Theoretical Computer Science (Bridging Course)

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Exercise Sheet 9

Due: 15th January 2014

Exercise 9.1 (P)

(a) Show that P is closed under union, complement, and concatenation.

(b) The complexity class *coP* contains all languages L whose complement is in P. Formally, $coP = \{L \mid \bar{L} \in P\}$. Is $P = coP$?

Exercise 9.2 (Reduction)

Given an undirected graph $\mathcal{G} := \langle G, E \rangle$ and an integer number $0 \leq k \leq |G|$, the following NP–complete problems have been introduced in the lectures (see `07.pdf`, slides 80-82-84):

Clique : Does $\mathcal{G}$ contain a *clique* of size at least k? That is, there exist a set $C \subseteq G$ so that $\langle u, v \rangle \in E$ for every $u, v \in C$ $(u \neq v)$ and $|C| \geq k$?

IndSet : Does $\mathcal{G}$ contain an *independent set* whose size is at least k? In other words, does G admit a subset $I \subseteq G$ with $|I| \geq k$ and such that there exists no edge $\langle u, v \rangle$ whenever u, v lie in I?

VertexCover : Does $\mathcal{G}$ contain a *vertex cover* of size at most k? That is, is it possible to find a set $C \subseteq G$ so that $|C| \leq k$ and for every edge $\langle u, v \rangle \in E$, $u \in C$ or $v \in C$?

Prove the following statements:

(a) Clique $\leq_P$ IndSet
Hint: consider the complement graph.

(b) IndSet $\leq_P$ VertexCover
Hint: consider the relation between vertex covers and independent sets.