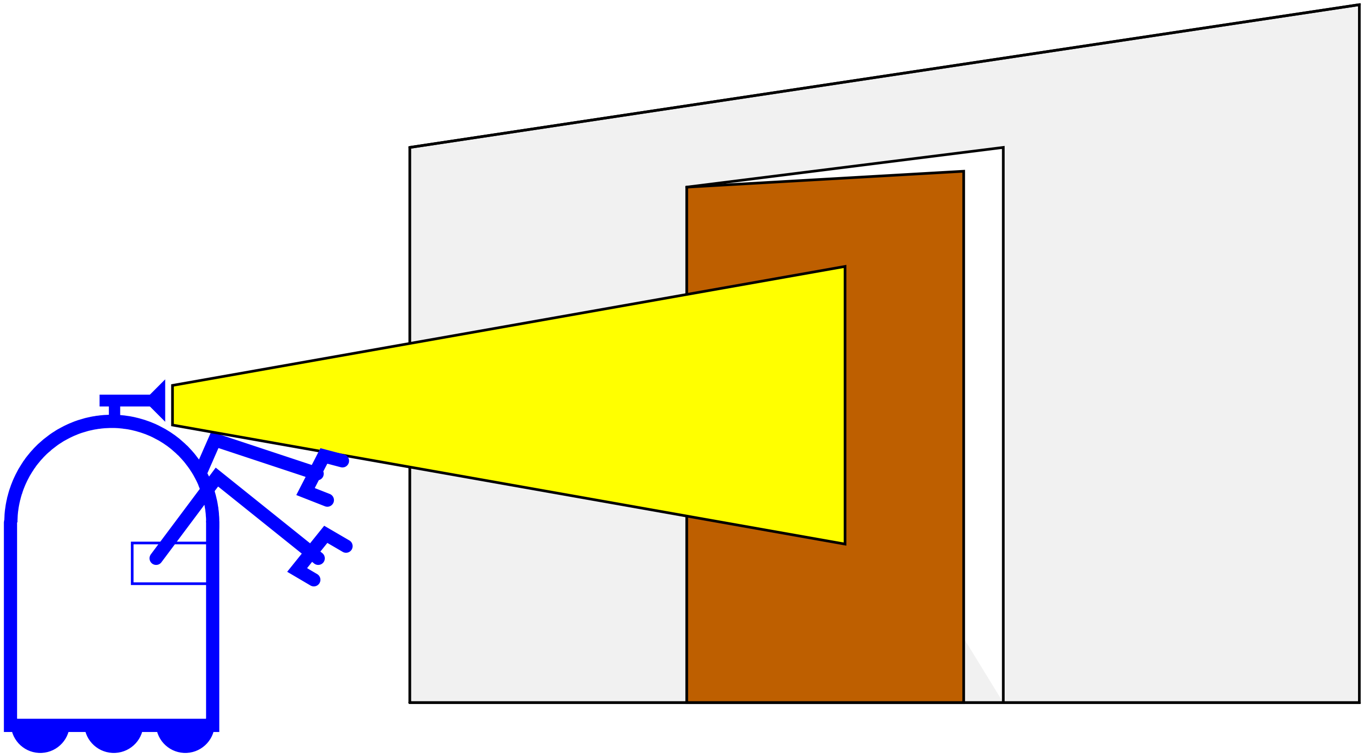
$$P(x, y) = P(x \mid y) P(y) = P(y \mid x) P(x)$$

$$\Rightarrow$$

$$\boxed{P(x \mid y) = \frac{P(y \mid x)\ P(x)}{P(y)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}}$$

Simple Example of State Estimation

- Suppose a robot obtains measurement z
- What is *P(open|z)?*



Causal vs. Diagnostic Reasoning

- *P(open|z)* is diagnostic.
- *P(z|open)* is causal.
- Often causal knowledge is easier to obtain.
- Bayes rule allows us to use causal knowledge:

count frequencies!

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z)}$$

Bayes Filters

z = observation
u = action
x = state

$$\boxed{Bel(x_t)} = P(x_t \mid u_1, z_1, \ldots, u_t, z_t)$$

Bayes $$= \eta\, P(z_t \mid x_t, u_1, z_1, \ldots, u_t)\, P(x_t \mid u_1, z_1, \ldots, u_t)$$

Markov $$= \eta\, P(z_t \mid x_t)\, P(x_t \mid u_1, z_1, \ldots, u_t)$$

Total prob. $$= \eta\, P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1})\, P(x_{t-1} \mid u_1, z_1, \ldots, u_t)\, dx_{t-1}$$

Markov $$= \eta\, P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1})\, P(x_{t-1} \mid u_1, z_1, \ldots, u_t)\, dx_{t-1}$$

Markov $$= \eta\, P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1})\, P(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1})\, dx_{t-1}$$

$$\boxed{= \eta\, P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1})\, Bel(x_{t-1})\, dx_{t-1}}$$

Bayes Filters are Familiar!

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

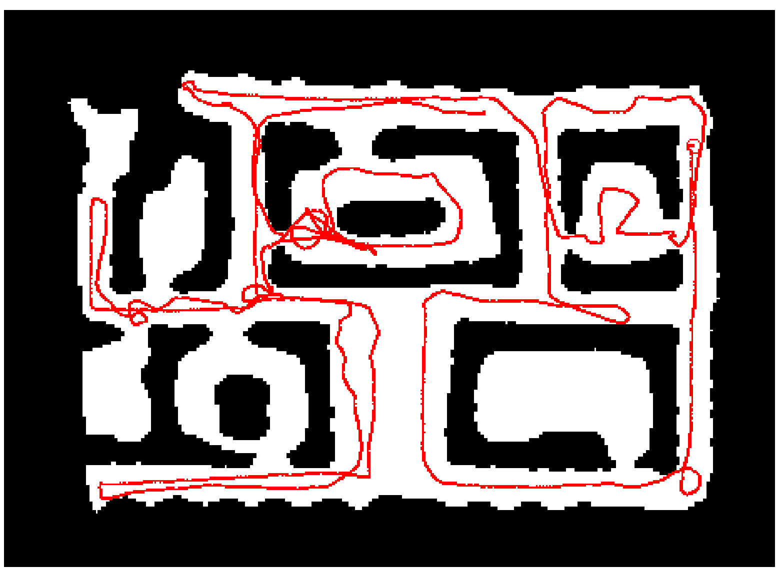
- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- …

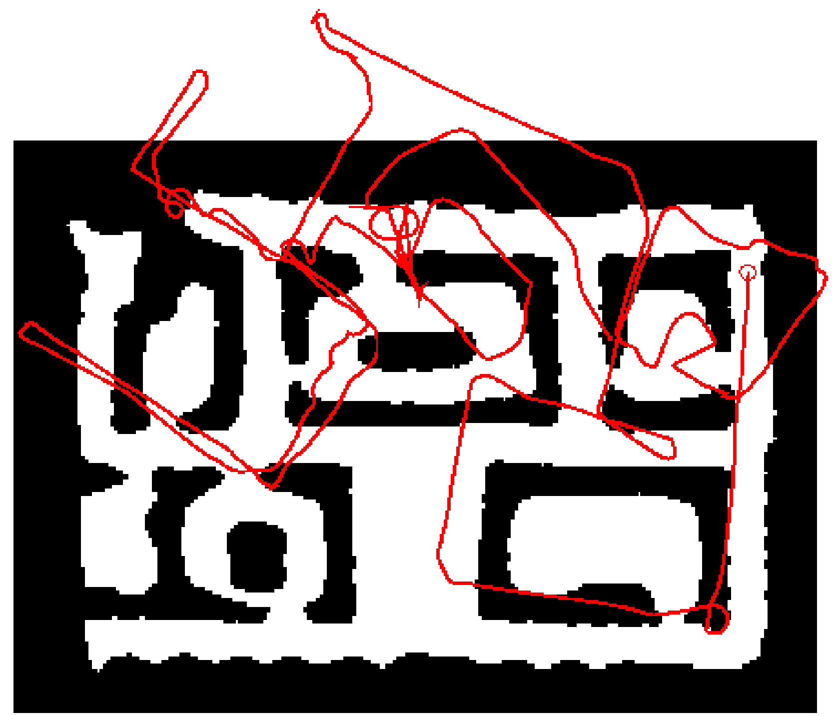
Sensor and Motion Models

$P(z \mid x, m)$ $P(x \mid x', u)$

Motion Models

- Robot motion is inherently uncertain.
- How can we model this uncertainty?





Probabilistic Motion Models

- To implement the Bayes Filter, we need the transition model $p(x \mid x', u)$.
- The term $p(x \mid x', u)$ specifies a posterior probability, that action u carries the robot from x' to x.

Typical Motion Models

- In practice, one often finds two types of motion models:
  - **Odometry-based**
  - **Velocity-based** (**dead reckoning**)
- Odometry-based models are used when systems are equipped with wheel encoders.
- Velocity-based models have to be applied when no wheel encoders are given.
- They calculate the new pose based on the velocities and the time elapsed.

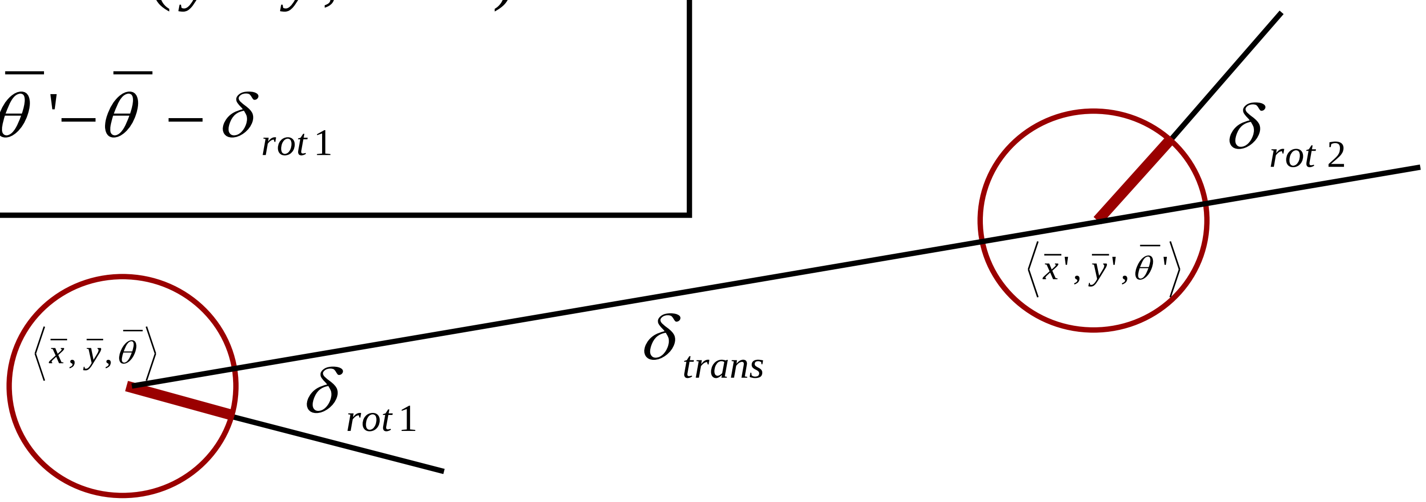
Odometry Model

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.
- Odometry information $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle$.

$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$

$$\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$



Sensors for Mobile Robots

- Contact sensors: Bumpers
- Internal sensors
  - Accelerometers (spring-mounted masses)
  - Gyroscopes (spinning mass, laser light)
  - Compasses, inclinometers (earth magnetic field, gravity)
- Proximity sensors
  - Sonar (time of flight)
  - Radar (phase and frequency)
  - Laser range-finders (triangulation, tof, phase)
  - Infrared (intensity)
- Visual sensors: Cameras
- Satellite-based sensors: GPS

Beam-based Sensor Model

- Scan z consists of K measurements.

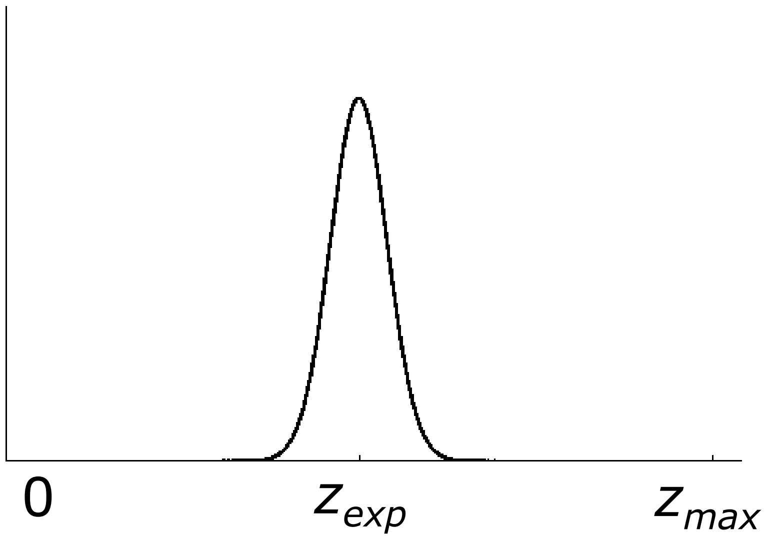
$$z = \{z_1, z_2, \ldots, z_K\}$$

- Individual measurements are independent given the robot position.

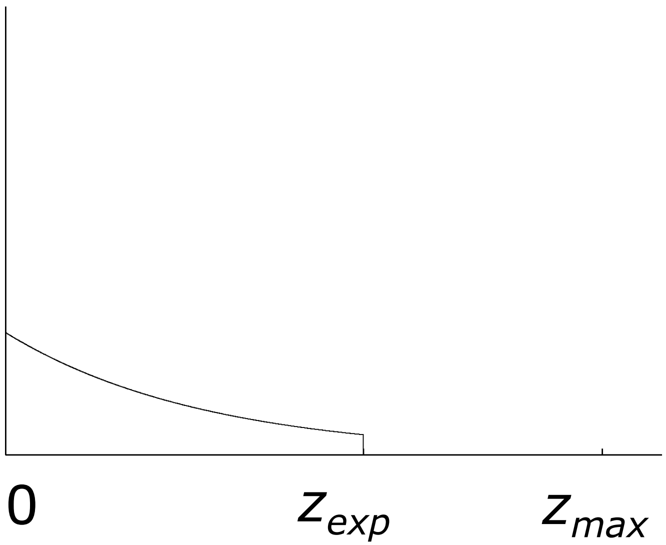
$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

Beam-based Proximity Model

Measurement noise



Unexpected obstacles

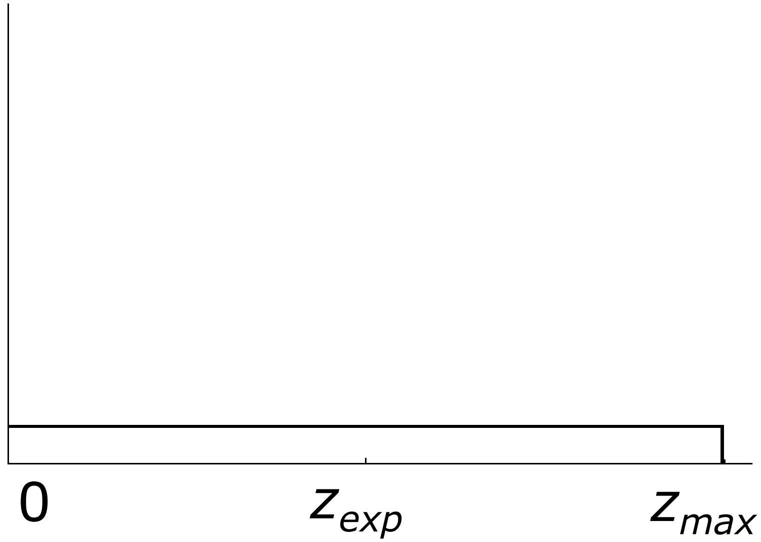


$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\frac{(z - z_{\exp})^2}{b}}$$

$$P_{\text{unexp}}(z \mid x, m) = \begin{Bmatrix} \eta \lambda e^{-\lambda z} & z < z_{\exp} \\ 0 & \mathit{otherwise} \end{Bmatrix}$$

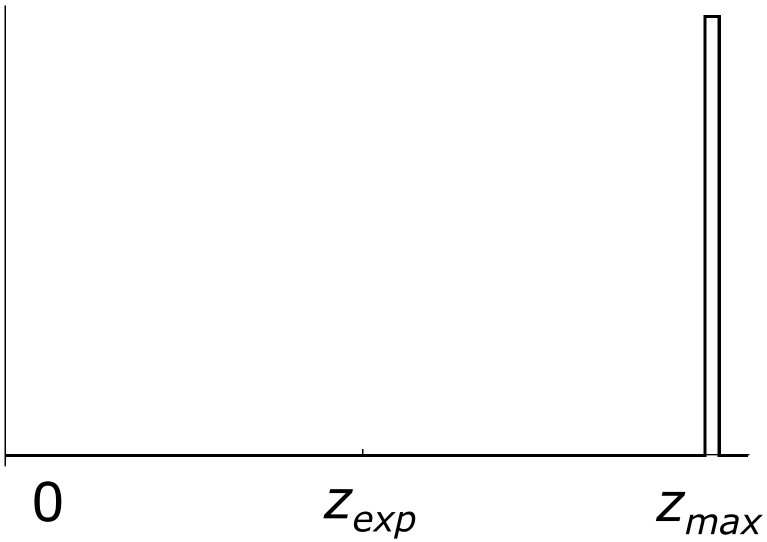
Beam-based Proximity Model

Random measurement



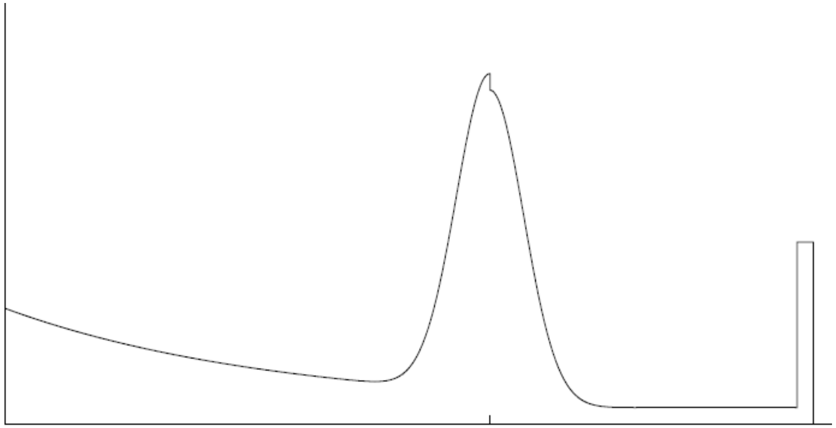
$$P_{rand}(z \mid x, m) = \eta \frac{1}{z_{\max}}$$

Max range



$$P_{\max}(z \mid x, m) = \eta \frac{1}{z_{small}}$$

Resulting Mixture Density



$$P(z \mid x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\text{max}} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\text{hit}}(z \mid x, m) \\ P_{\text{unexp}}(z \mid x, m) \\ P_{\max}(z \mid x, m) \\ P_{\text{rand}}(z \mid x, m) \end{pmatrix}$$

How can we determine the model parameters?

Bayes Filter in Robotics

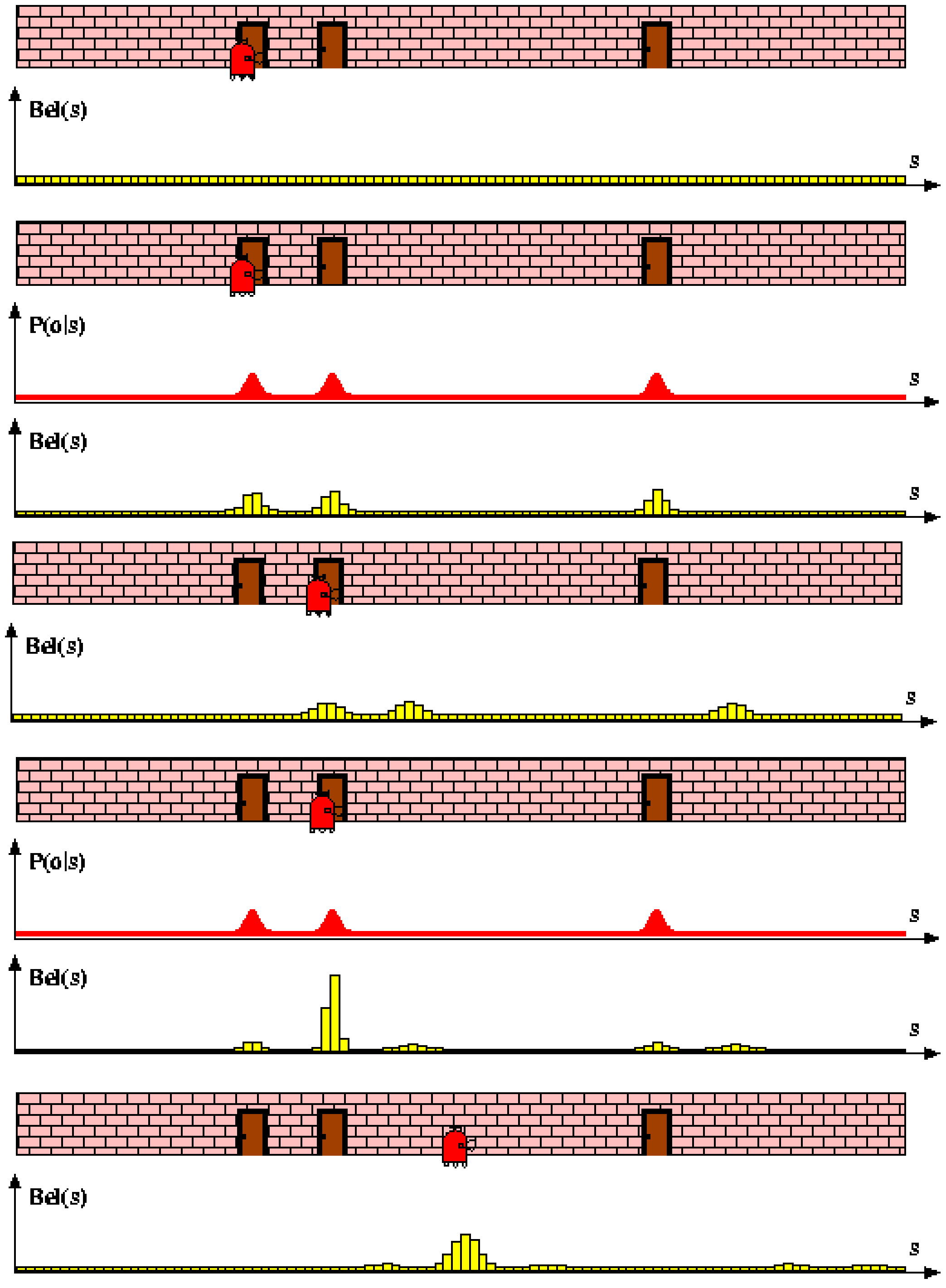
Bayes Filters in Action

- Discrete filters
- Kalman filters
- Particle filters

Discrete Filter

- The belief is typically stored in a histogram / grid representation
- To update the belief upon sensory input and to carry out the normalization one has to iterate over all cells of the grid

Piecewise Constant



Kalman Filter

- Optimal for linear Gaussian systems!
- Most robotics systems are nonlinear!
- Polynomial in measurement dimensionality k and state dimensionality n:

$$O(k^{2.376} + n^2)$$

Kalman Filter Algorithm

1. Algorithm **Kalman_filter**(μ_{t-1}, Σ_{t-1}, u_t, z_t):

2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + Q_t$

5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + R_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

Extended Kalman Filter

- Approach to handle non-linear models
- Performs a linearization in each step
- Not optimal
- Can diverge if nonlinearities are large!
- Works surprisingly well even when all assumptions are violated!
- Same complexity than the KF

Particle Filter

- Basic principle
  - Set of state hypotheses (“particles”)
  - Survival-of-the-fittest

- Particle filters are a way to efficiently represent non-Gaussian distributions

Mathematical Description

- Set of weighted samples

$$S = \left\{ \left\langle s^{[i]}, w^{[i]} \right\rangle \mid i = 1, \dots, N \right\}$$

State hypothesis Importance weight

- The samples represent the posterior

$$p(x) = \sum_{i=1}^{N} w_i \cdot \delta_{s^{[i]}}(x)$$

Particle Filter Algorithm in Brief

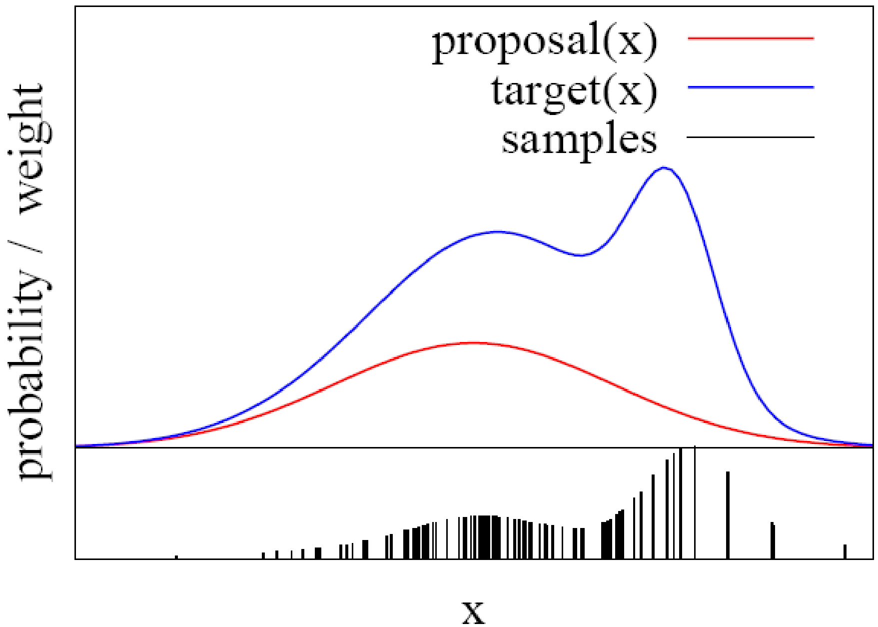
- Sample the next generation for particles using the proposal distribution

- Compute the importance weights :
  $weight = target\ distribution\ /\ proposal\ distribution$

- Resampling: “Replace unlikely samples by more likely ones”

Importance Sampling Principle

- We can even use a different distribution g to generate samples from f
- By introducing an importance weight w, we can account for the "differences between g and f "
- $w = f / g$
- f is often called target
- g is often called proposal
- Pre-condition:
  $f(x)>0 \rightarrow g(x)>0$



Particle Filter Algorithm

1. Algorithm **particle_filter**(S_{t-1}, u_{t-1} z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \ldots n$ ***Generate new samples***
4. Sample index *j(i)* from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}
6. $w_t^i = p(z_t \mid x_t^i)$ ***Compute importance weight***
7. $\eta = \eta + w_t^i$ ***Update normalization factor***
8. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$ ***Insert***
9. **For** $i = 1 \ldots n$
10. $w_t^i = w_t^i / \eta$ ***Normalize weights***

Particle Filter Algorithm

$$Bel(x_t) = \eta\ p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\ Bel(x_{t-1})\ dx_{t-1}$$

draw x^i_{t-1} from $Bel(\mathrm{x}_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

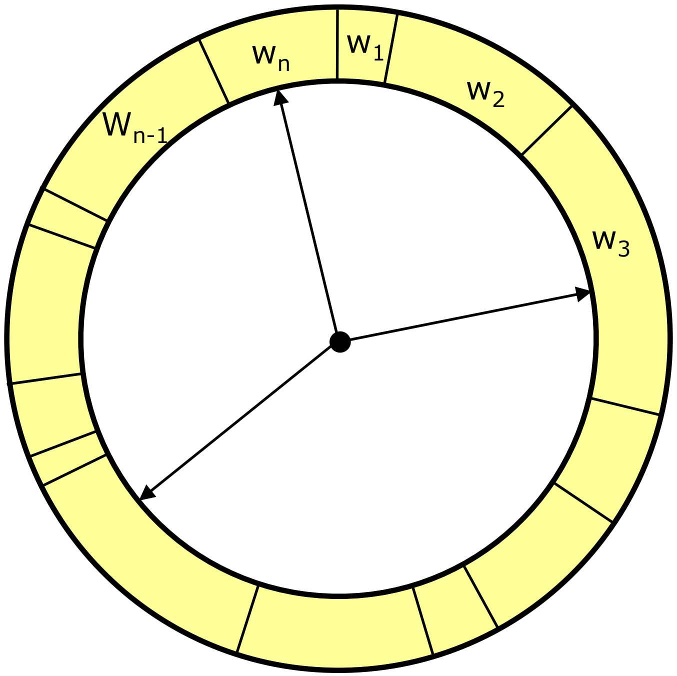
Importance factor for x^i_t:

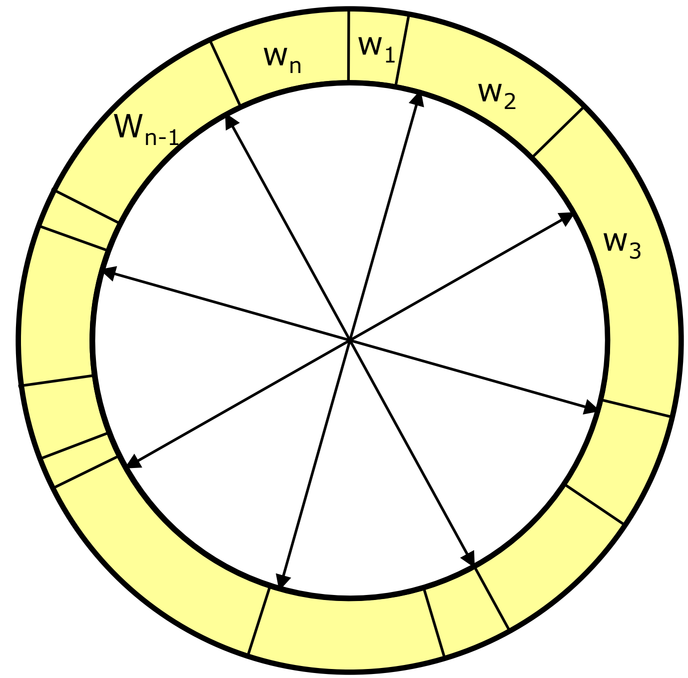
$$w_t^i = \frac{\text{target distribution}}{\text{proposal distribution}}$$

$$= \frac{\eta\, p(z_t \mid x_t)\, p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})}$$

$$\propto p(z_t \mid x_t)$$

Resampling

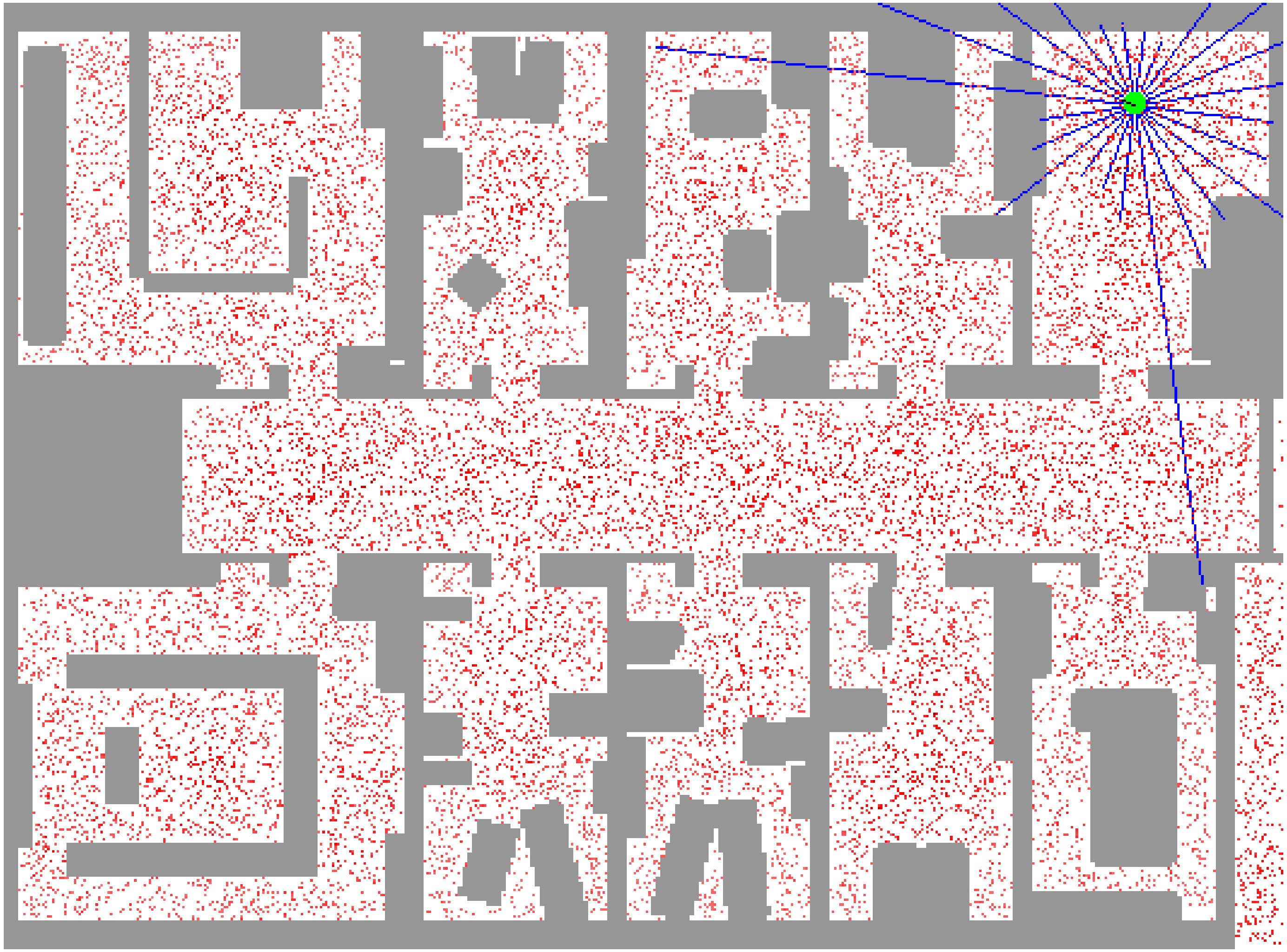




- Roulette wheel
- Binary search, n log n

- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

MCL Example



Mapping

Why Mapping?

- Learning maps is one of the fundamental problems in mobile robotics
- Maps allow robots to efficiently carry out their tasks, allow localization …
- Successful robot systems rely on maps for localization, path planning, activity planning etc

Occupancy Grid Maps

- Discretize the world into equally spaced cells
- Each cells stores the probability that the corresponding area is occupied by an obstacle
- The cells are assumed to be conditionally independent
- If the pose of the robot is know, mapping is easy

Updating Occupancy Grid Maps

- Update the map cells using the inverse sensor model

$$Bel\left(m_t^{[xy]}\right) = 1 - \left(1 + \frac{P\left(m_t^{[xy]} \mid z_t, u_{t-1}\right)}{1 - P\left(m_t^{[xy]} \mid z_t, u_{t-1}\right)} \cdot \frac{1 - P\left(m_t^{[xy]}\right)}{P\left(m_t^{[xy]}\right)} \cdot \frac{Bel\left(m_{t-1}^{[xy]}\right)}{1 - Bel\left(m_{t-1}^{[xy]}\right)}\right)^{-1}$$

- Or use the log-odds representation

$$\begin{aligned} \overline{B}\left(m_t^{[xy]}\right) = \log odds\left(m_t^{[xy]} \mid z_t, u_{t-1}\right) \\ - \log odds\left(m_t^{[xy]}\right) \\ + \overline{B}\left(m_{t-1}^{[xy]}\right) \end{aligned}$$

$$\overline{B}\left(m_t^{[xy]}\right) = \log odds\left(m_t^{[xy]}\right)$$

$$odds(x) = \left(\frac{P(x)}{1 - P(x)}\right)$$

Reflection Probability Maps

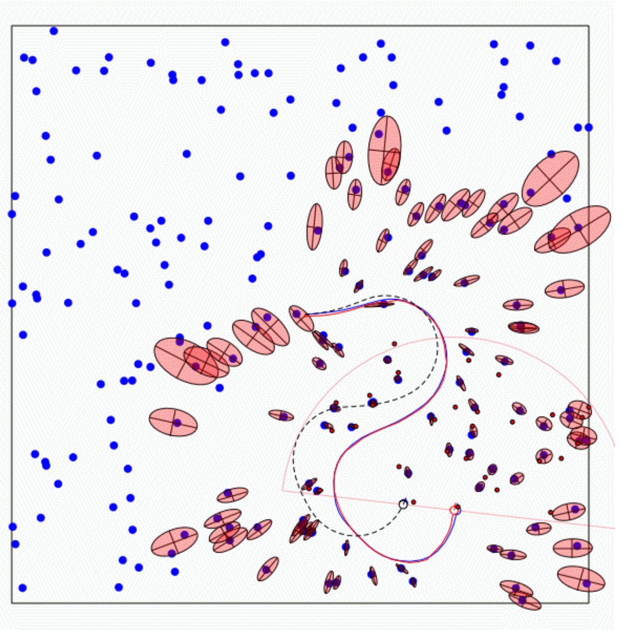
- Value of interest: $P(reflects(x,y))$
- For every cell count
  - hits(x,y): number of cases where a beam ended at <x,y>
  - misses(x,y): number of cases where a beam passed through <x,y>

$$\boxed{Bel(m^{[xy]}) = \frac{\text{hits}(x,y)}{\text{hits}(x,y) + \text{misses}(x,y)}}$$

SLAM

The SLAM Problem

A robot is exploring an unknown, static environment.



Given:

- The robot’s controls
- Observations of nearby features

Estimate:

- Map of features
- Path of the robot

Chicken-and-Egg-Problem

- SLAM is a chicken-and-egg problem
  - A map is needed for localizing a robot
  - A good pose estimate is needed to build a map
- Thus, SLAM is regarded as a hard problem in robotics
- A variety of different approaches to address the SLAM problem have been presented
- Probabilistic methods outperform most other techniques

SLAM:
Simultaneous Localization and Mapping

- Full SLAM: $p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$

Estimates entire path and map!

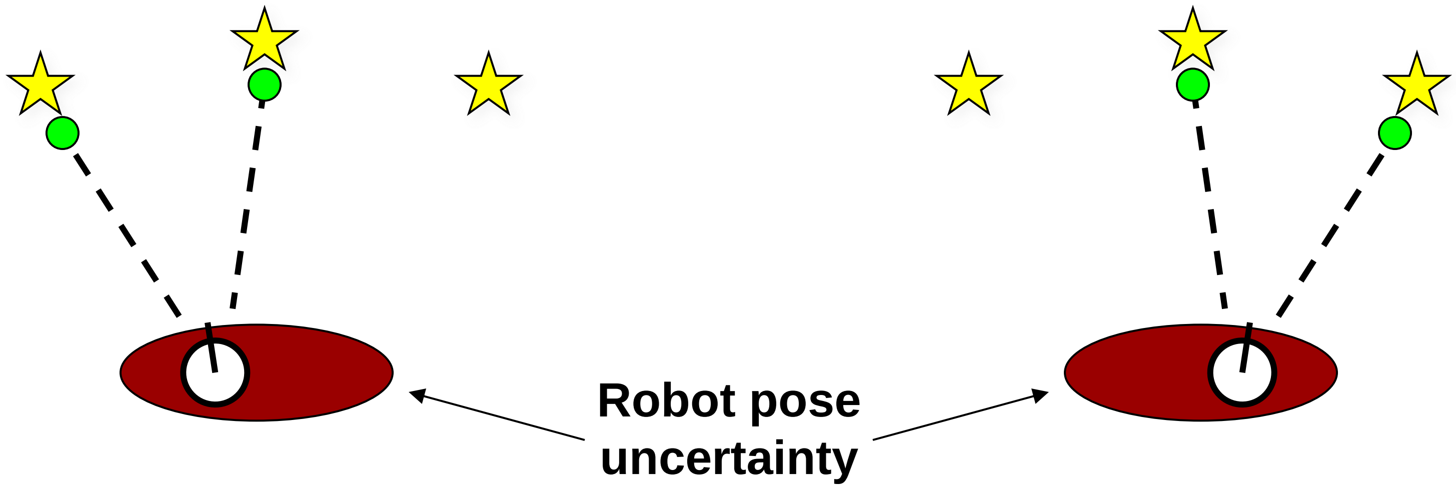
- Online SLAM:

$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \dots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) dx_1 dx_2 \dots dx_{t-1}$$

Integrations typically done one at a time

Estimates most recent pose and map!

Why is SLAM a hard problem?



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations

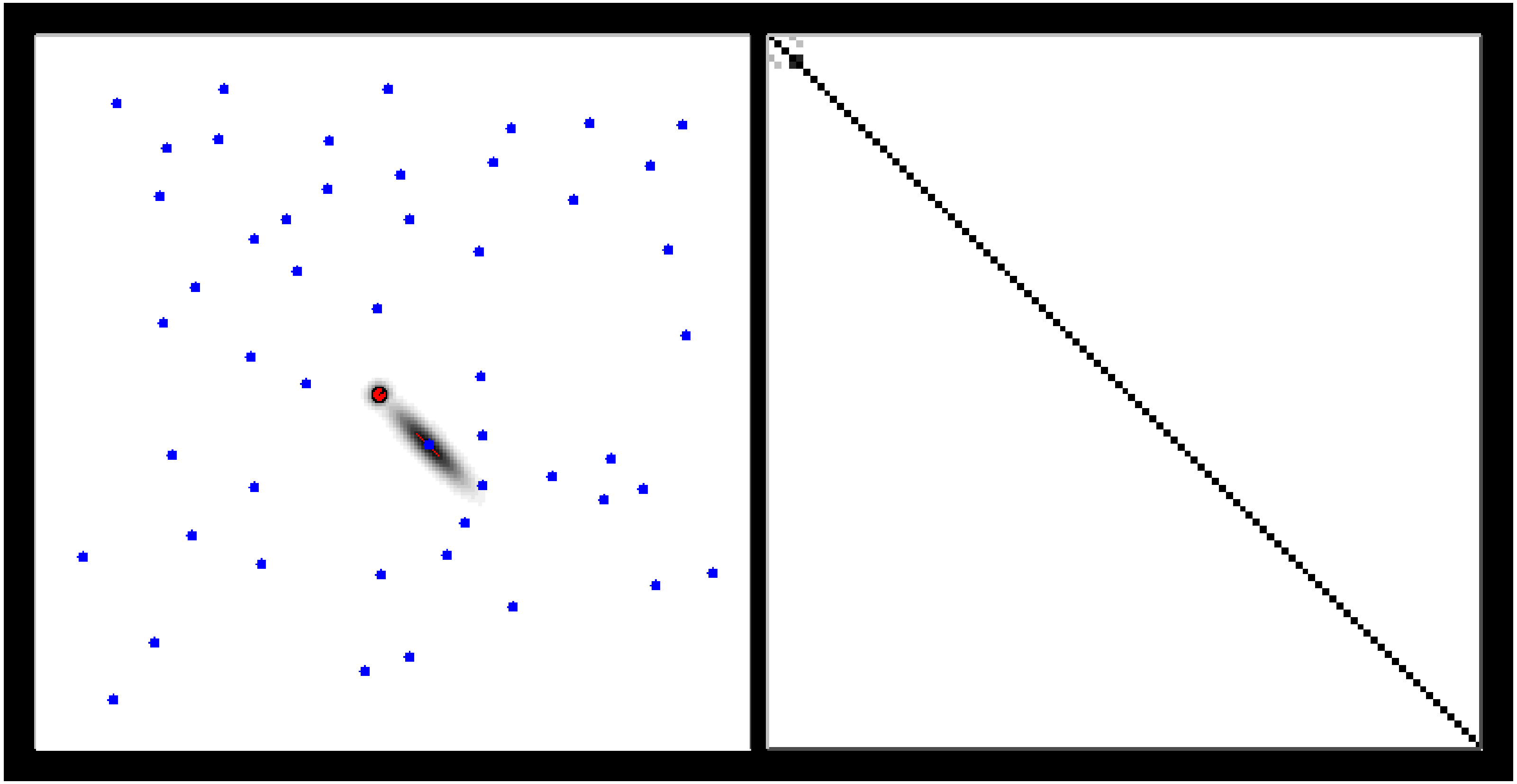
(E)KF-SLAM

- Map with N landmarks:(3+2N)-dimensional Gaussian

$$Bel(x_t, m_t) = \left\langle \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix} \right\rangle$$

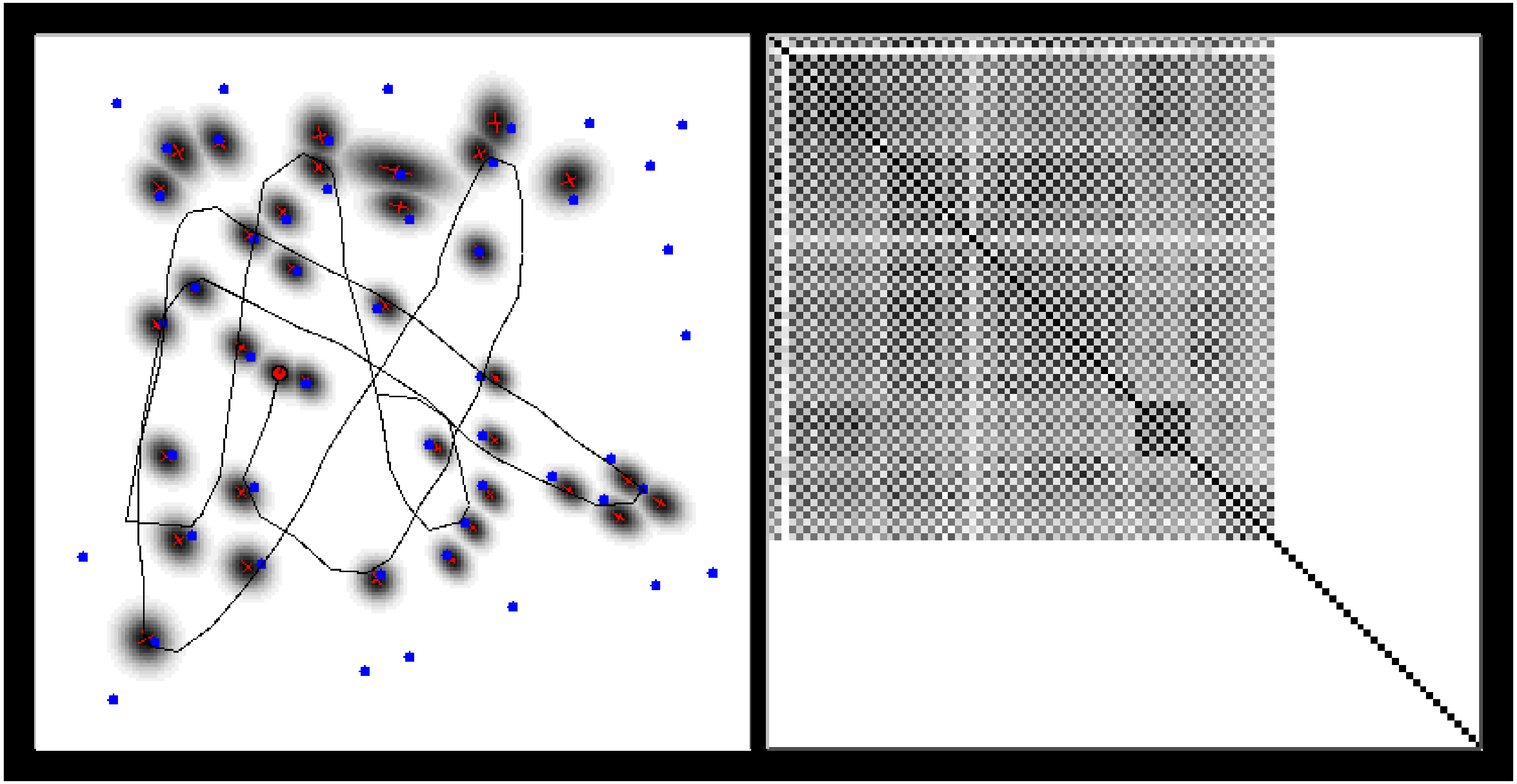
- Can handle hundreds of dimensions

EKF-SLAM



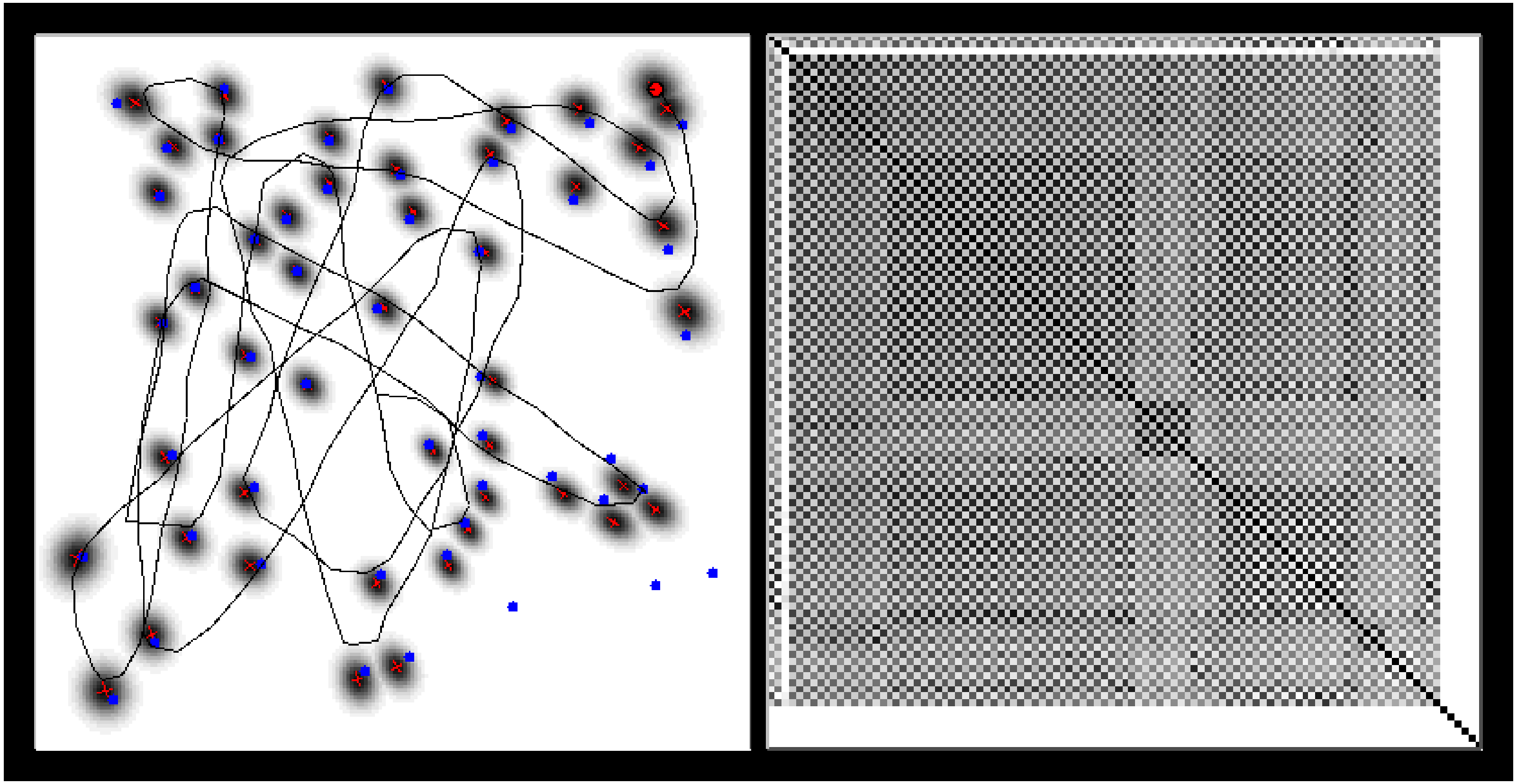
Map Correlation matrix

EKF-SLAM



Map Correlation matrix

EKF-SLAM



Map Correlation matrix

FastSLAM

- Use a particle filter for map learning
- Problem: the map is high-dimensional
- Solution: separate the estimation of the robot's trajectory from the one of the map of the environment
- This is done by means of a factorization in the SLAM posterior often called Rao-Blackwellization

Rao-Blackwellization

poses map observations & movements

$$p(x_{1:t}, m \mid z_{1:t}, u_{0:t-1}) =$$

$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(m \mid x_{1:t}, z_{1:t})$$

SLAM posterior

Robot path posterior

Mapping with known poses

Factorization first introduced by Murphy in 1999

Rao-Blackwellized Mapping

- Each particle represents a possible trajectory of the robot

- Each particle
  - maintains its own map and
  - updates it upon "mapping with known poses"

- Each particle survives with a probability proportional to the likelihood of the observations relative to its own map

FastSLAM

- Rao-Blackwellized particle filtering based on landmarks
- Each landmark is represented by a 2x2 Extended Kalman Filter (EKF)
- Each particle therefore has to maintain *M* EKFs

Particle #1	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**
Particle #2	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**
⋮					
Particle N	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**

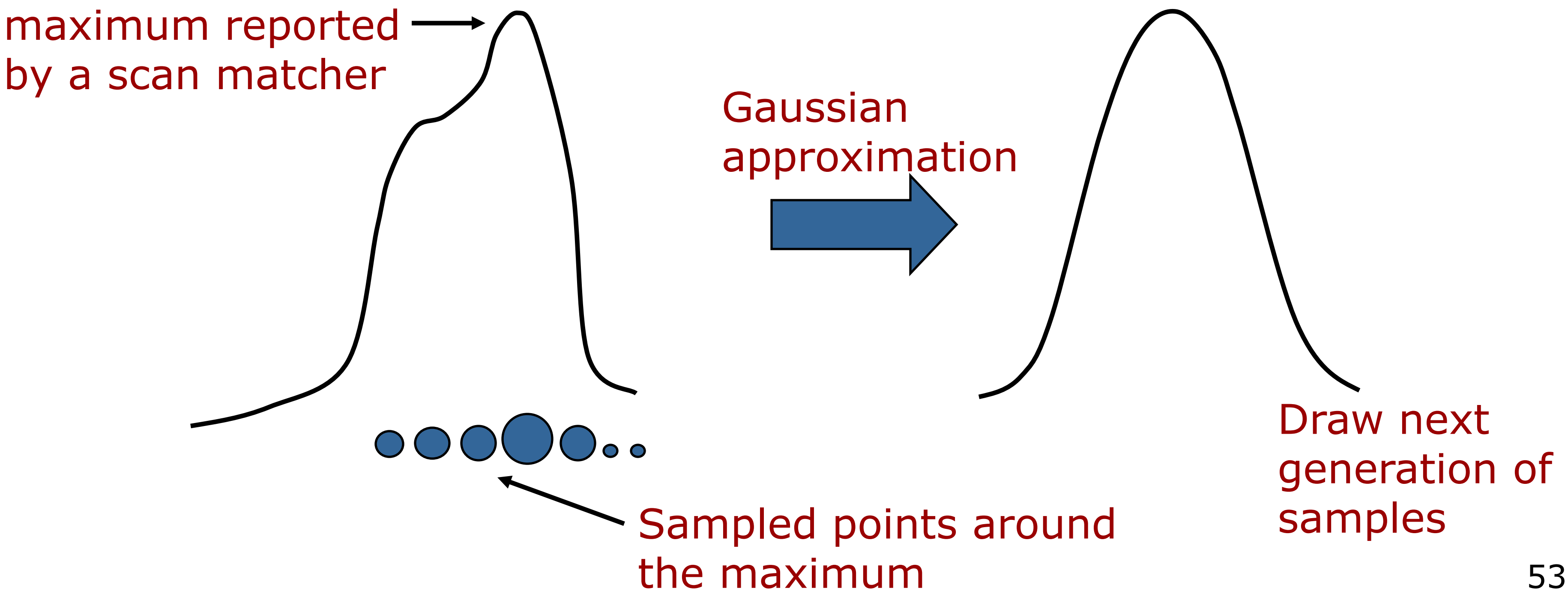
Grid-based FastSLAM

- Similar ideas can be used to learn grid maps
- To obtain a practical solution, an efficiently computable, informed proposal distribution is needed
- Idea: in the SLAM posterior, the observation model dominates the motion model (given an accurate sensor)

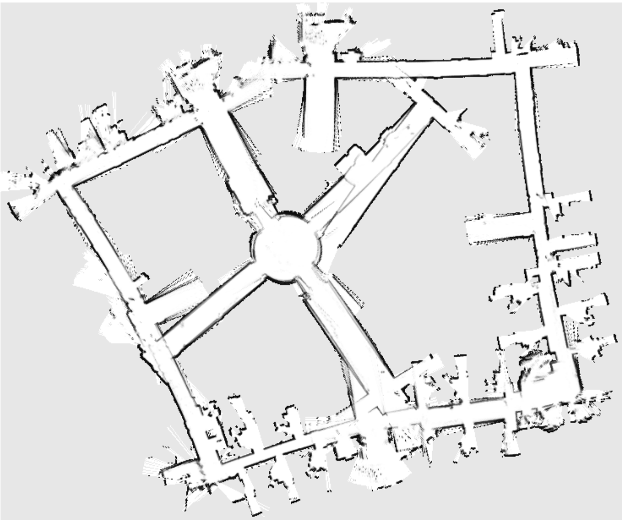
Proposal Distribution

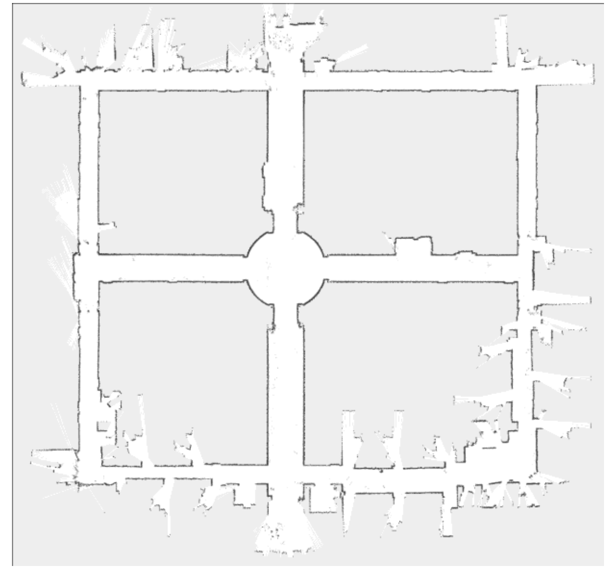
$$p(x_t|x_{t-1}^{(i)}, m^{(i)}, z_t, u_t) \simeq \frac{p(z_t|x_t, m^{(i)})}{\int_{x_t \in \{x|p(z_t|x,m^{(i)})>\epsilon\}} p(z_t|x_t, m^{(i)}) dx_t}$$

Approximate this equation by a Gaussian:



Typical Results





Robot Motion

Robot Motion Planning

Latombe (1991): "... eminently necessary since, by definition, a robot accomplishes tasks by moving in the real world."

Goals:

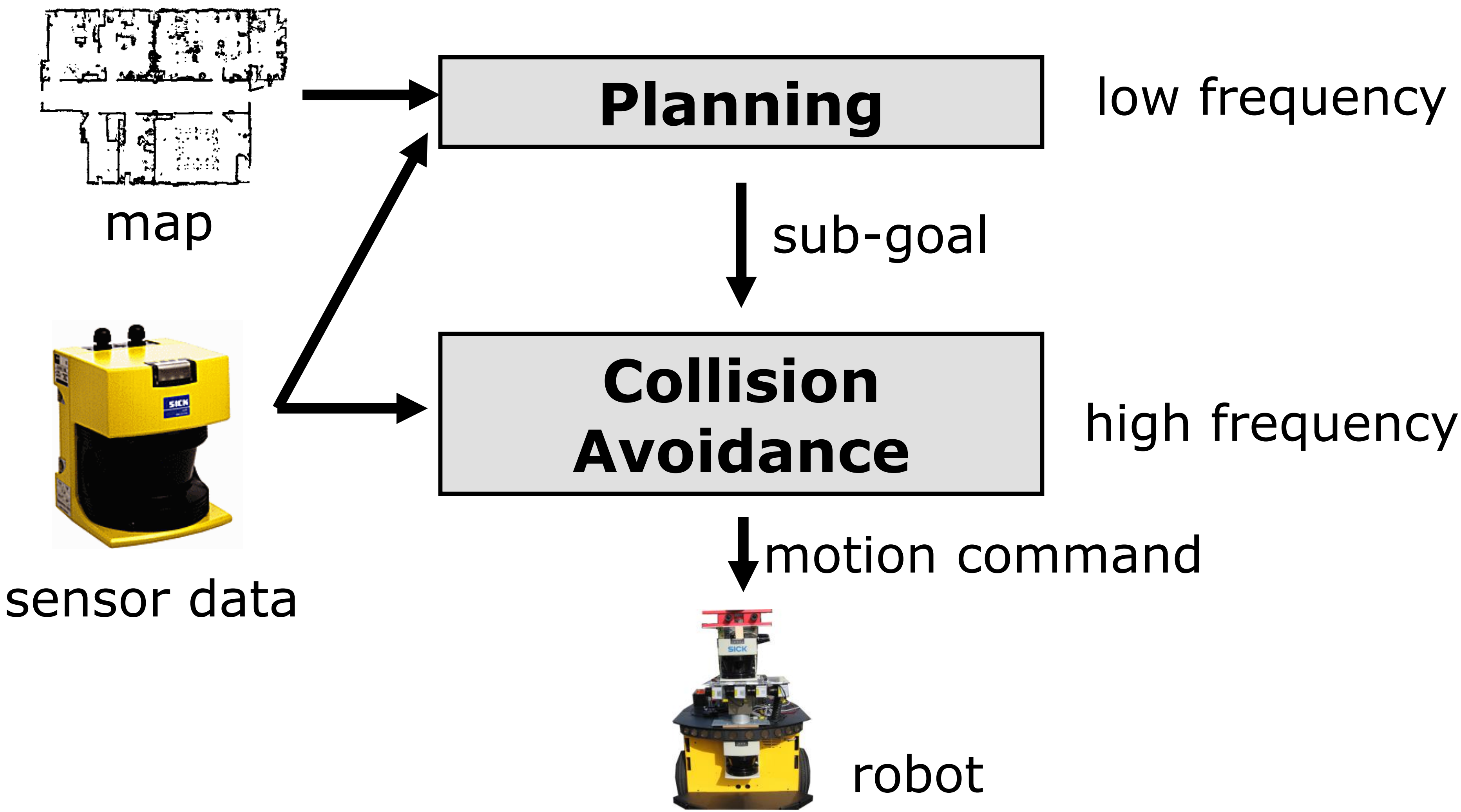
- Collision-free trajectories.
- Robot should reach the goal location as fast as possible.

Two Challenges

- Calculate the optimal path taking potential uncertainties in the actions into account

- Quickly generate actions in the case of unforeseen objects

Classic Two-layered Architecture



Information Gain-based Exploration

- SLAM is typically passive, because it consumes incoming sensor data
- Exploration actively guides the robot to cover the environment with its sensors
- Exploration in combination with SLAM: Acting under pose and map uncertainty
- Uncertainty should/needs to be taken into account when selecting an action
- Key question: **Where to move next?**

Mutual Information

- The mutual information I is given by the reduction of entropy in the belief

action to be carried out

$I(X,M;Z^a) =$ uncertainty of the filter –
"uncertainty of the filter
after carrying out action a"

Integrating Over Observations

- Computing the mutual information requires to integrate over potential observations

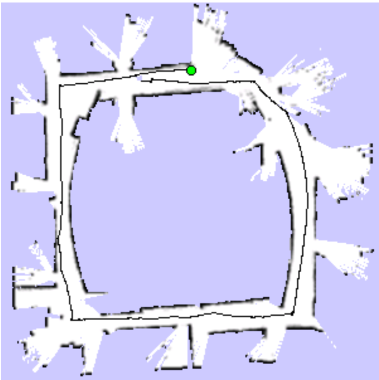
$$I(X,M;Z^a) = H(X,M) - H(X,M \mid Z^a)$$

$$H(X,M \mid Z^a) = \int_z p(z \mid a) H(X,M \mid Z^a = z)\, dz$$

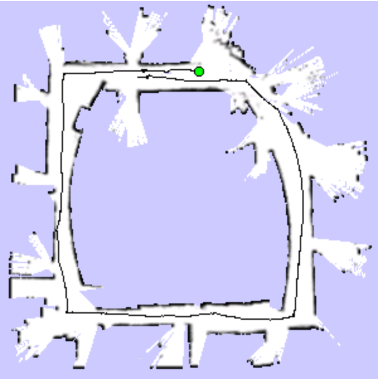
potential observation sequences

Integral Approximation

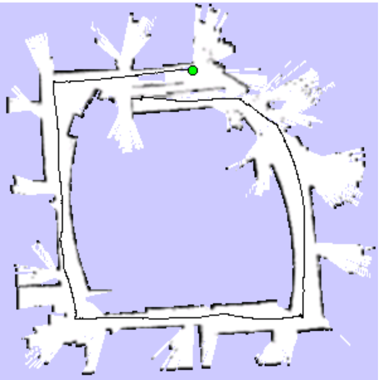
- The particle filter represents a posterior about possible maps



map of particle 1



map of particle 2



map of particle 3

...

Integral Approximation

- The particle filter represents a posterior about possible maps
- Simulate laser measurements in the maps of the particles

$$H(X,M \mid Z^a) = \sum_z p(z \mid a) H(X,M \mid Z^a = z)$$

measurement sequences simulated in the maps

likelihood (particle weight)

$$= \sum_i \omega^{[i]} H(X,M \mid Z^a = z_{sim_a}^{[i]})$$

Summary on Information Gain-based Exploration

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions

The Exam is Approaching …

- This lecture gave a short overview over the most important topics addressed in this course
- For the exam, you need to know at least the basic formulas (e.g., Bayes filter, MCL eqs., Rao-Blackwellization, entropy, …)

Good luck for the exam!