# Introduction to Mobile Robotics

## SLAM – Grid-based FastSLAM

Wolfram Burgard, Diego Tipaldi

# The SLAM Problem

- SLAM stands for simultaneous localization and mapping
- The task of building a map while estimating the pose of the robot relative to this map

- Why is SLAM hard?
Chicken and egg problem:
a map is needed to localize the robot and
a pose estimate is needed to build a map

# Mapping using Raw Odometry

# Grid-based SLAM

- Can we solve the SLAM problem if no pre-defined landmarks are available?
- Can we use the ideas of FastSLAM to build grid maps?
- As with landmarks, the map depends on the poses of the robot during data acquisition
- If the poses are known, grid-based mapping is easy ("mapping with known poses")

# Rao-Blackwellization

poses map observations & movements

$$p(x_{1:t}, m \mid z_{1:t}, u_{0:t-1}) =$$
$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(m \mid x_{1:t}, z_{1:t})$$

Factorization first introduced by Murphy in 1999

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SLAM posterior

Robot path posterior

Mapping with known poses

Factorization first introduced by Murphy in 1999

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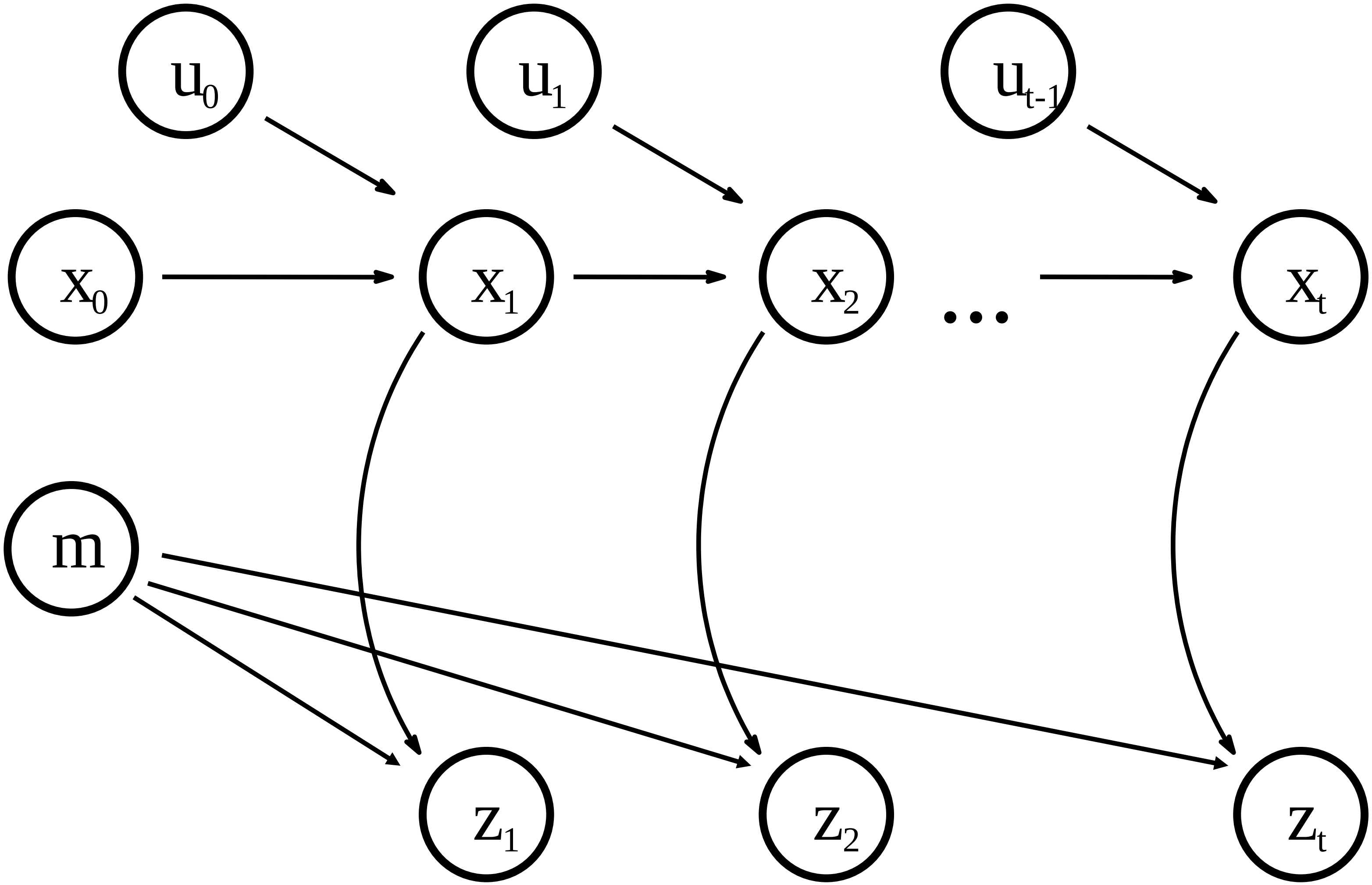
This is localization, use MCL

Use the pose estimate from the MCL and apply mapping with known poses

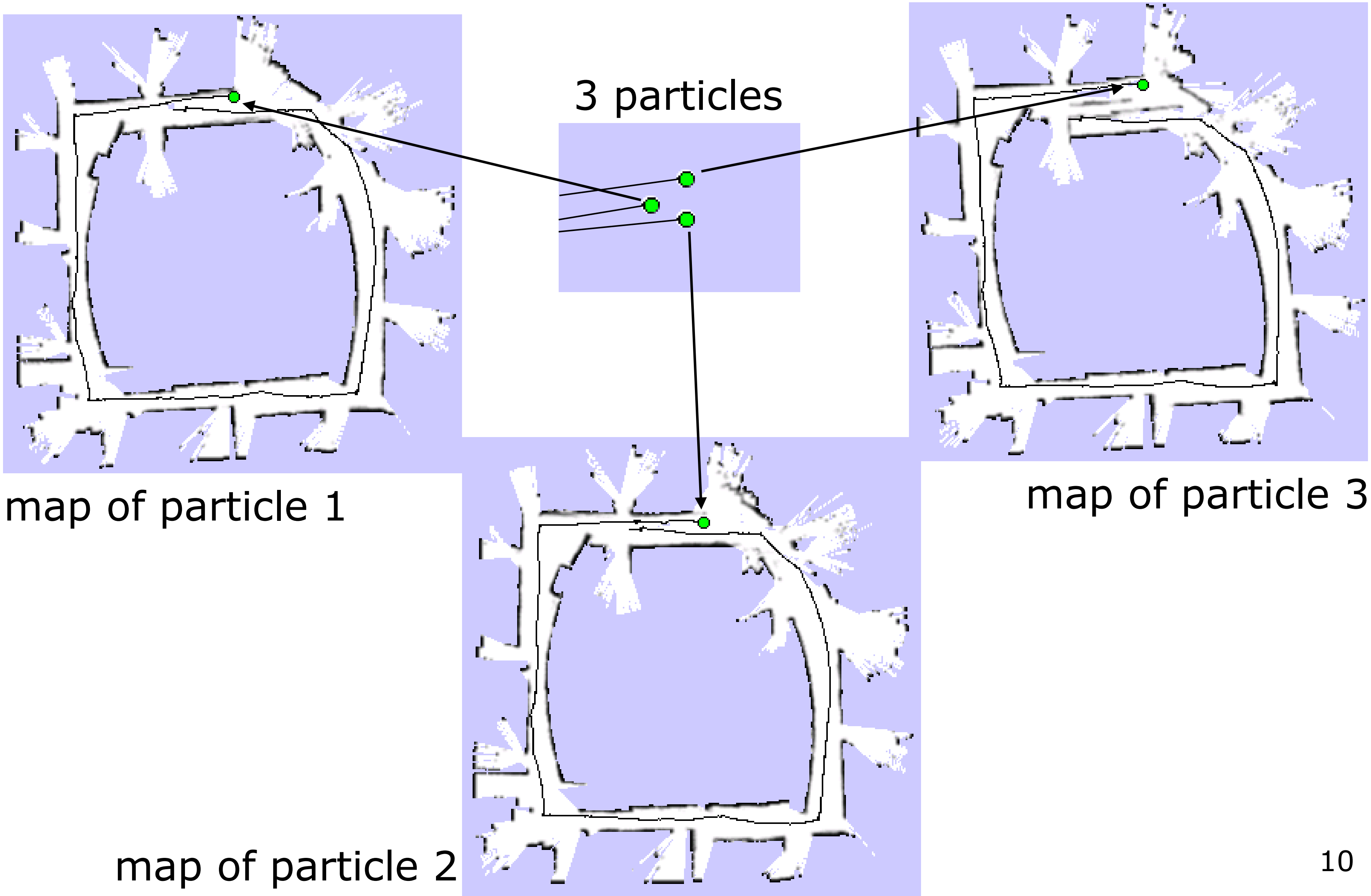
# A Graphical Model of Mapping with Rao-Blackwellized PFs



# Mapping with Rao-Blackwellized Particle Filters

- Each particle represents a possible trajectory of the robot
- Each particle
  - maintains its own map and
  - updates it upon "mapping with known poses"
- Each particle survives with a probability proportional to the likelihood of the observations relative to its own map

# Particle Filter Example



# Problem

- Each map is quite big in case of grid maps
- Since each particle maintains its own map
- Therefore, one needs to keep the number of particles small

- **Solution**:
  Compute better proposal distributions!
- **Idea**:
  Improve the pose estimate **before** applying the particle filter

# Pose Correction Using Scan Matching

Maximize the likelihood of the i-th pose and map relative to the (i-1)-th pose and map

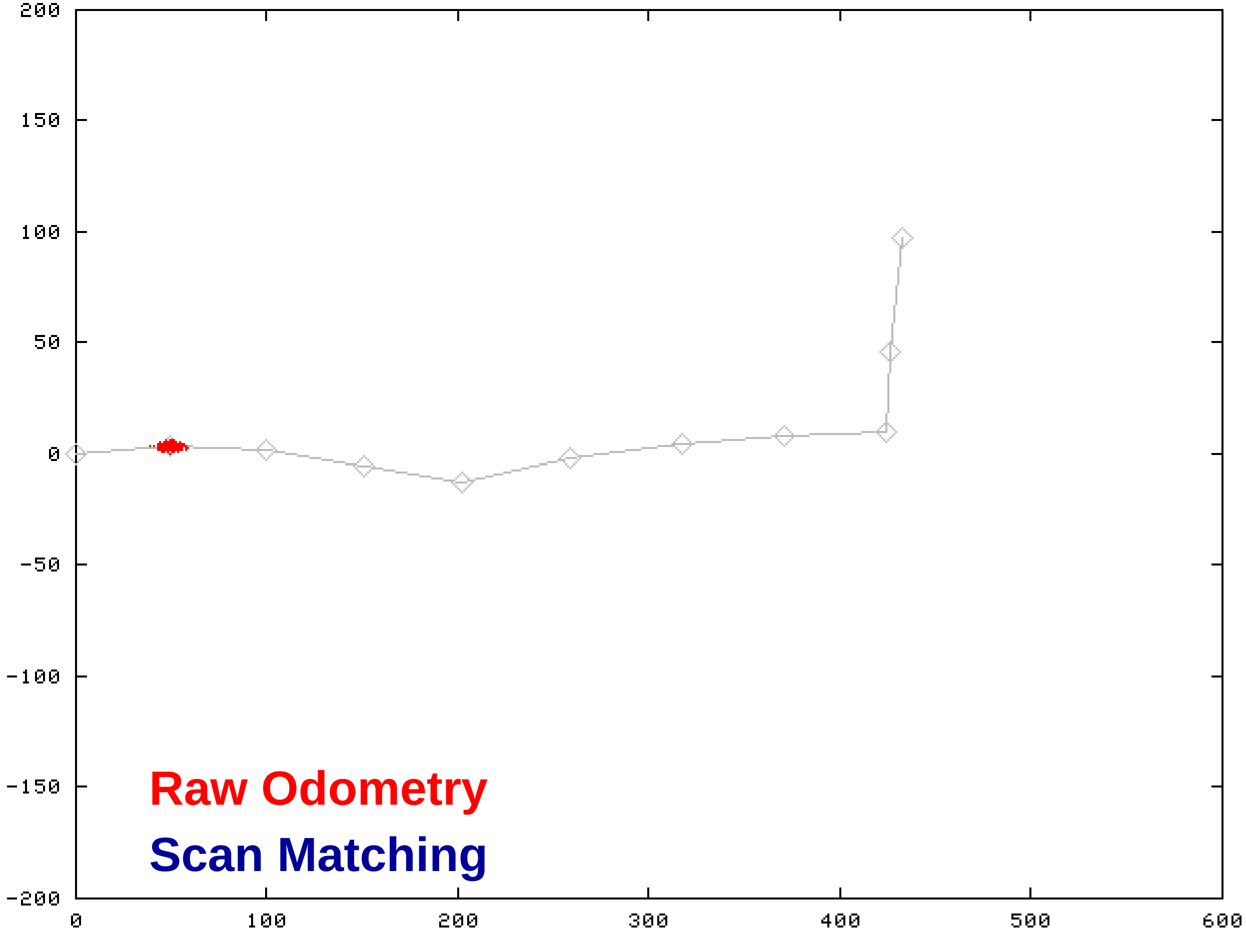
$$\hat{x}_t = \underset{x_t}{\operatorname{argmax}} \{ \underbrace{p(z_t \mid x_t, \hat{m}_{t-1})}_{\text{current measurement, map constructed so far}} \cdot \underbrace{p(x_t \mid u_{t-1}, \hat{x}_{t-1})}_{\text{robot motion}} \}$$

current measurement

robot motion

map constructed so far

# Motion Model for Scan Matching



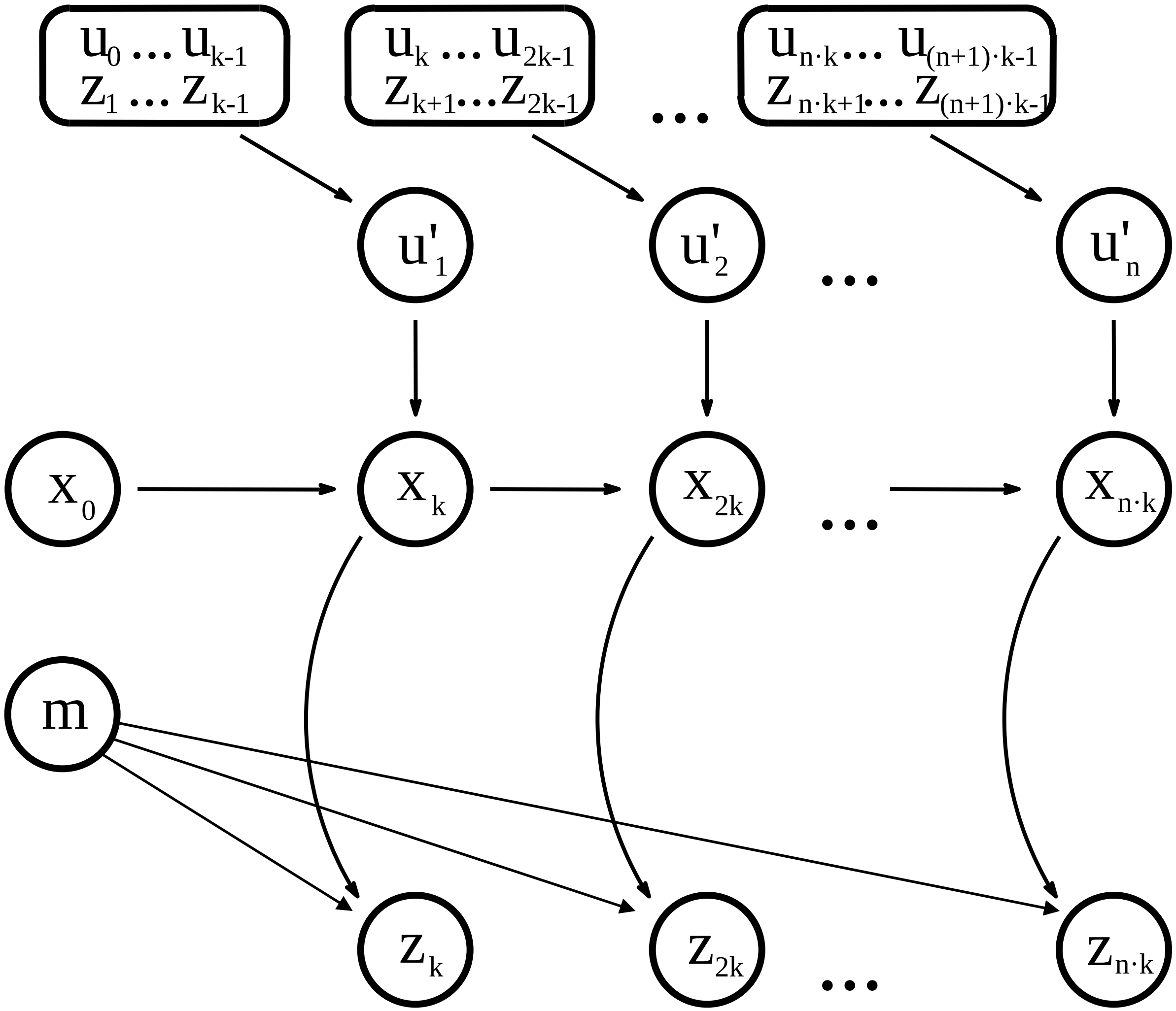
# Mapping using Scan Matching

# FastSLAM with Improved Odometry

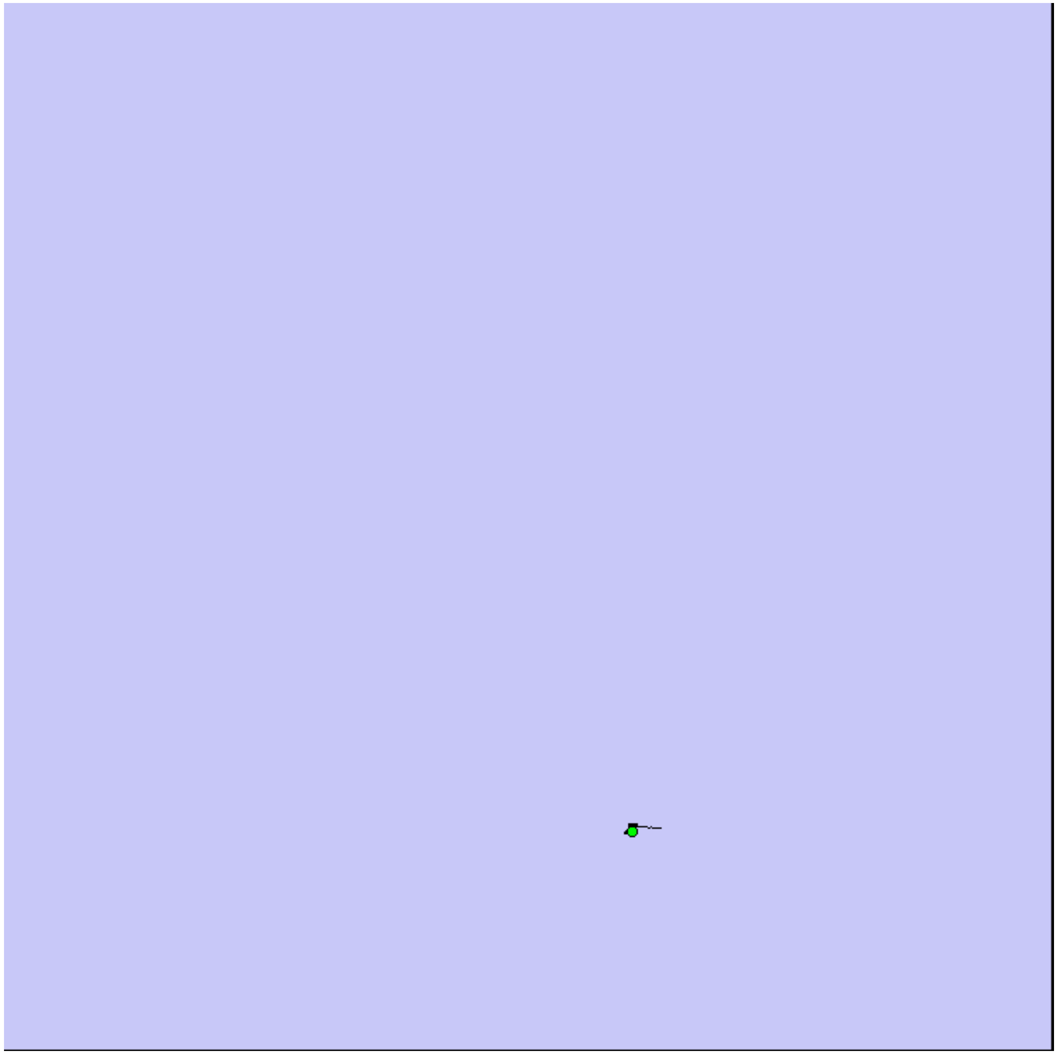
- Scan-matching provides a **locally consistent** pose correction
- Pre-correct short odometry sequences using scan-matching and use them as input to FastSLAM
- Fewer particles are needed, since the error in the input in smaller

[Haehnel et al., 2003]

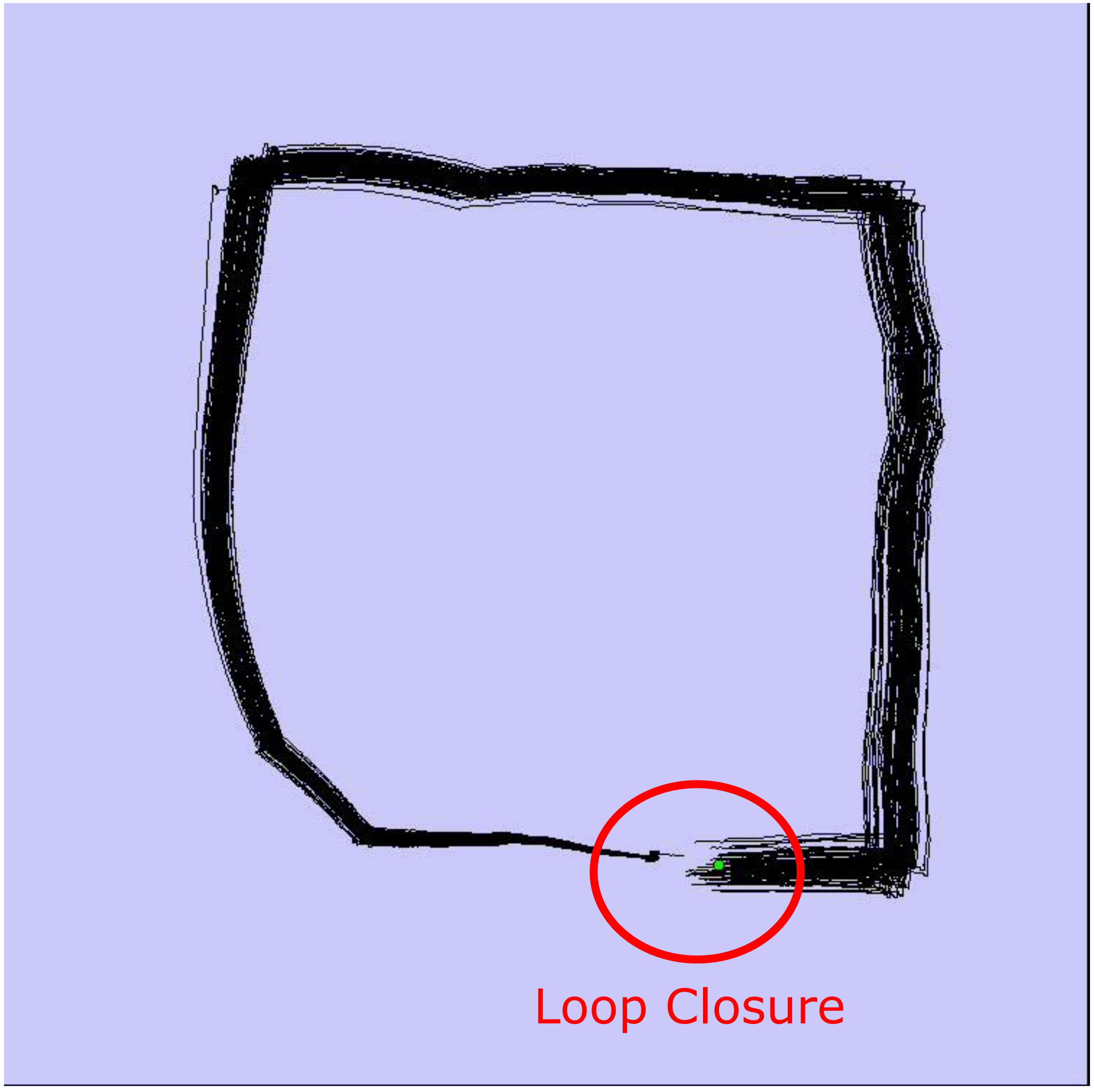
# Graphical Model for Mapping with Improved Odometry



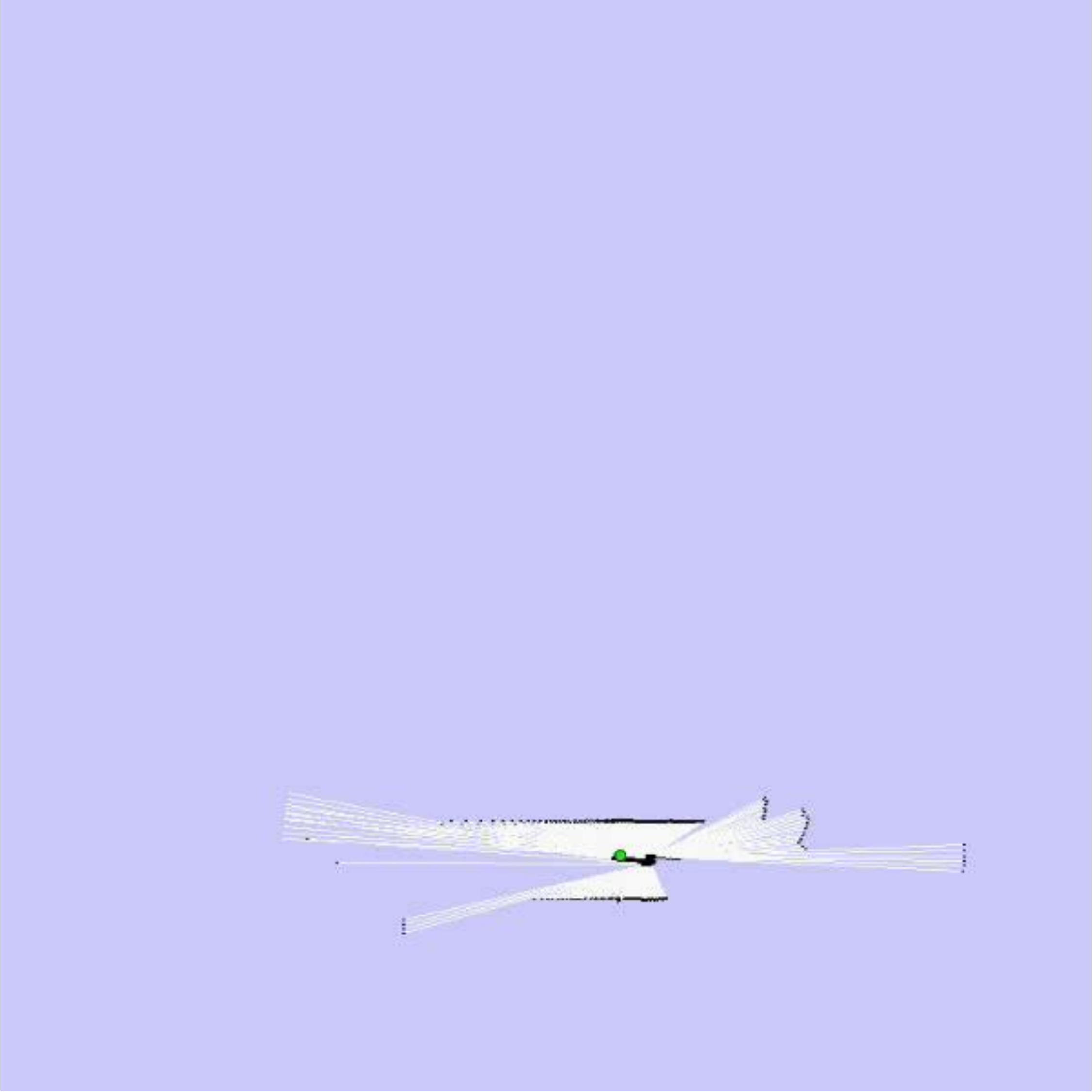
# FastSLAM with Scan-Matching



# FastSLAM with Scan-Matching



# FastSLAM with Scan-Matching



Map: Intel Research Lab Seattle

# Comparison to Standard FastSLAM

- Same model for observations
- Odometry instead of scan matching as input
- Number of particles varying from 500 to 2.000
- Typical result:

# Conclusion (thus far …)

- The presented approach is a highly efficient algorithm for SLAM combining ideas of scan matching and FastSLAM
- Scan matching is used to transform sequences of laser measurements into odometry measurements
- This version of grid-based FastSLAM can handle larger environments than before in "real time"

# What's Next?

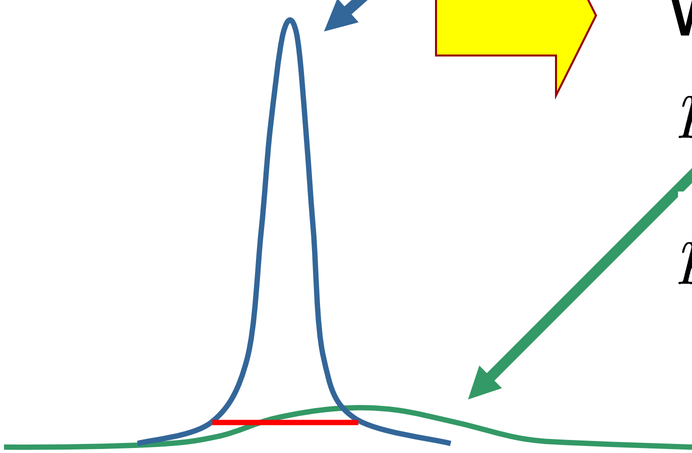
- Further reduce the number of particles
- Improved proposals will lead to more accurate maps
- Use the properties of our sensor when drawing the next generation of particles

# The Optimal Proposal Distribution

$$p(x_t|x_{t-1}^{(i)}, m^{(i)}, z_t, u_t) = \frac{p(z_t|x_t, m^{(i)})p(x_t|x_{t-1}^{(i)}, u_t)}{\int p(z_t|x_t, m^{(i)})p(x_t|x_{t-1}^{(i)}, u_t)dx_t}$$

[Arulampalam et al., 01]

For lasers $p(z_t|x_t, m^{(i)})$ is extremely peaked and dominates the product.



We can safely approximate $p(x_t|x_{t-1}^{(i)}, u_t)$ by a constant:

$$p(x_t|x_{t-1}^{(i)}, u_t)\ |_{x_t:p(z_t|x_t,m^{(i)})>\epsilon} = c$$

# Resulting Proposal Distribution

$$p(x_t|x_{t-1}^{(i)}, m^{(i)}, z_t, u_t) \simeq \frac{p(z_t|x_t, m^{(i)})}{\int_{x_t \in \{x | p(z_t|x, m^{(i)}) > \epsilon\}} p(z_t|x_t, m^{(i)}) dx_t}$$

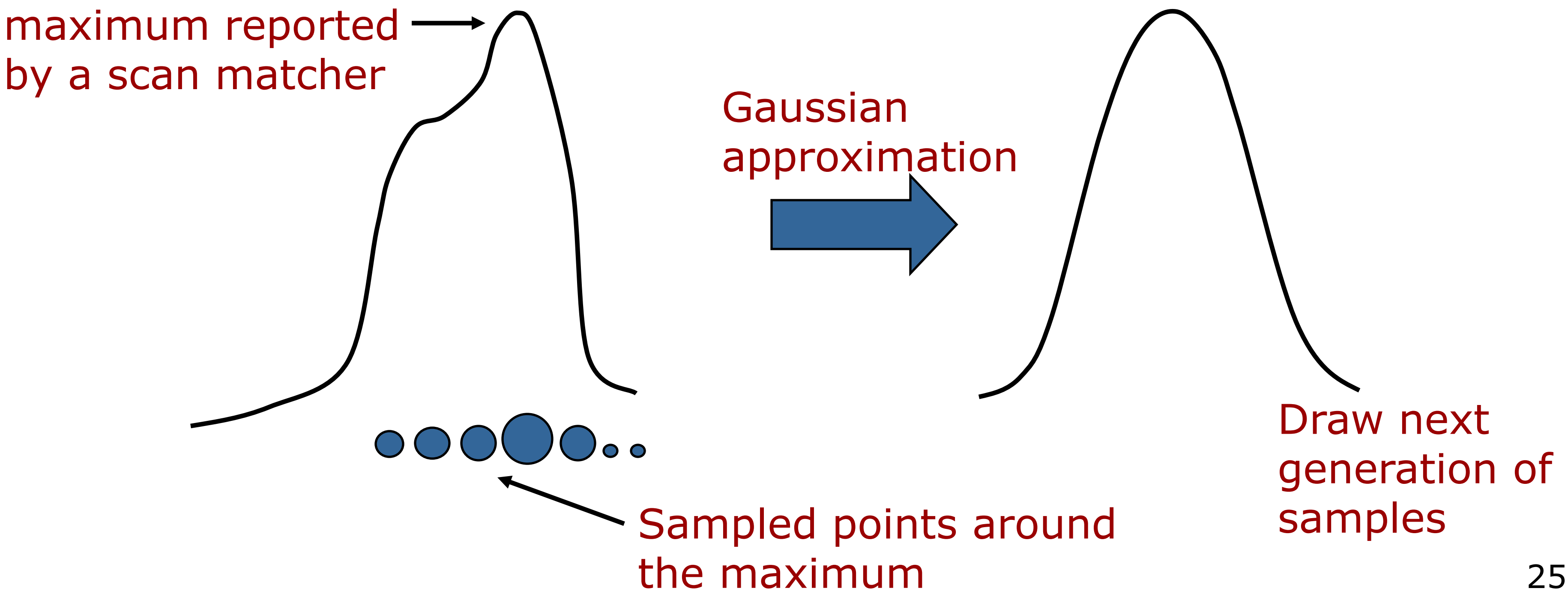
Gaussian approximation:

$$p(x_t|x_{t-1}^{(i)}, m^{(i)}, z_t, u_t) \simeq \mathcal{N}(\mu^{(i)}, \Sigma^{(i)})$$

# Resulting Proposal Distribution

$$p(x_t|x_{t-1}^{(i)}, m^{(i)}, z_t, u_t) \simeq \frac{p(z_t|x_t, m^{(i)})}{\int_{x_t \in \{x|p(z_t|x,m^{(i)})>\epsilon\}} p(z_t|x_t, m^{(i)}) dx_t}$$

Approximate this equation by a Gaussian:



# Estimating the Parameters of the Gaussian for each Particle

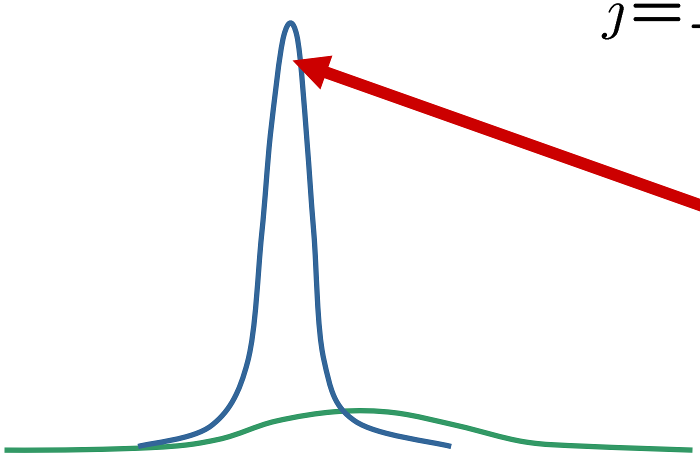
$$\mu^{(i)} = \frac{1}{\eta} \sum_{j=1}^{K} x_j \, p(z_t | x_j, m^{(i)})$$

$$\Sigma^{(i)} = \frac{1}{\eta} \sum_{j=1}^{K} (x_j - \mu^{(i)})(x_j - \mu^{(i)})^T p(z_t | x_j, m^{(i)})$$

- $x_j$ are a set of sample points around the point $x^*$ the scan matching has converged to.
- $\eta$ is a normalizing constant

# Computing the Importance Weight

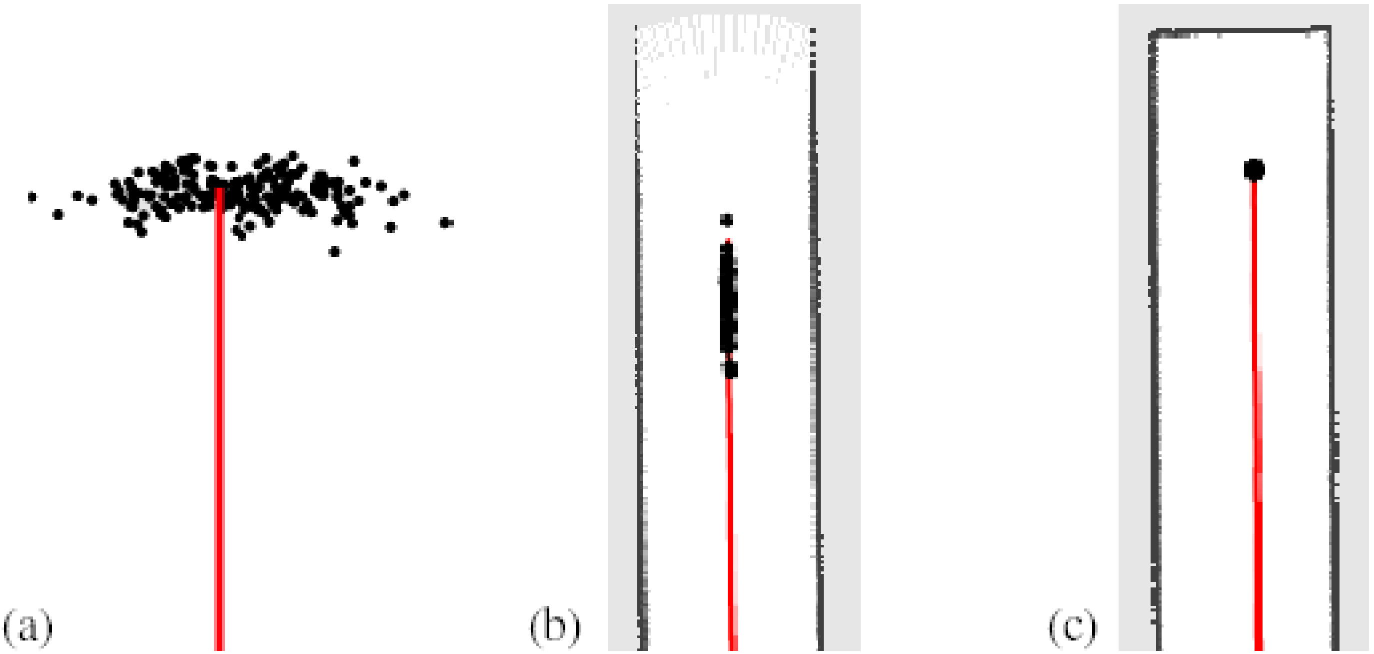
$$
\begin{aligned}
w_t^{(i)} &= w_{t-1}^{(i)} p(z_t | x_{t-1}^{(i)}, m^{(i)}, u_t) \\
&\simeq w_{t-1}^{(i)} \int p(z_t | x_t, m^{(i)}) p(x_t | x_{t-1}^{(i)}, u_t) d\, x_t \\
&\simeq w_{t-1}^{(i)} c \int_{x_t \in \{x | p(z_t | x, m^{(i)}) > \epsilon\}} p(z_t | x_t, m^{(i)}) dx_t \\
&\simeq w_{t-1}^{(i)} c \sum_{j=1}^{K} p(z_t | x_j, m^{(i)})
\end{aligned}
$$



Sampled points around the maximum of the observation likelihood

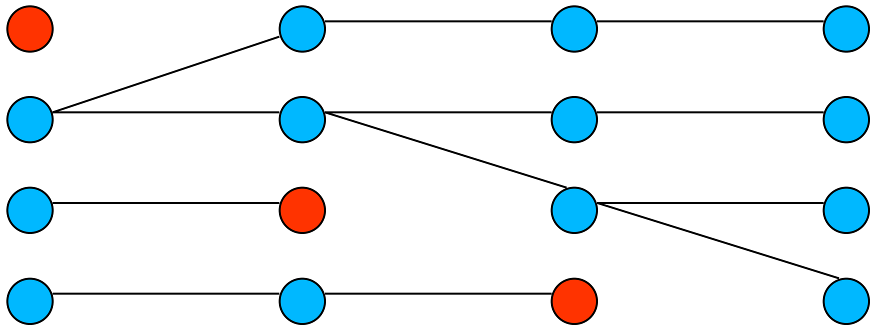
# Improved Proposal

- The proposal adapts to the structure of the environment



# Resampling

- Sampling from an improved proposal reduces the effects of resampling
- However, resampling at each step limits the “memory” of our filter
- Supposed we loose at each frame 25% of the particles, in the worst case we have a memory of only 4 steps.



**Goal: reduce the number of resampling actions**

# Selective Re-sampling

- Re-sampling is dangerous, since important samples might get lost
(particle depletion problem)

- In case of suboptimal proposal distributions re-sampling is necessary to achieve convergence.

- Key question: When should we re-sample?

# Number of Effective Particles

$$n_{eff} = \frac{1}{\sum_i \left(w_t^{(i)}\right)^2}$$

- Empirical measure of how well the goal distribution is approximated by samples
drawn from the proposal
- $n_{eff}$ describes "the variance of the particle weights"
- $n_{eff}$ is maximal for equal weights. In this case, the distribution is close to the proposal

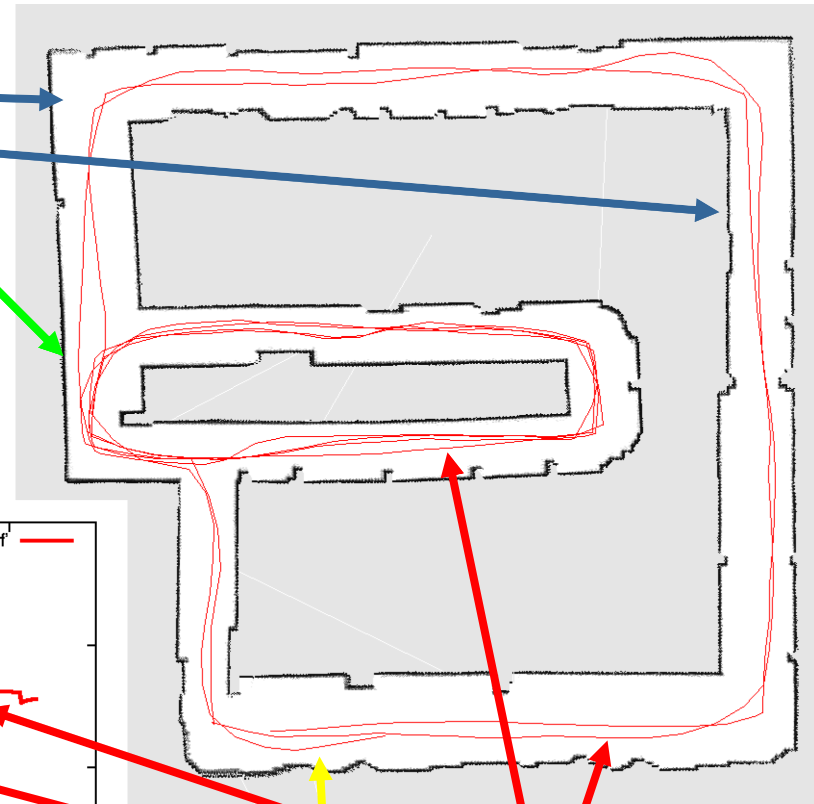
# Resampling with $n_{eff}$

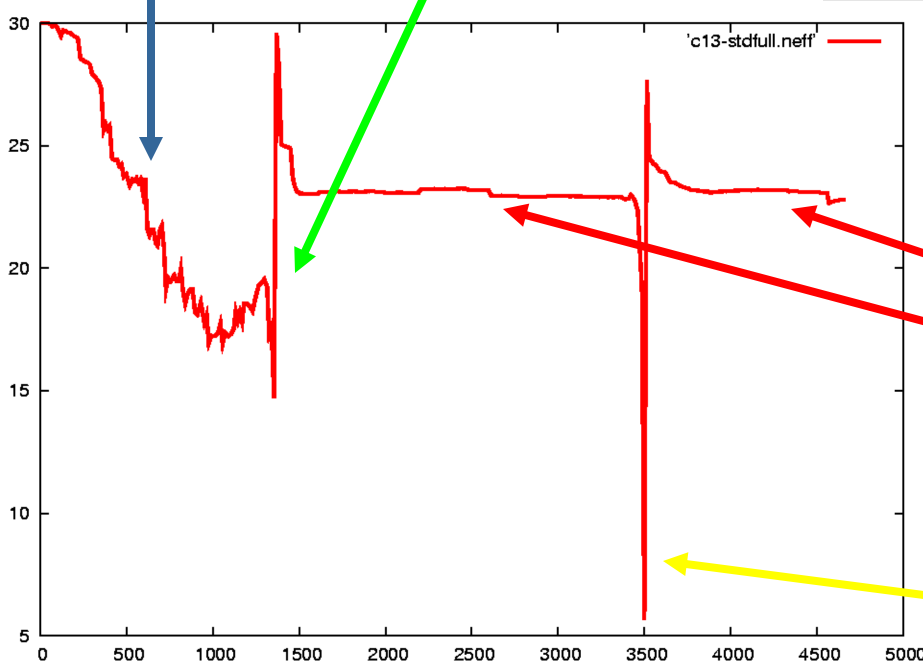
- If our approximation is close to the proposal, no resampling is needed
- We only re-sample when $n_{eff}$ drops below a given threshold (n/2)
- See [Doucet, ’98; Arulampalam, ’01]

# Typical Evolution of $n_{eff}$

visiting new areas

closing the first loop





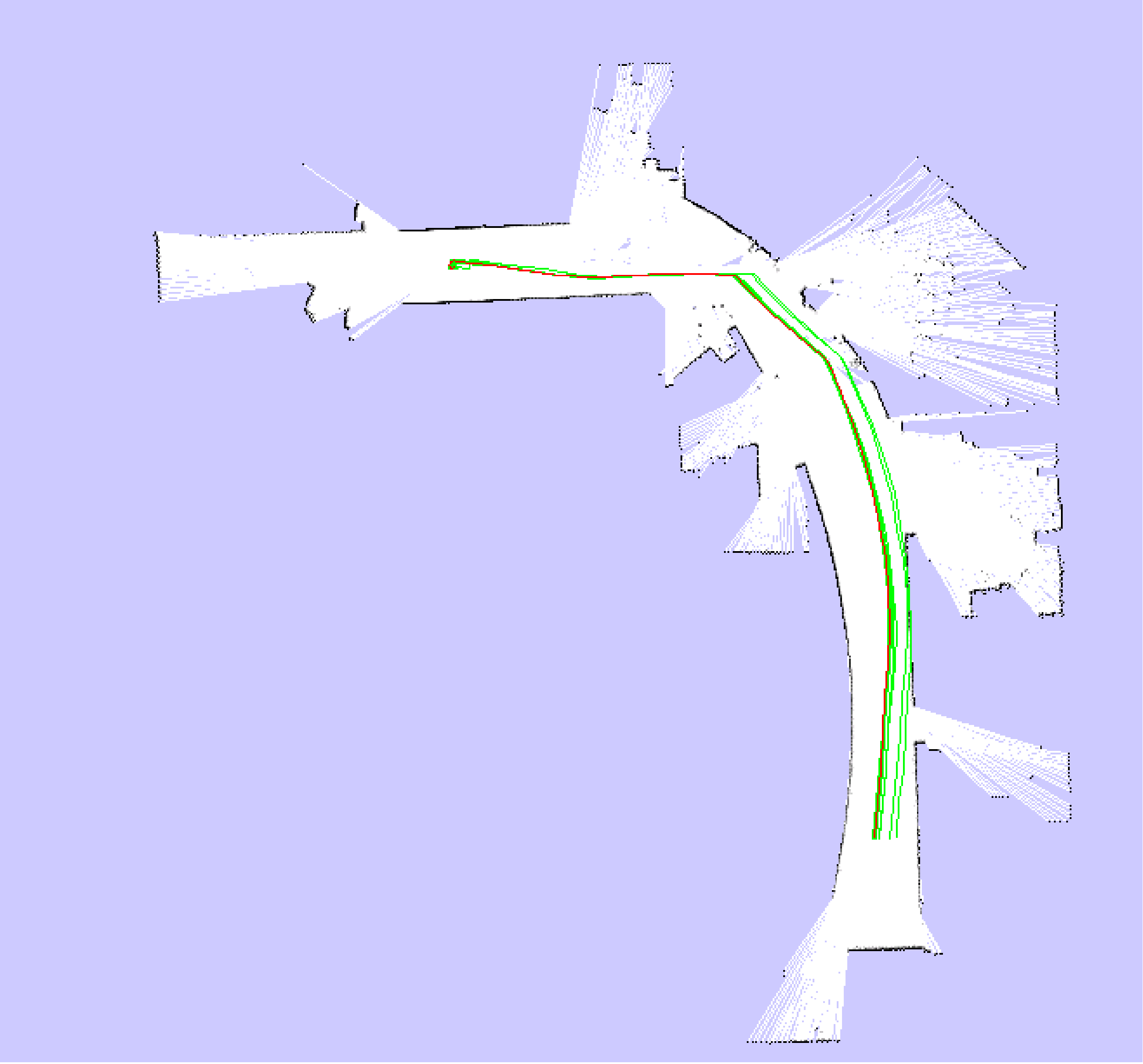

visiting known areas

second loop closure

# Intel Lab

- **15 particles**
- four times faster than real-time P4, 2.8GHz
- 5cm resolution during scan matching
- 1cm resolution in final map

# Intel Lab

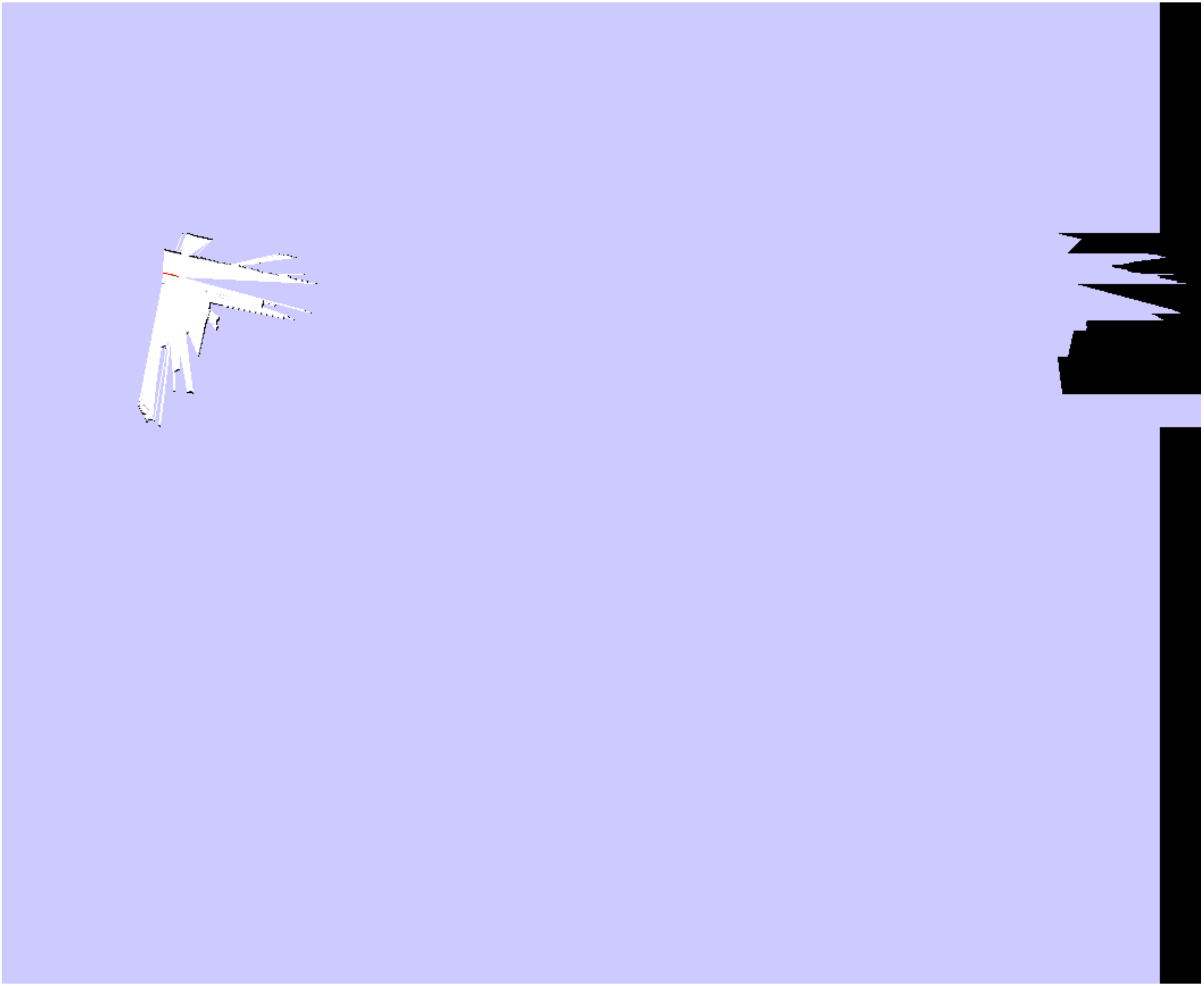


- **15 particles**
- Compared to FastSLAM with Scan-Matching, the particles are propagated closer to the true distribution

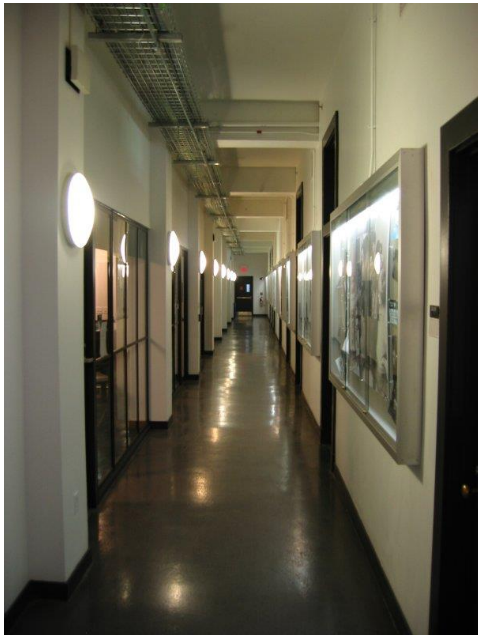
# Outdoor Campus Map

- **30 particles**
- 250x250m$^2$
- 1.088 miles (odometry)
- 20cm resolution during scan matching
- 30cm resolution in final map

# Outdoor Campus Map - Video

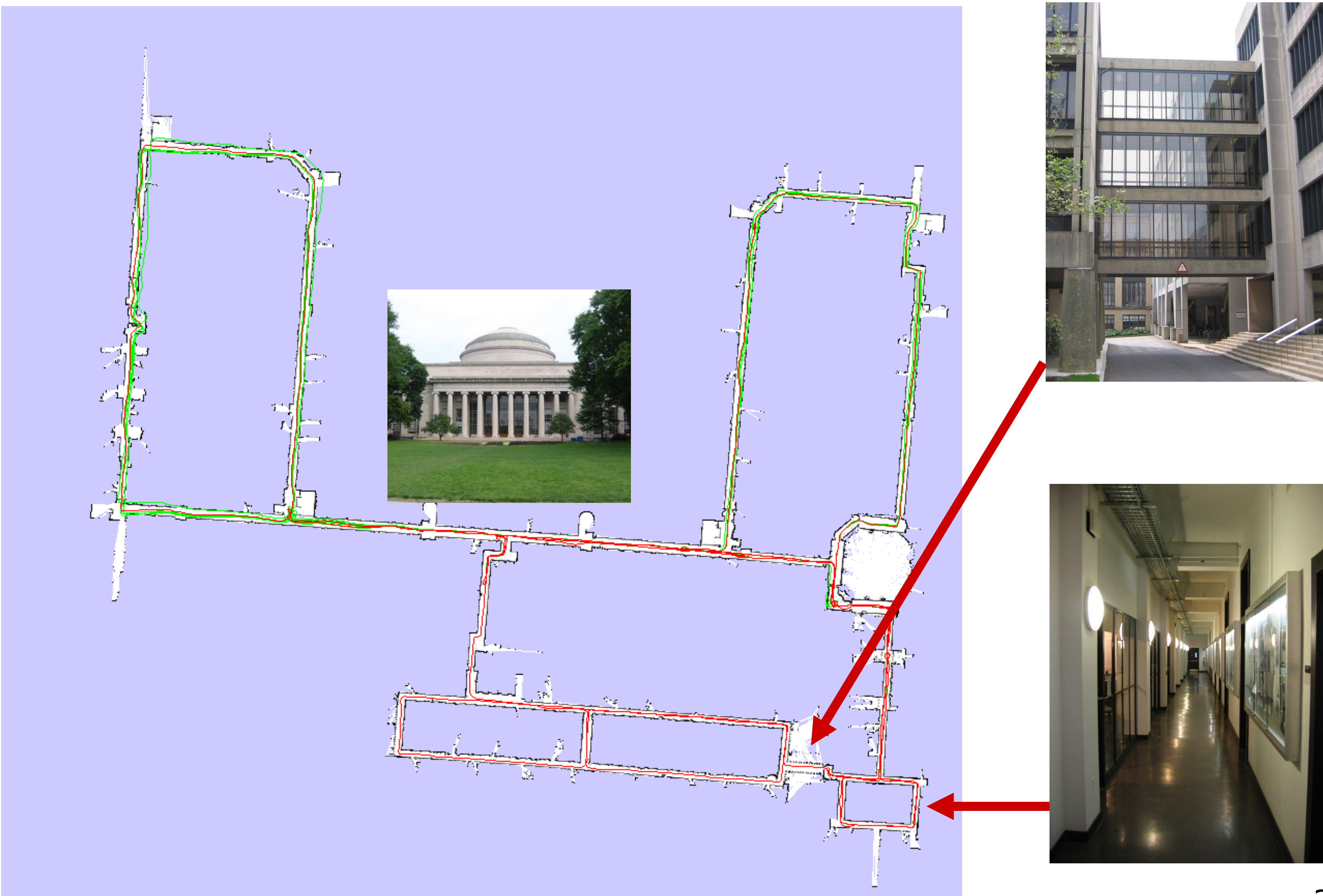


# MIT Killian Court




- The **“infinite-corridor-dataset”** at MIT

# MIT Killian Court



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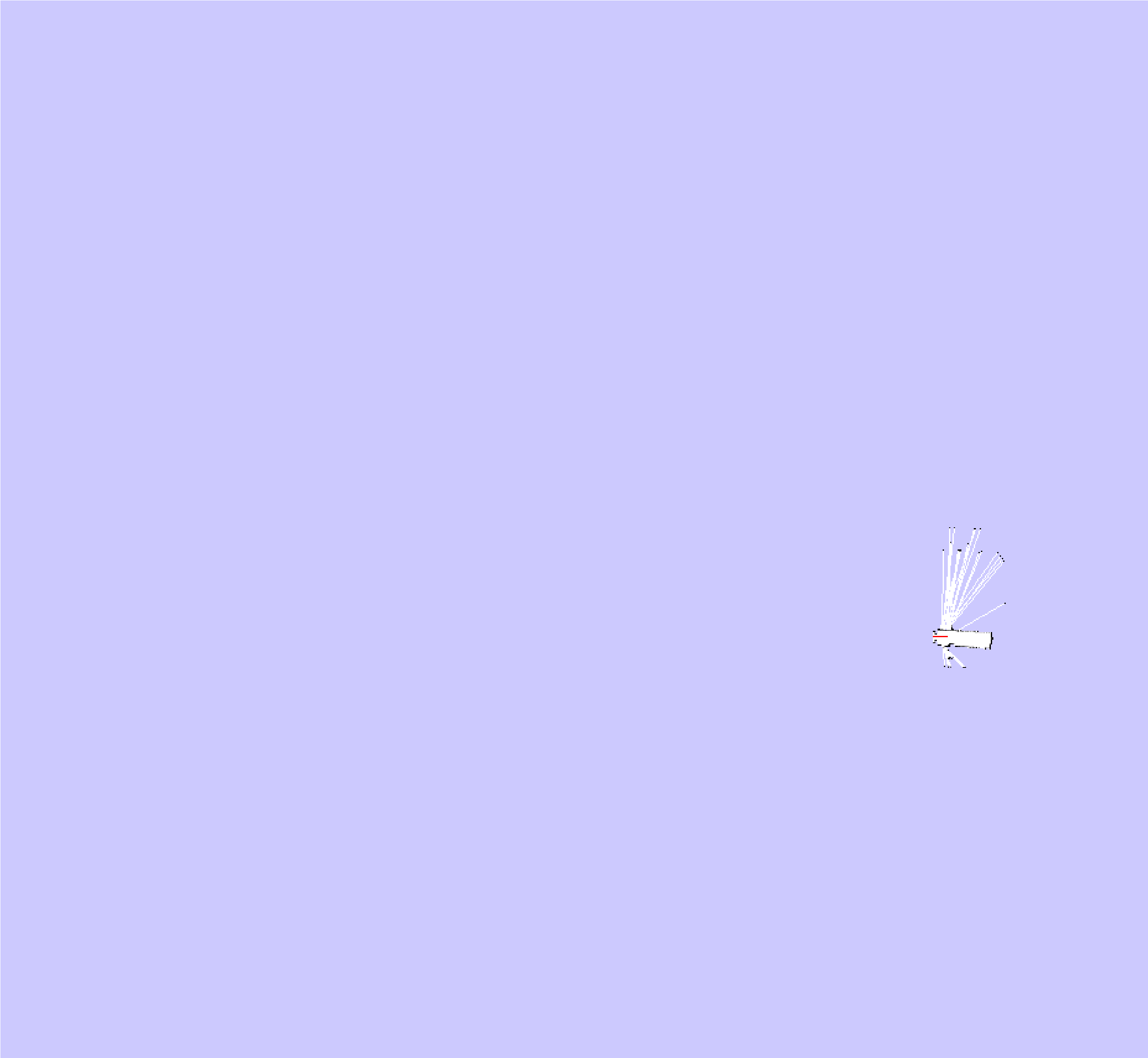

# Conclusion

- The ideas of FastSLAM can also be applied in the context of grid maps
- Utilizing accurate sensor observation leads to good proposals and highly efficient filters
- It is similar to scan-matching on a per-particle base
- The number of necessary particles and re-sampling steps can seriously be reduced
- Improved versions of grid-based FastSLAM can handle larger environments than naïve implementations in "real time" since they need one order of magnitude fewer samples

# More Details on FastSLAM

- M. Montemerlo, S. Thrun, D. Koller, and B. Wegbreit. FastSLAM: A factored solution to simultaneous localization and mapping, *AAAI02* *(The classic FastSLAM paper with landmarks)*

- D. Haehnel, W. Burgard, D. Fox, and S. Thrun. An efcient FastSLAM algorithm for generating maps of large-scale cyclic environments from raw laser range measurements, IROS03
*(FastSLAM on grid-maps using scan-matched input)*

- G. Grisetti, C. Stachniss, and W. Burgard. Improving grid-based SLAM with Rao-Blackwellized particle filters by adaptive proposals and selective resampling, ICRA05
*(Proposal using laser observation, adaptive resampling)*

- A. Eliazar and R. Parr. DP-SLAM: Fast, robust simultaneous localization and mapping without predetermined landmarks, IJCAI03
*(An approach to handle big particle sets)*