Introduction to Mobile Robotics

Path Planning and Collision Avoidance

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Motion Planning

Latombe (1991):

"… is eminently necessary since, by definition, a robot accomplishes tasks by moving in the real world."

Goals:

- Collision-free trajectories
- Robot should reach the goal location as quickly as possible

Optimality

… in Dynamic Environments

- How to react to unforeseen obstacles?
  - efficiency
  - reliability

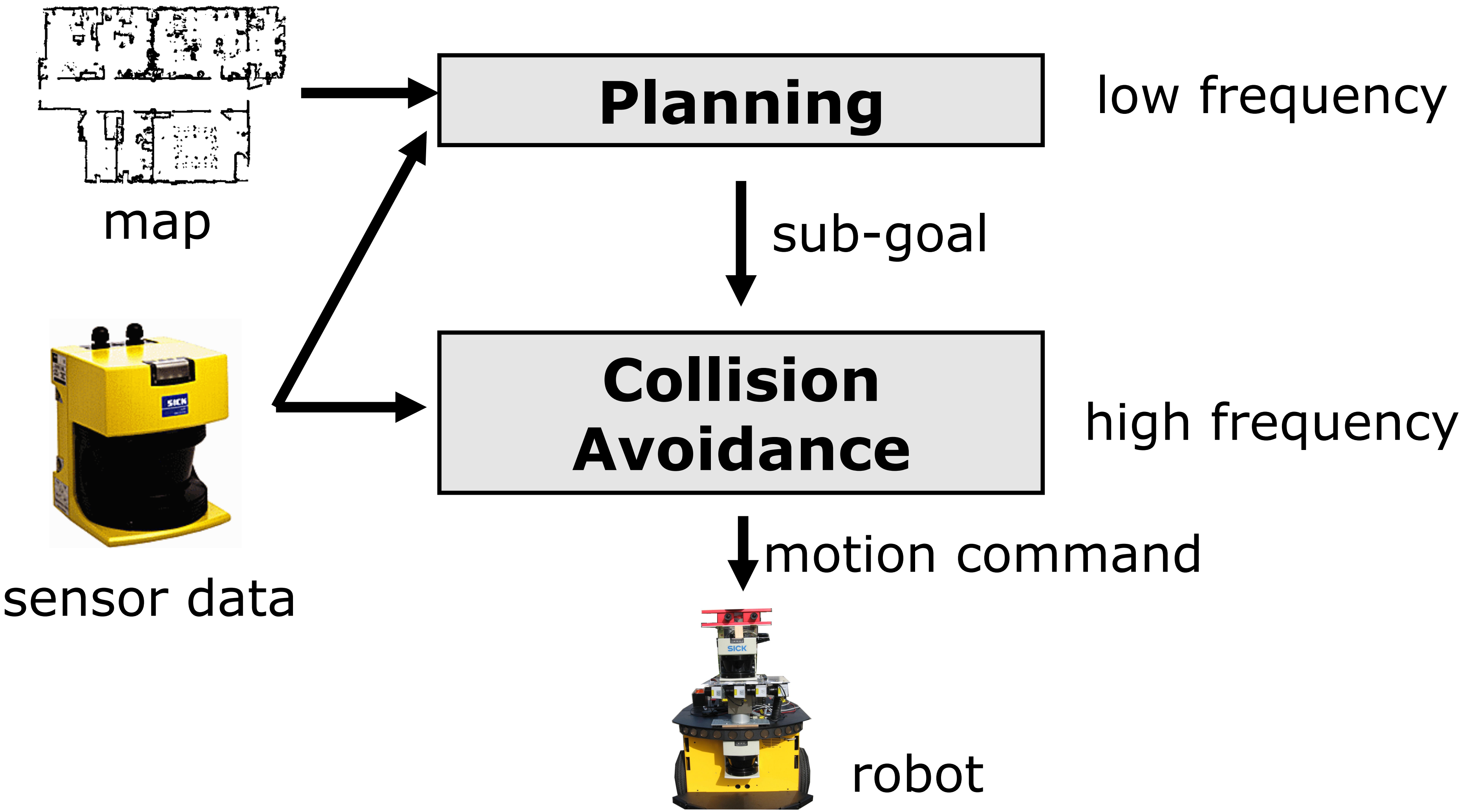
- Dynamic Window Approaches
  [Simmons, 96], [Fox et al., 97], [Brock & Khatib, 99]
- Grid map based planning
  [Konolige, 00]
- Nearness Diagram Navigation
  [Minguez at al., 2001, 2002]
- Vector-Field-Histogram+
  [Ulrich & Borenstein, 98]
- A*, D*, D* Lite, ARA*, …

Two Challenges

- Calculate the optimal path taking potential uncertainties in the actions into account

- Quickly generate actions in the case of unforeseen objects

Classic Two-Layered Architecture



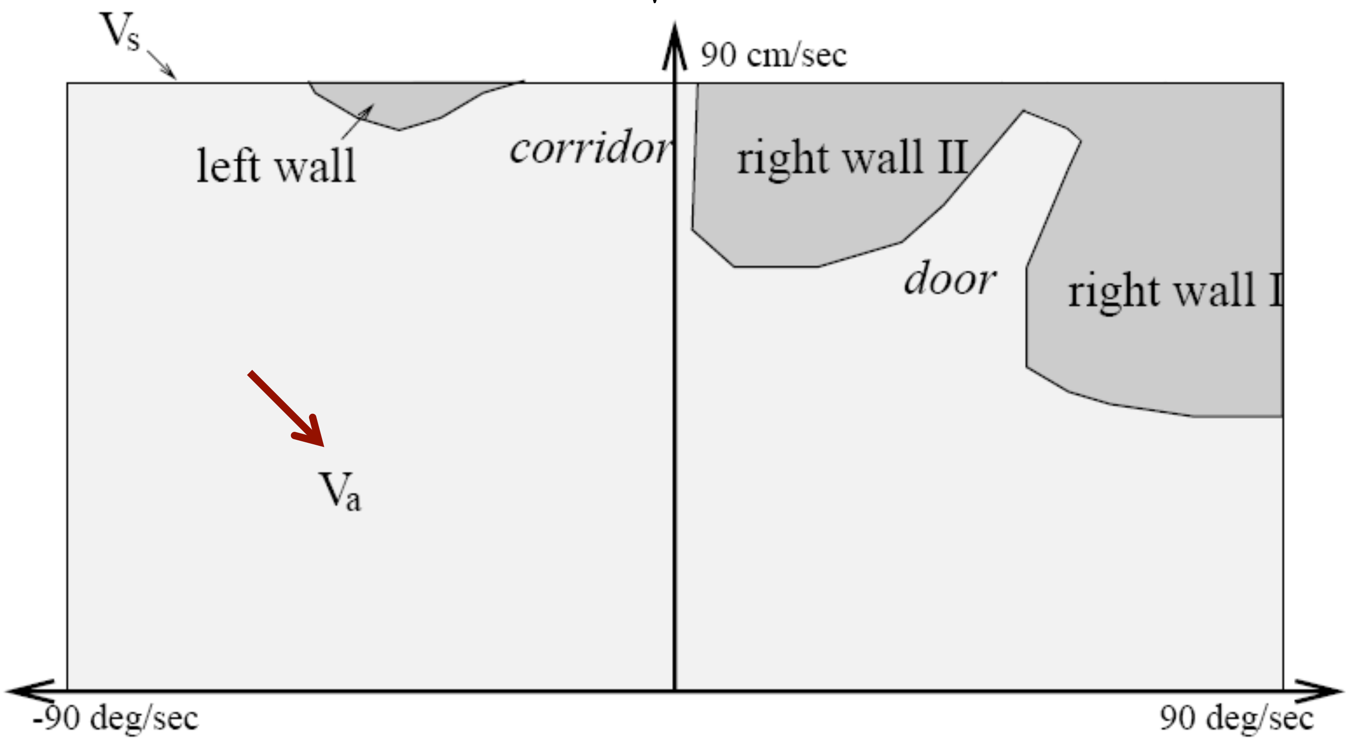
Dynamic Window Approach

- **Collision avoidance:** Determine collision-free trajectories using geometric operations
- Here: Robot moves on circular arcs
- Motion commands (v,ω)
- Which (v,ω) are admissible and reachable?

Admissible Velocities

- A speed is admissible if the robot is able to stop before colliding with an obstacle

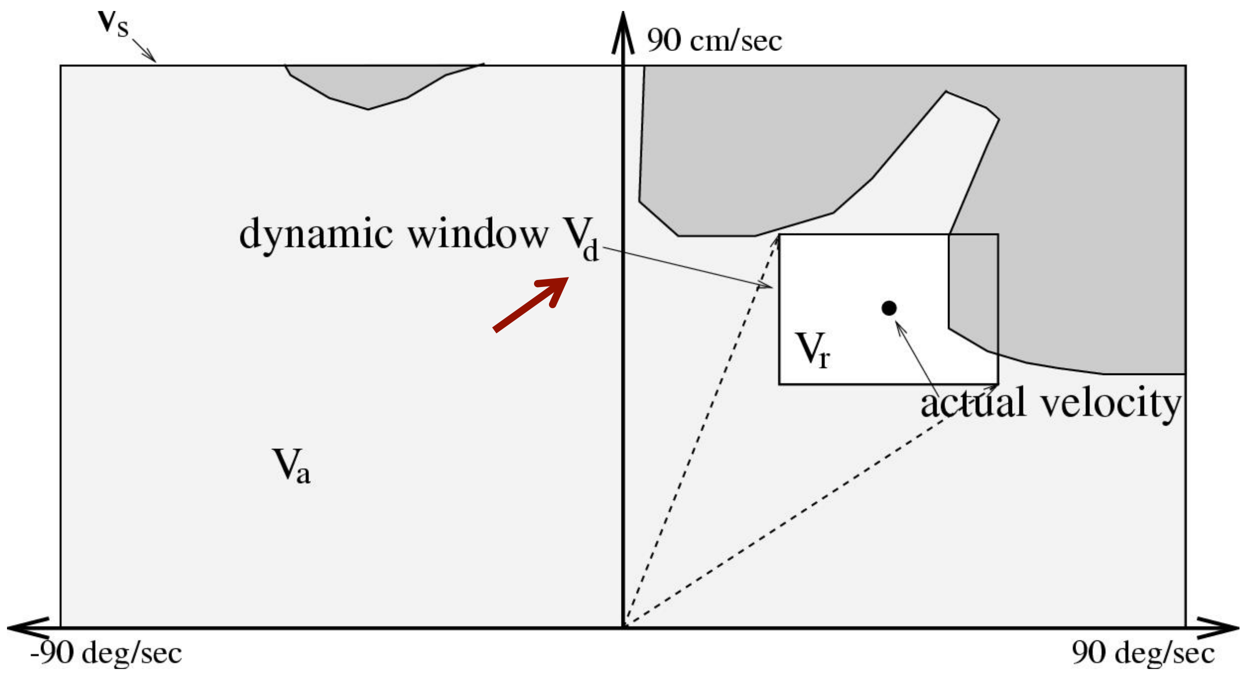
$$V_a = \{(v,\omega) \mid v \leq \sqrt{2\text{dist}(v,\omega)a_{trans}} \quad \wedge \quad \omega \leq \sqrt{2\text{dist}(v,\omega)a_{rot}}\}$$



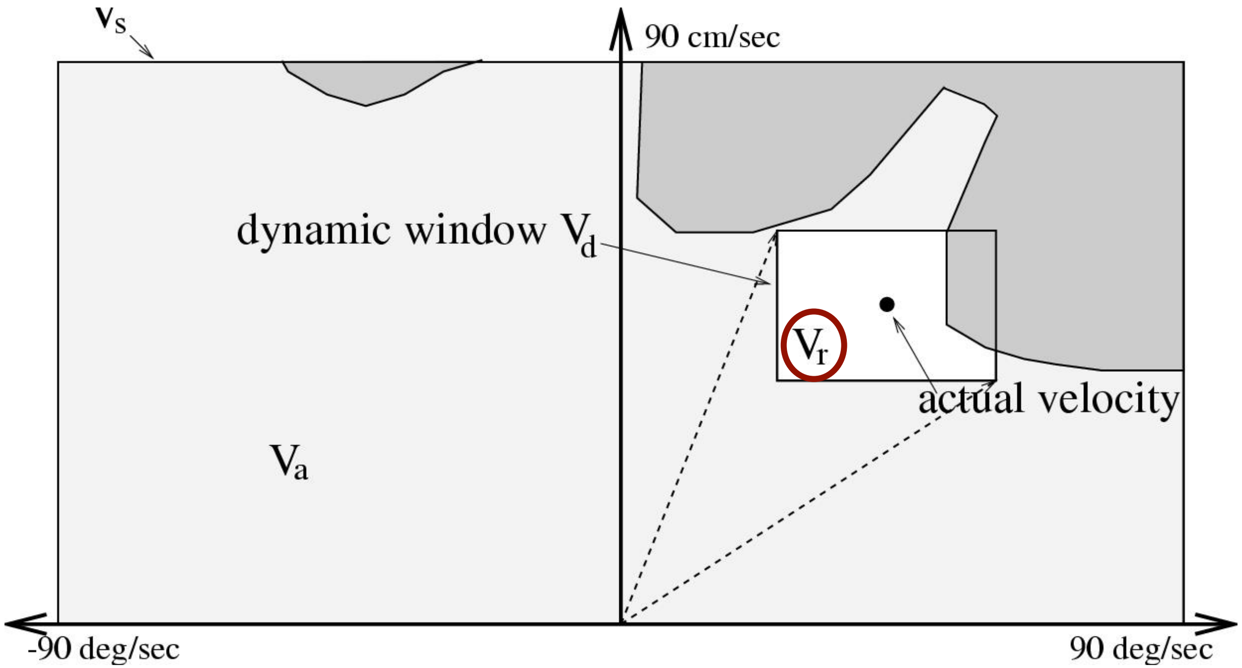
Reachable Velocities

- Speeds that are reachable by acceleration within the time period t:

$$V_d = \{(v, \omega) \;\; | \;\; v \in [v - a_{trans}t, v + a_{trans}t] \;\; \wedge \\ \omega \in [\omega - a_{rot}t, \omega + a_{rot}t]\}$$



DWA Search Space



- V_s = all possible speeds of the robot
- V_a = obstacle free area
- V_d = speeds reachable within a certain time frame based on possible accelerations

$$V_r = V_s \cap V_a \cap V_d$$

Dynamic Window Approach

- How to choose <v,ω>?
- Steering commands are chosen by a heuristic navigation function
- This function tries to minimize the travel-time by:
  "**driving fast** in the **right direction**"

Dynamic Window Approach

- Heuristic navigation function
- Planning restricted to <x,y>-space
- No planning in the velocity space

Navigation Function: [Brock & Khatib, 99]

$$NF = \alpha \cdot vel + \beta \cdot nf + \gamma \cdot \Delta nf + \delta \cdot goal$$

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Maximizes velocity.

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Maximizes velocity.

Considers cost to reach the goal.

Dynamic Window Approach

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$$NF = \alpha \cdot vel + \beta \cdot nf + \gamma \cdot \Delta nf + \delta \cdot goal$$

Maximizes velocity.

Considers cost to reach the goal.

Follows grid based path computed by A*.

Dynamic Window Approach

- Heuristic navigation function
- Planning restricted to <x,y>-space
- No planning in the velocity space

Navigation Function:

Goal nearness.

$$NF = \alpha \cdot vel + \beta \cdot nf + \gamma \cdot \Delta nf + \delta \cdot goal$$

Maximizes velocity.

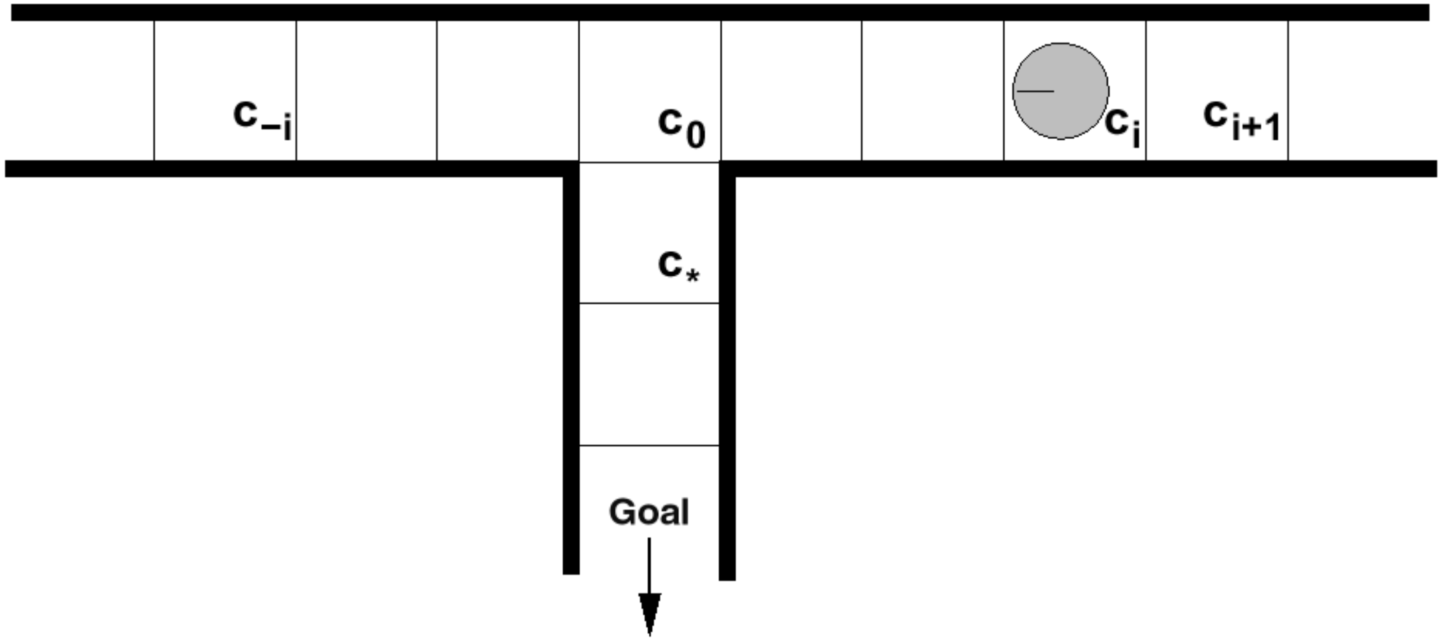
Considers cost to reach the goal.

Follows grid based path computed by A*.

Dynamic Window Approach

- Reacts quickly
- Low CPU power requirements
- Guides a robot on a collision-free path
- Successfully used in a lot of real-world scenarios
- Resulting trajectories sometimes sub-optimal
- Local minima might prevent the robot from reaching the goal location

Problems of DWAs



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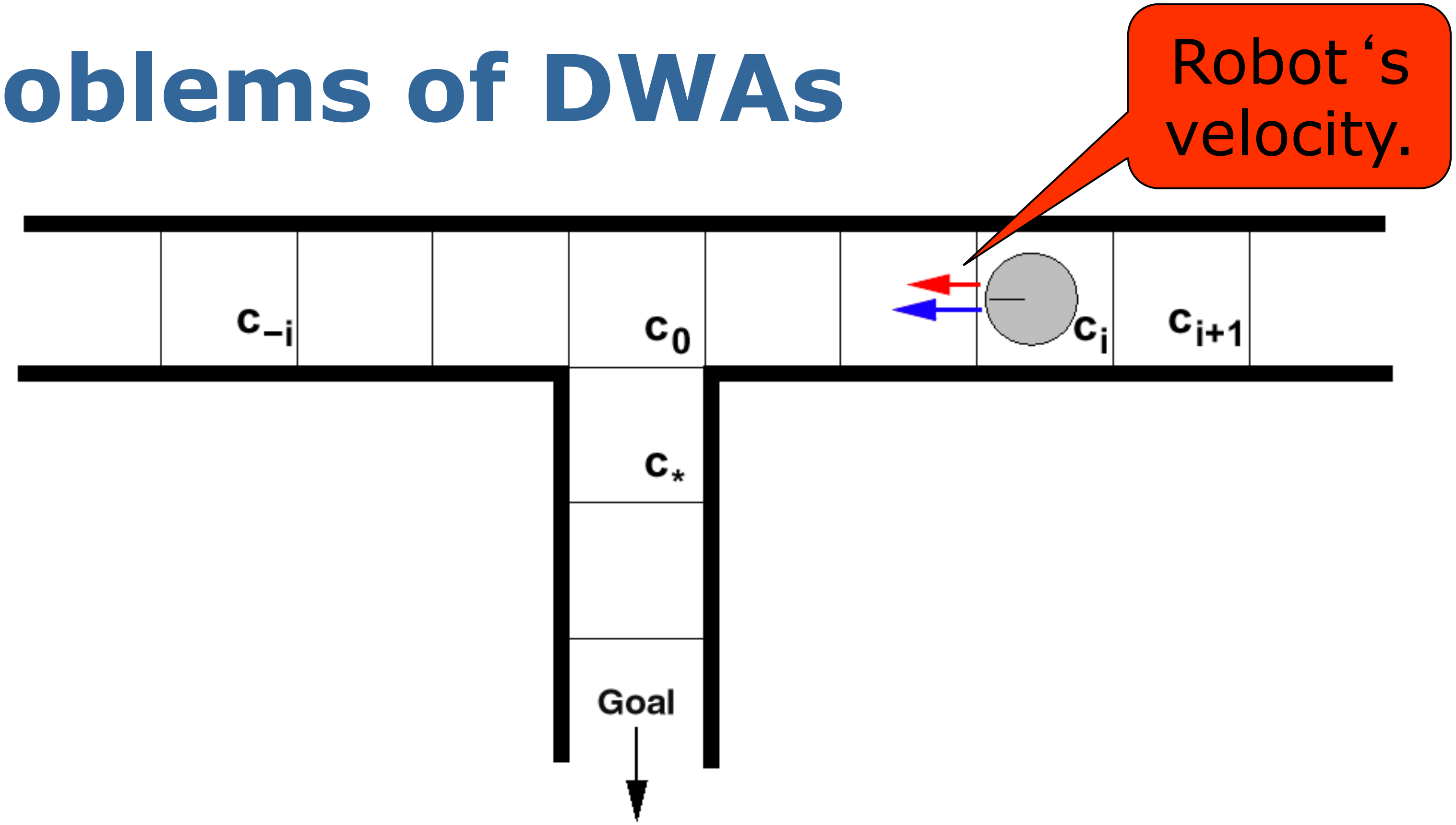

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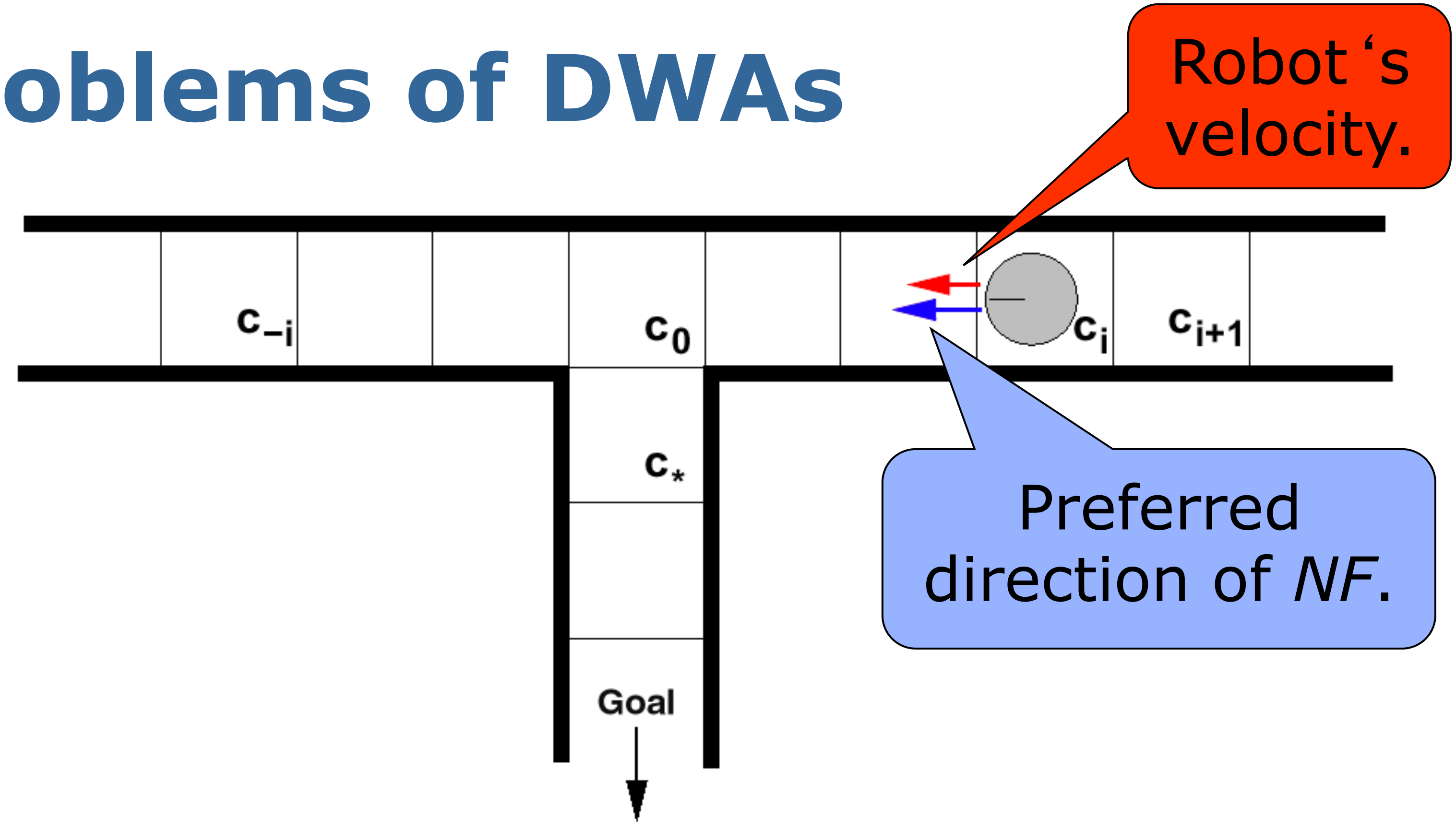

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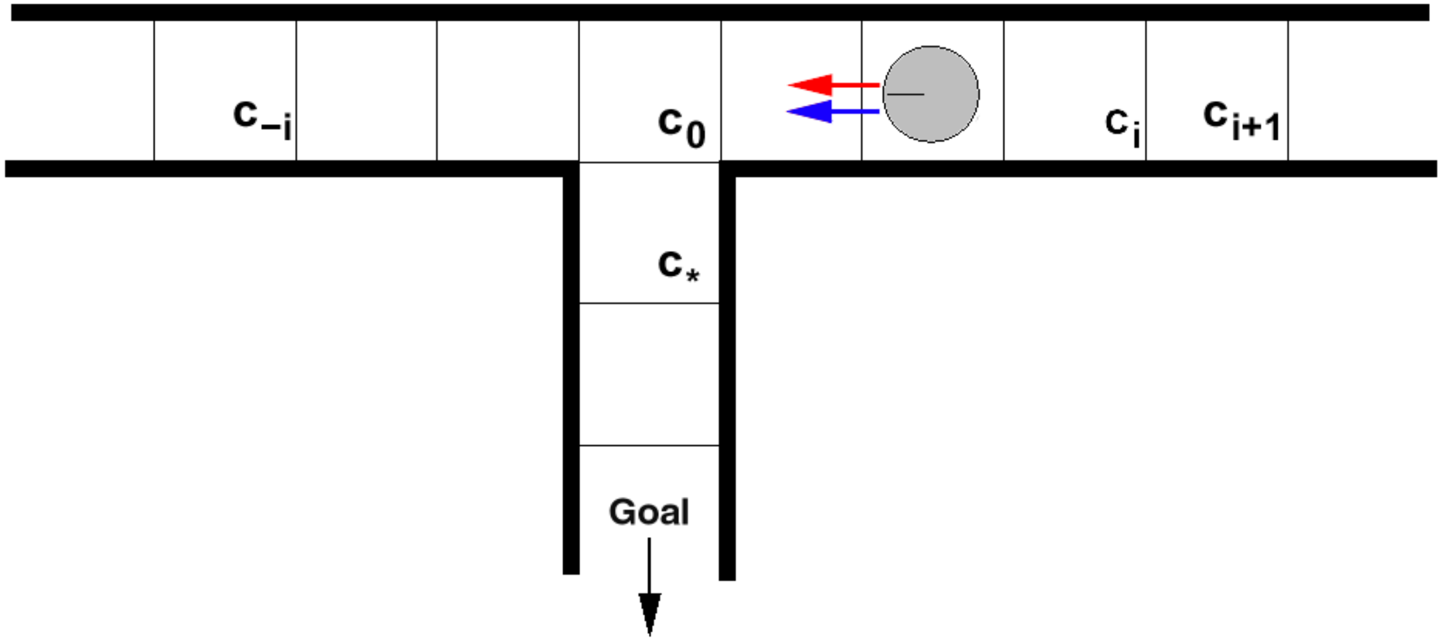

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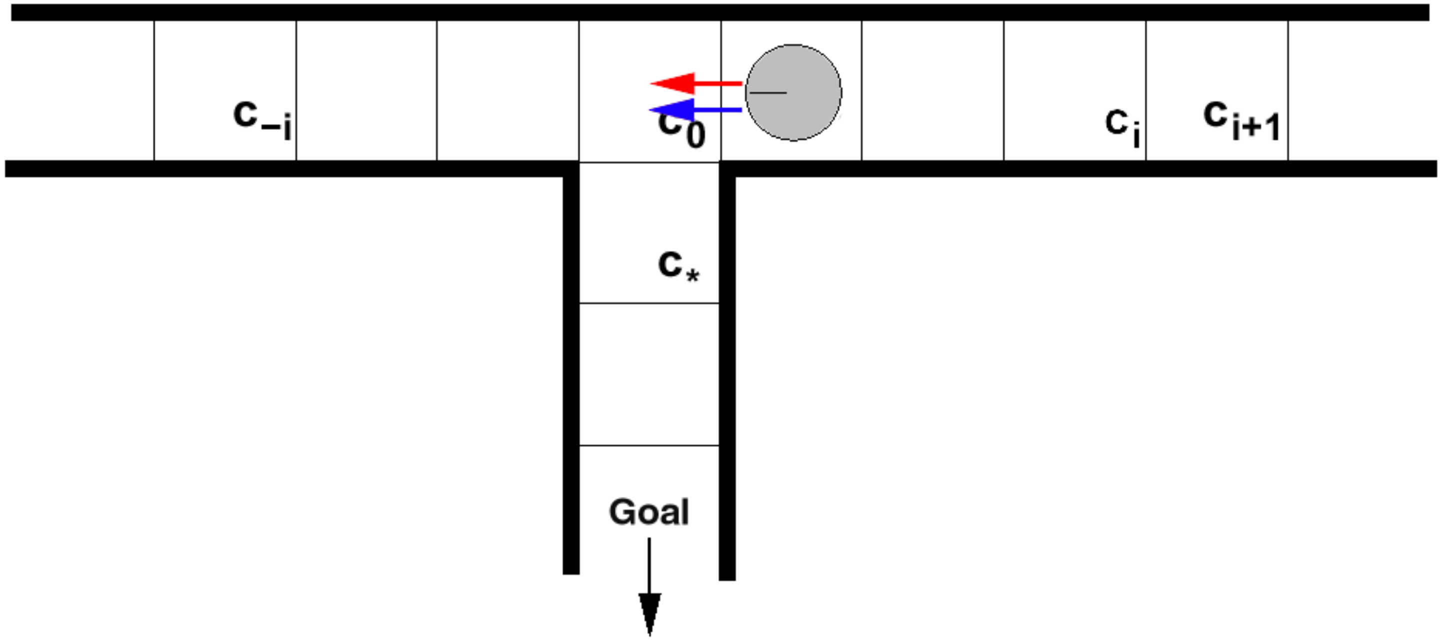

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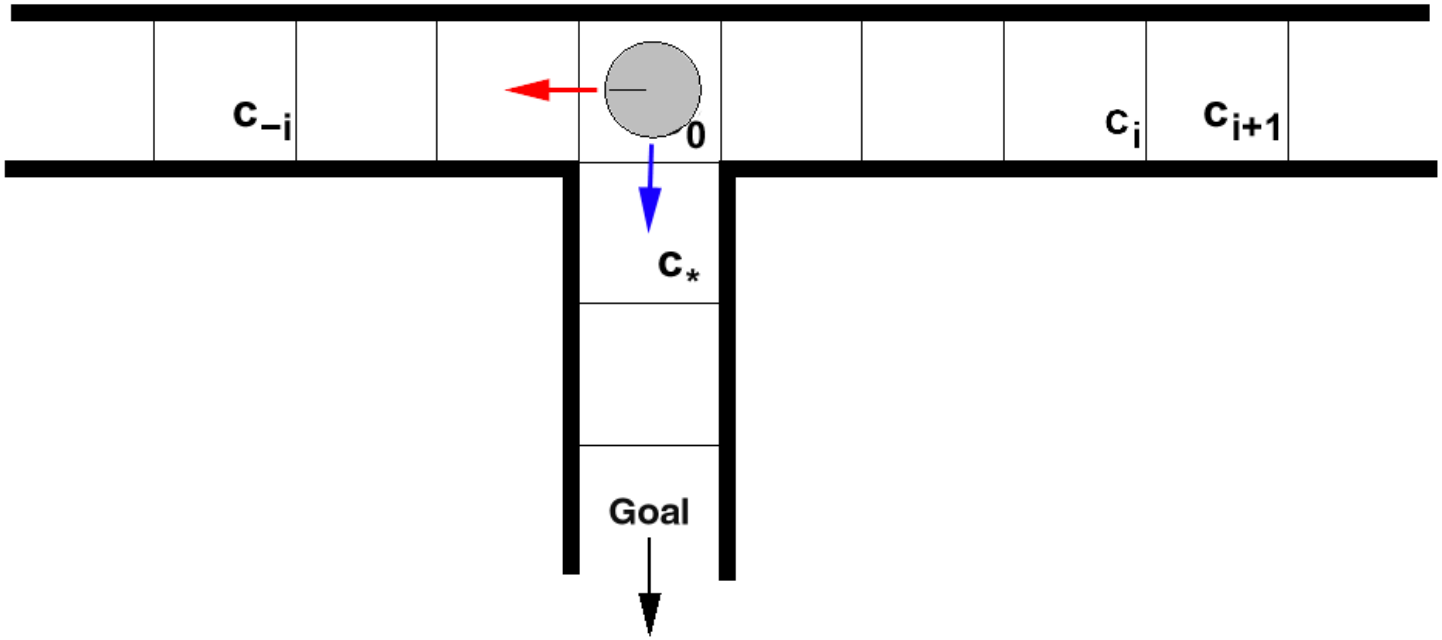
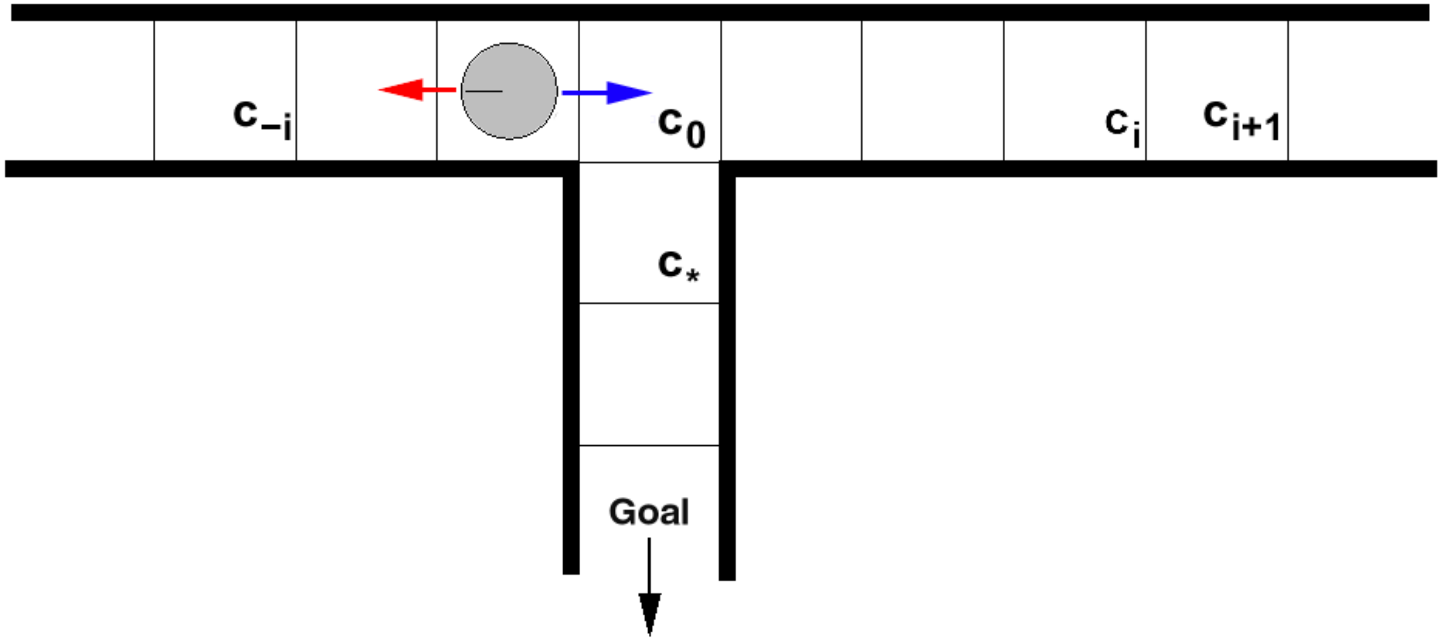
Problems of DWAs



$$NF = \alpha \cdot vel + \beta \cdot nf + \gamma \cdot \Delta nf + \delta \cdot goal$$

- The robot drives too fast at c_0 to enter the corridor facing south.

Problems of DWAs



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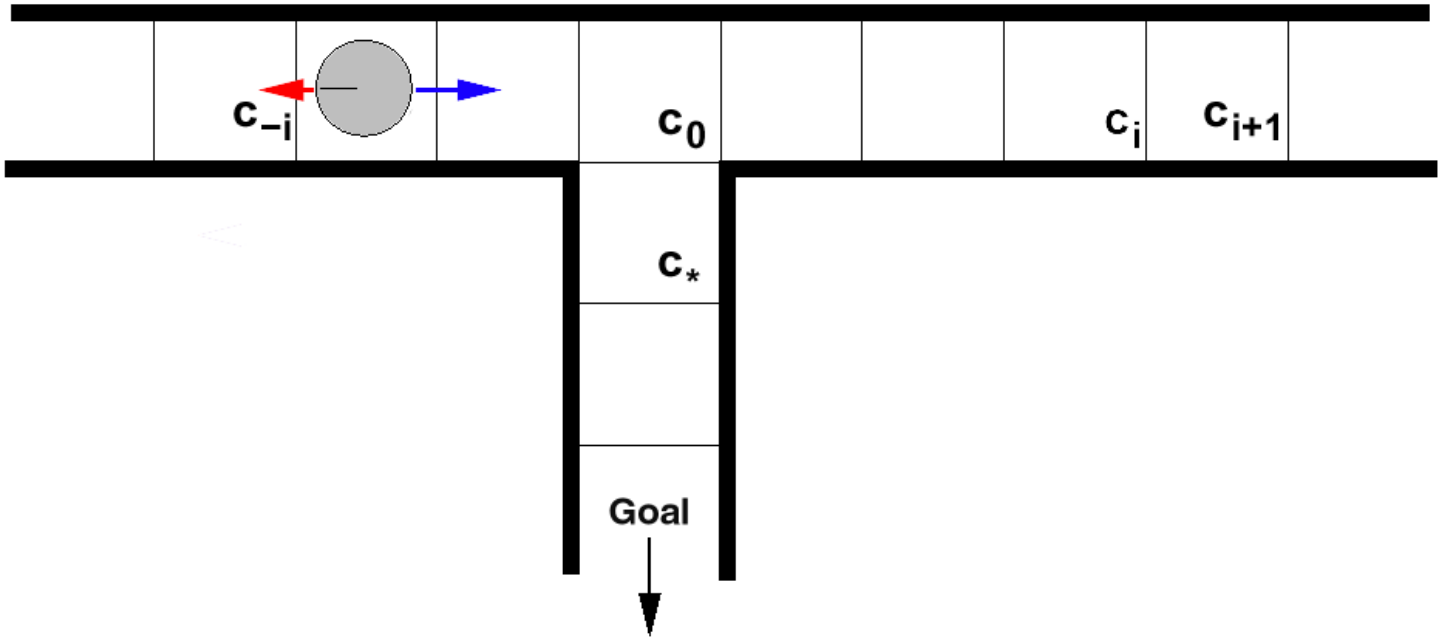

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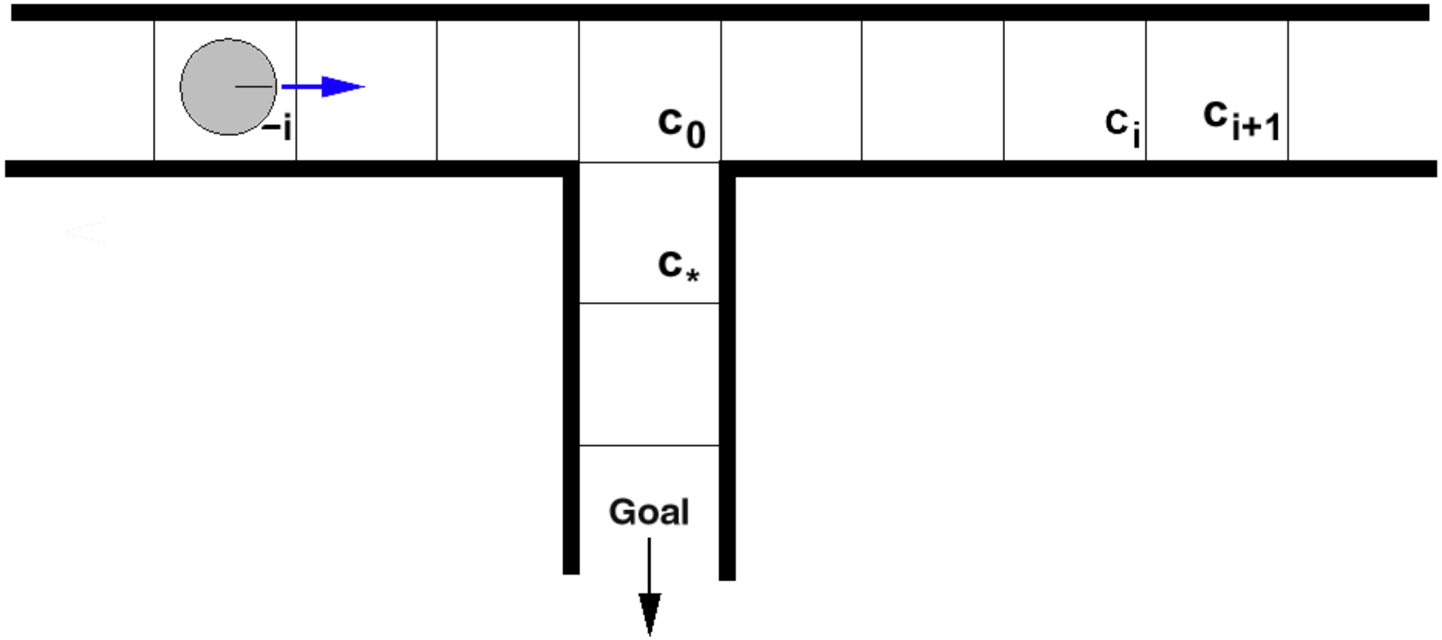
Problems of DWAs



- Same situation as in the beginning

  ➔ DWAs might not be able to reach the goal location.

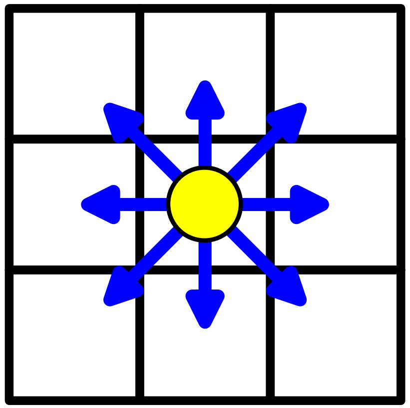
Problems of DWAs

- Typical problem in a real world situation:

- Robot does not slow down early enough to enter the doorway

Robot Path Planning with A*

- Finds the shortest path
- Requires a graph structure
- Limited number of edges
- In robotics: Often planning using a 2D occupancy grid map



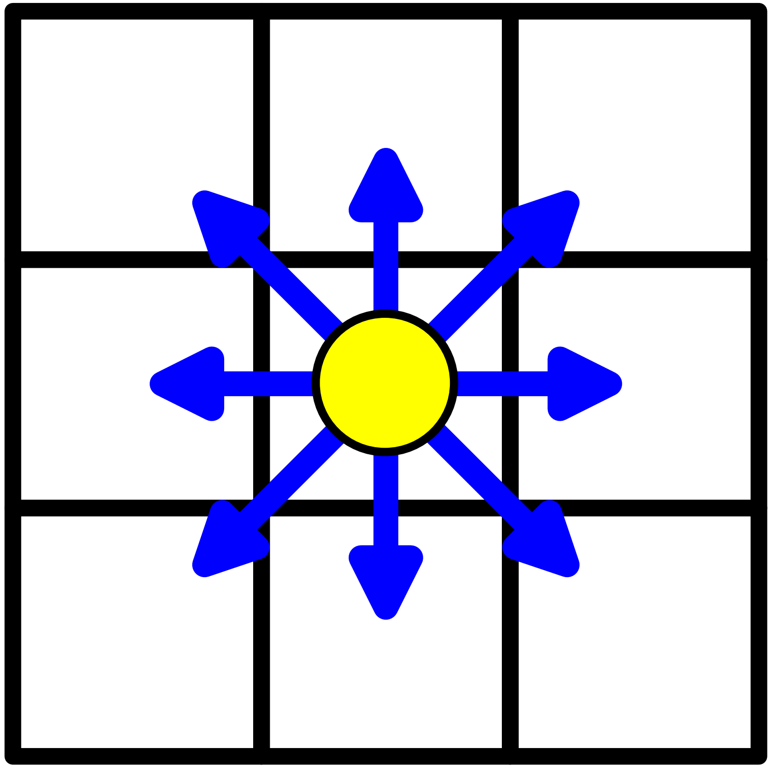
Reminder: A*

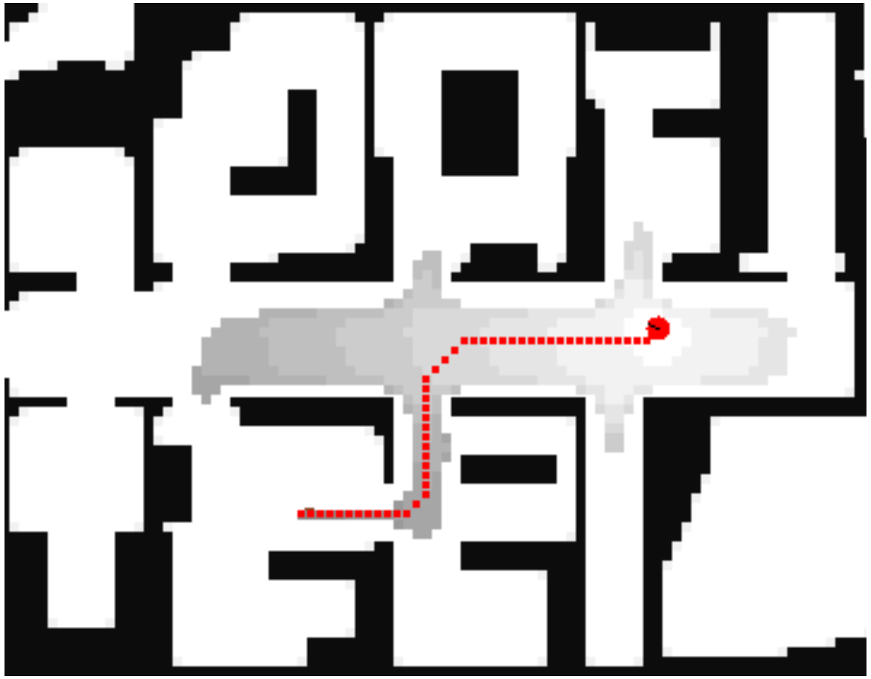
- $g(n)$ = actual cost from the initial state to n
- $h(n)$ = estimated cost from n to the next goal
- $f(n) = g(n) + h(n)$, the estimated cost of the cheapest solution through n
- Let h*(n) be the actual cost of the optimal path from n to the next goal
- h is admissible if the following holds for all n :

$$h(n) \leq h^*(n)$$

- A* yields the optimal path if h is admissible (the straight-line distance is admissible in the Euclidean Space)

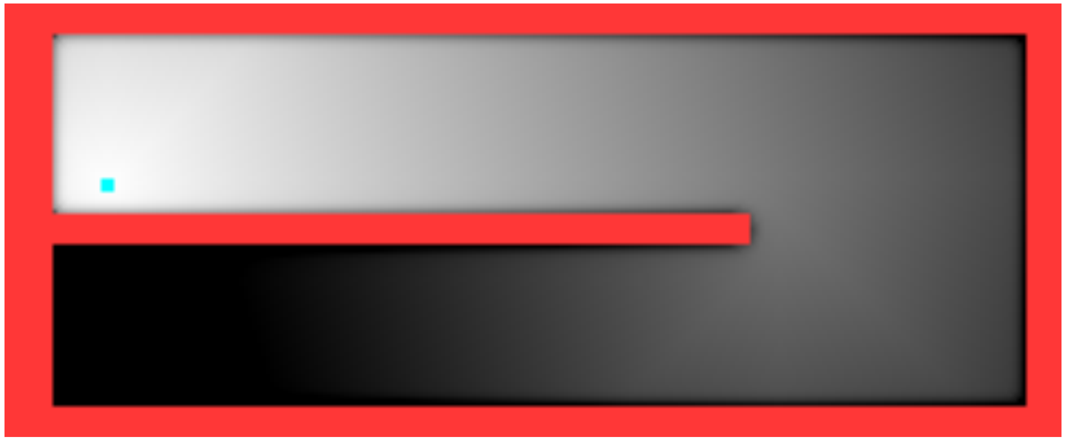
Example: Path Planning with A* for Robots in a Grid-World





Deterministic Value Iteration

- To compute the shortest path from every state to one goal state, use (deterministic) value iteration
- Very similar to Dijkstra's Algorithm
- Such a cost distribution is the optimal heuristic for A*



Typical Assumption in Robotics for A* Path Planning

- The robot is assumed to be localized
- The robot computes its path based on an occupancy grid
- Motion commands are executed accurately

Is this always true?

Problems

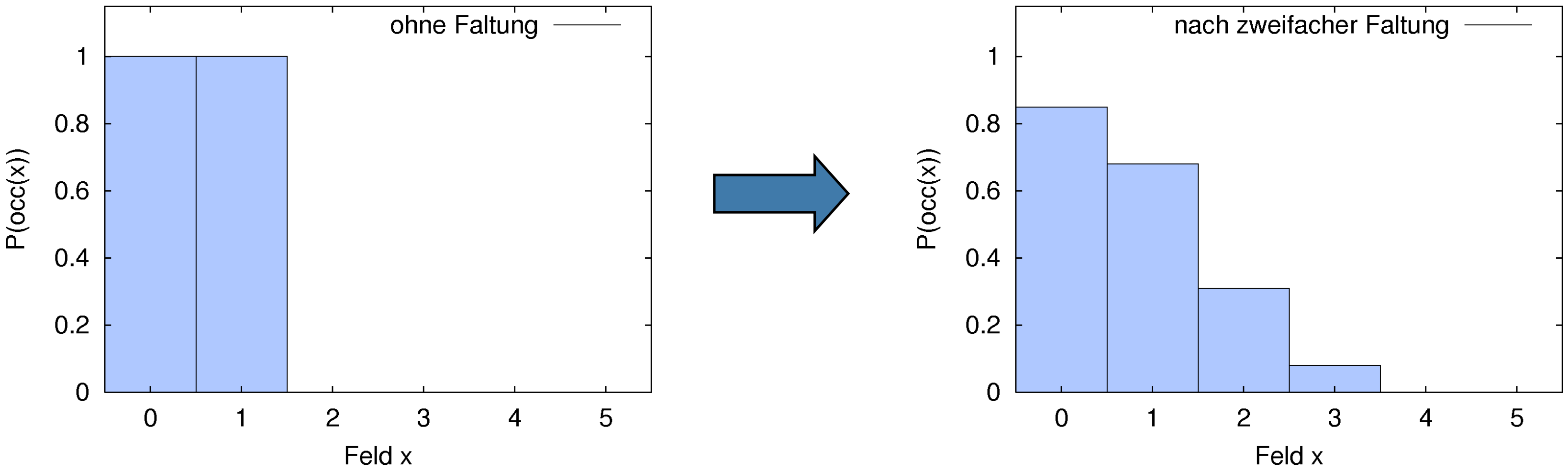
- What if the robot is slightly delocalized?
- Moving on the shortest path guides often the robot on a trajectory close to obstacles
- Trajectory aligned to the grid structure

Convolution of the Grid Map

- Convolution blurs the map
- Obstacles are assumed to be bigger than in reality
- Perform an A* search in such a convolved map
- Robot increases distance to obstacles and moves on a short path!

Example: Map Convolution

- 1-d environment, cells $c_0, \ldots, c_5$



- Cells before and after 2 convolution runs

Convolution

- Consider an occupancy map. The convolution is defined as:

$$P(occ_{x_i,y}) = \frac{1}{4} \cdot P(occ_{x_{i-1},y}) + \frac{1}{2} \cdot P(occ_{x_i,y}) + \frac{1}{4} \cdot P(occ_{x_{i+1},y})$$

$$P(occ_{x_0,y}) = \frac{2}{3} \cdot P(occ_{x_0,y}) + \frac{1}{3} \cdot P(occ_{x_1,y})$$

$$P(occ_{x_{n-1},y}) = \frac{1}{3} \cdot P(occ_{x_{n-2},y}) + \frac{2}{3} \cdot P(occ_{x_{n-1},y})$$

- This is done for each row and each column of the map
- "Gaussian blur"

A* in Convolved Maps

- The costs are a product of path length and occupancy probability of the cells
- Cells with higher probability (e.g., caused by convolution) are avoided by the robot
- Thus, it keeps distance to obstacles
- This technique is fast and quite reliable

5D-Planning – An Alternative to the Two-layered Architecture

- Plans in the full $<x,y,\theta,v,\omega>$-configuration space using A^*
  - ➔ Considers the robot's kinematic constraints
- Generates a sequence of steering commands to reach the goal location
- Maximizes trade-off between driving time and distance to obstacles

The Search Space (1)

- What is a state in this space?
  $<x,y,\theta,v,\omega>$ = current position and velocities of the robot

- How does a state transition look like?
  $<x_1,y_1,\theta_1,v_1,\omega_1> \rightarrow <x_2,y_2,\theta_2,v_2,\omega_2>$
  with motion command (v_2,ω_2) and
  $|v_1-v_2| < a_v,\ |\omega_1-\omega_2| < a_\omega$
- Pose of the robot is a result of the motion equations

The Search Space (2)

Idea: Search in the discretized <x,y,θ,v,ω>-space

Problem: The search space is too huge to be explored within the time constraints (5+ Hz for online motion plannig)

Solution: Restrict the full search space

The Main Steps of Our Algorithm

1. Update (static) grid map based on sensory input

2. Use A^* to find a trajectory in the <x,y>-space using the updated grid map

3. Determine a restricted 5d-configuration space based on step 2

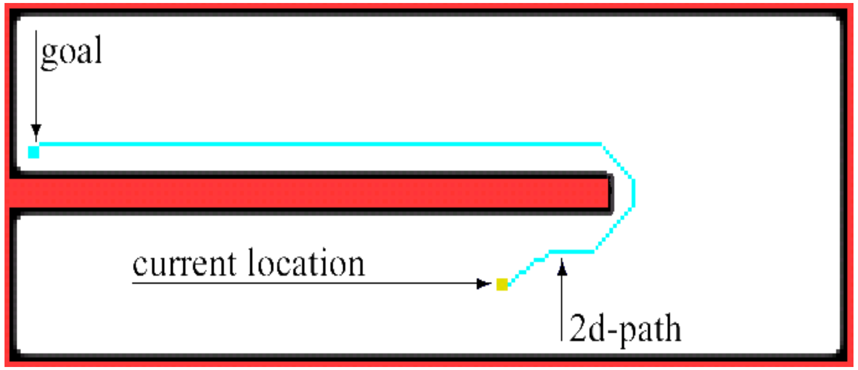
4. Find a trajectory by planning in the restricted $\langle x,y,\theta,v,\omega \rangle$-space

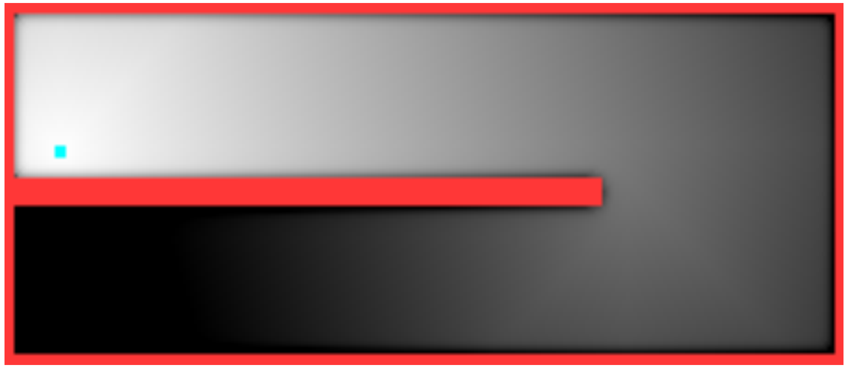
Updating the Grid Map

- The environment is represented as a 2d-occupency grid map
- Convolution of the map increases security distance
- Detected obstacles are added
- Cells discovered free are cleared

Find a Path in the 2d-Space

- Use A^* to search for the optimal path in the 2d-grid map
- Use heuristic based on a deterministic value iteration within the static map





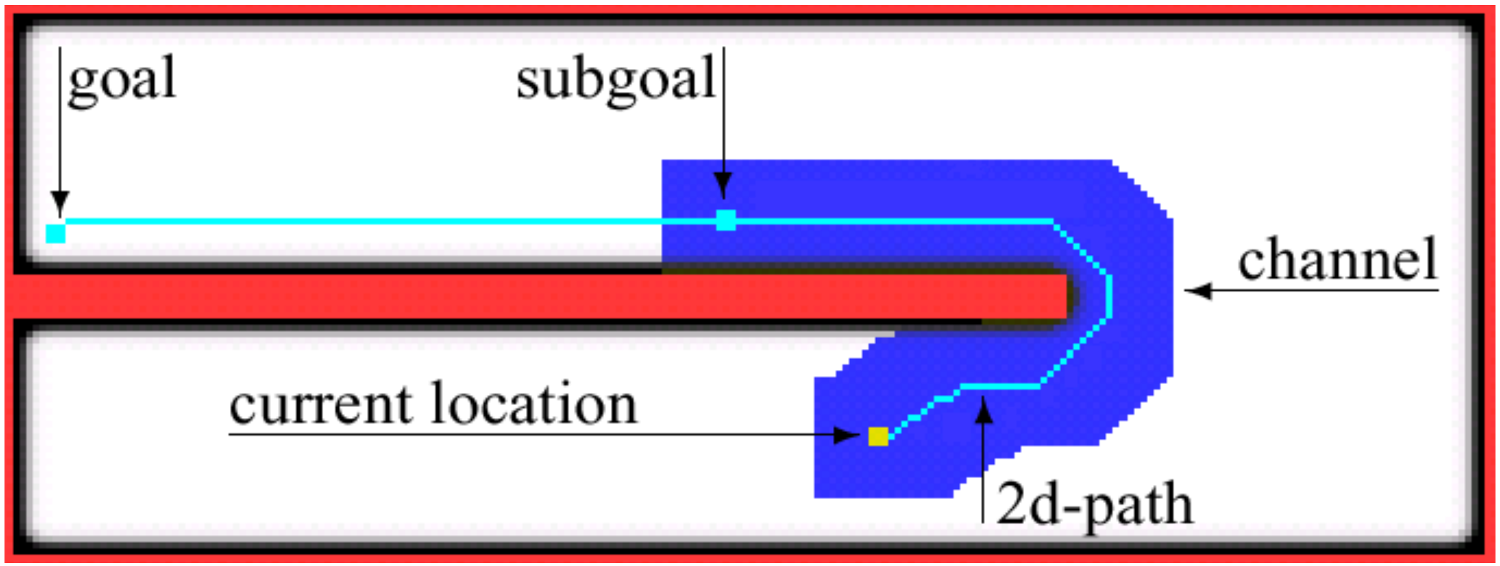
Restricting the Search Space

Assumption: The projection of the optimal 5d-path onto the <x,y>-space lies close to the optimal 2d-path

Therefore: Construct a restricted search space (channel) based on the 2d-path

Space Restriction

- Resulting search space = <x, y, θ, v, ω> with (x,y) ϵ channel
- Choose a sub-goal lying on the 2d-path within the channel

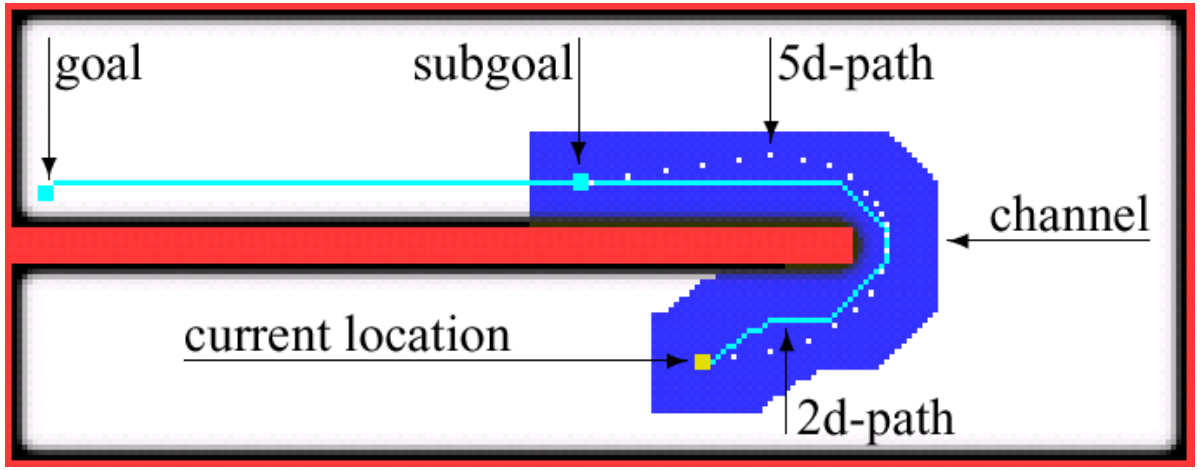


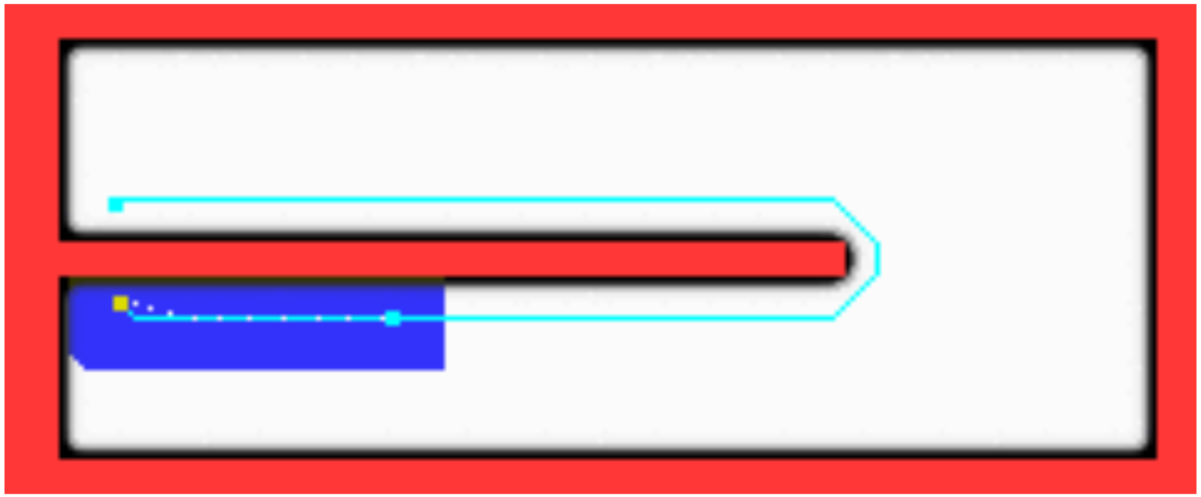
Find a Path in the 5d-Space

- Use A^* in the restricted 5d-space to find a sequence of steering commands to reach the sub-goal

- To estimate cell costs: Perform a deterministic 2d-value iteration within the channel

Examples





Timeouts

- Steering a robot online requires to set a new steering command every .25 secs

➔ Abort search after .25 secs.

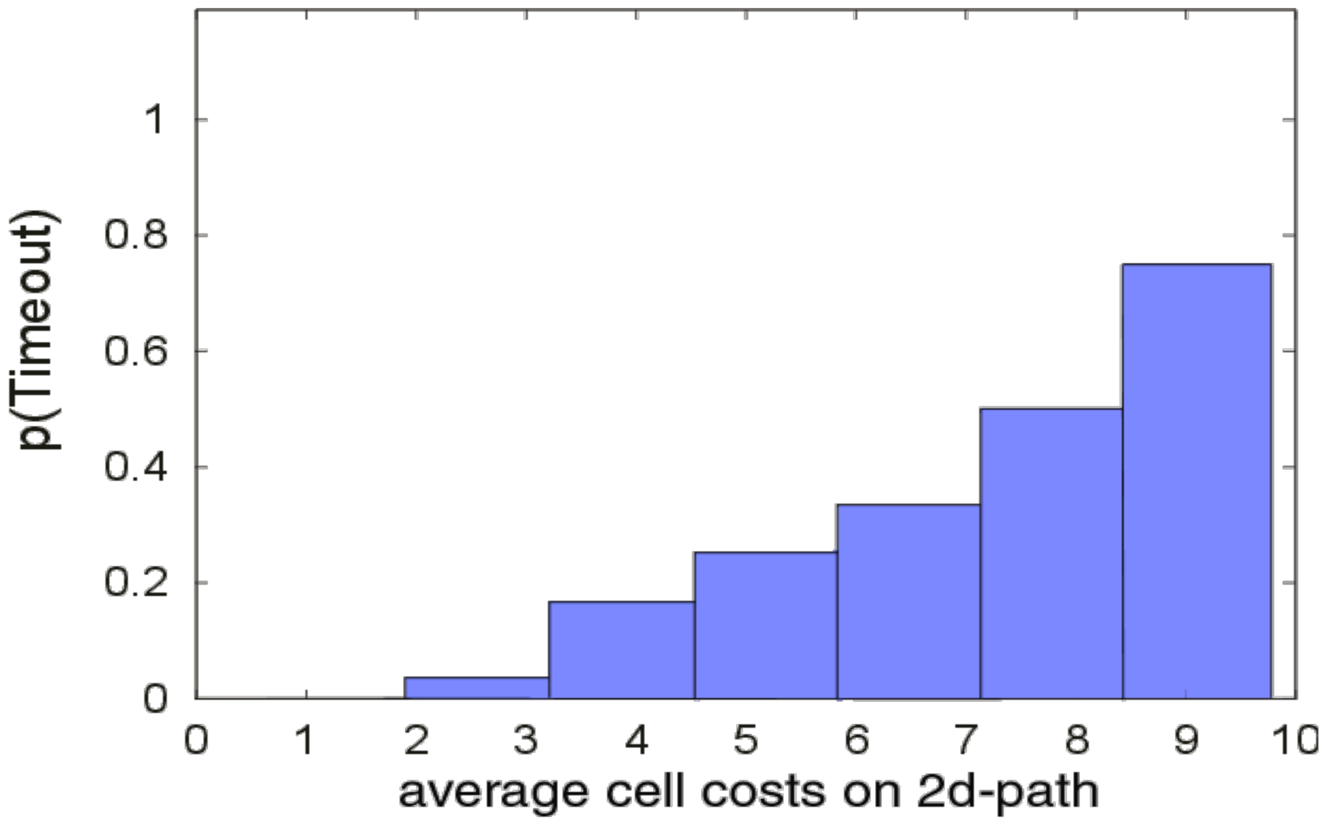
How to find an admissible steering command?

Alternative Steering Command

- Previous trajectory still admissible?
  → **OK**

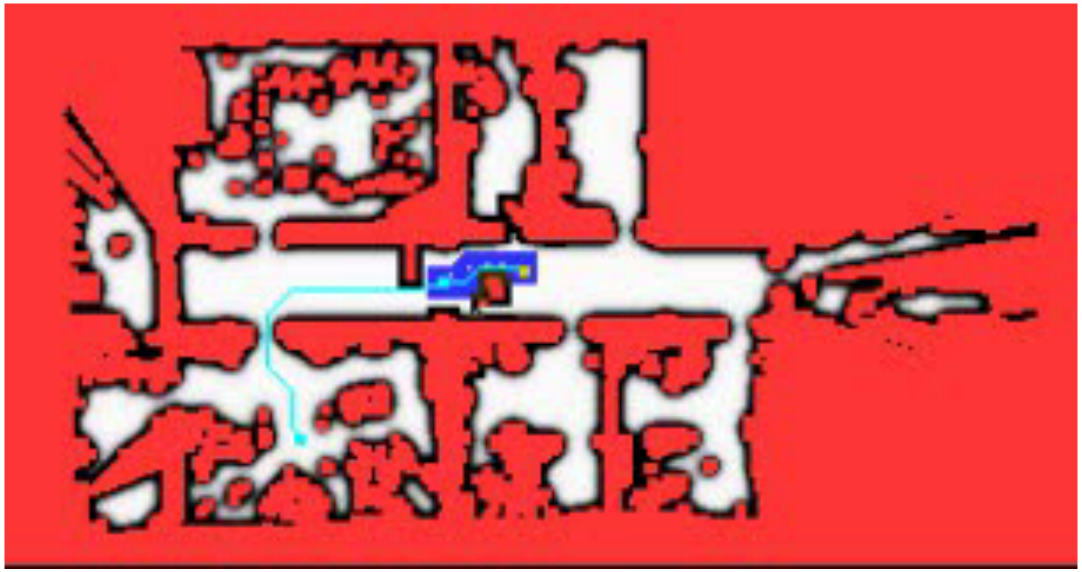
- If not, drive on the 2d-path or use DWA to find new command

Timeout Avoidance



➔ Reduce the size of the channel if the 2d-path has high cost

Example



Robot Albert

Planning state

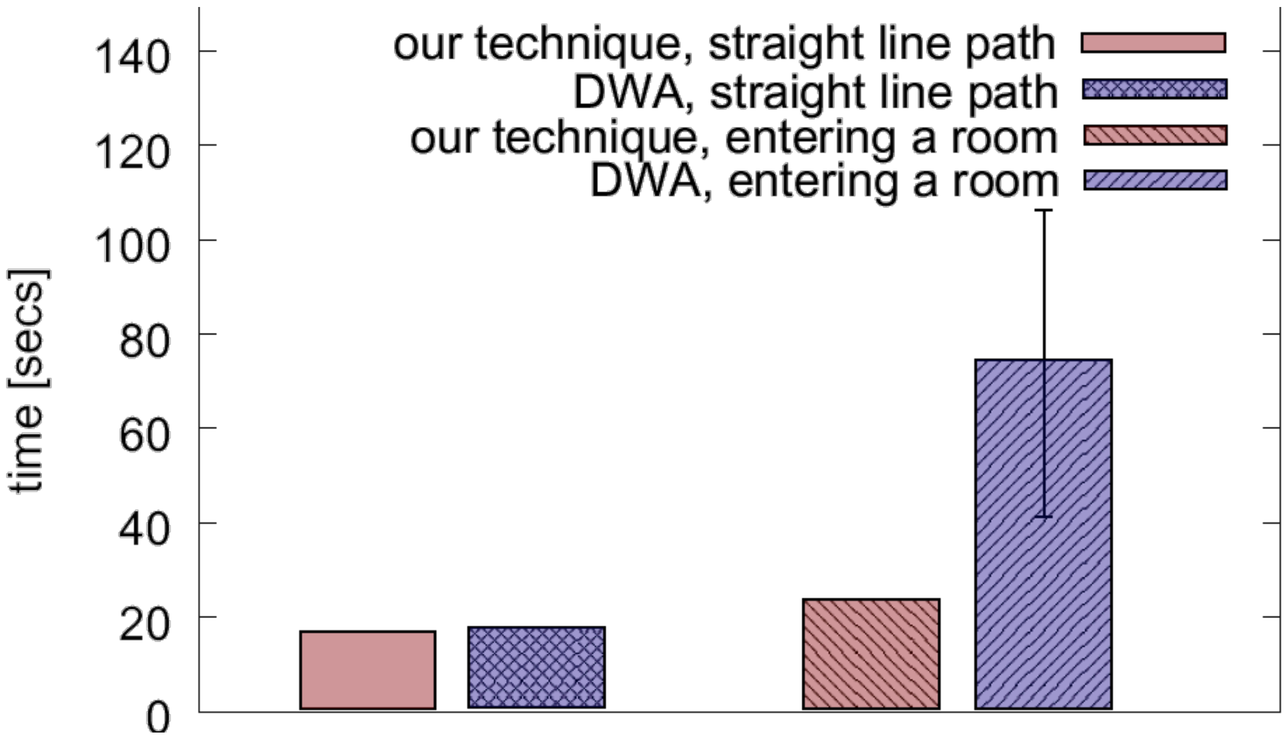
Comparison to the DWA (1)

- DWAs often have problems entering narrow passages

DWA planned path.

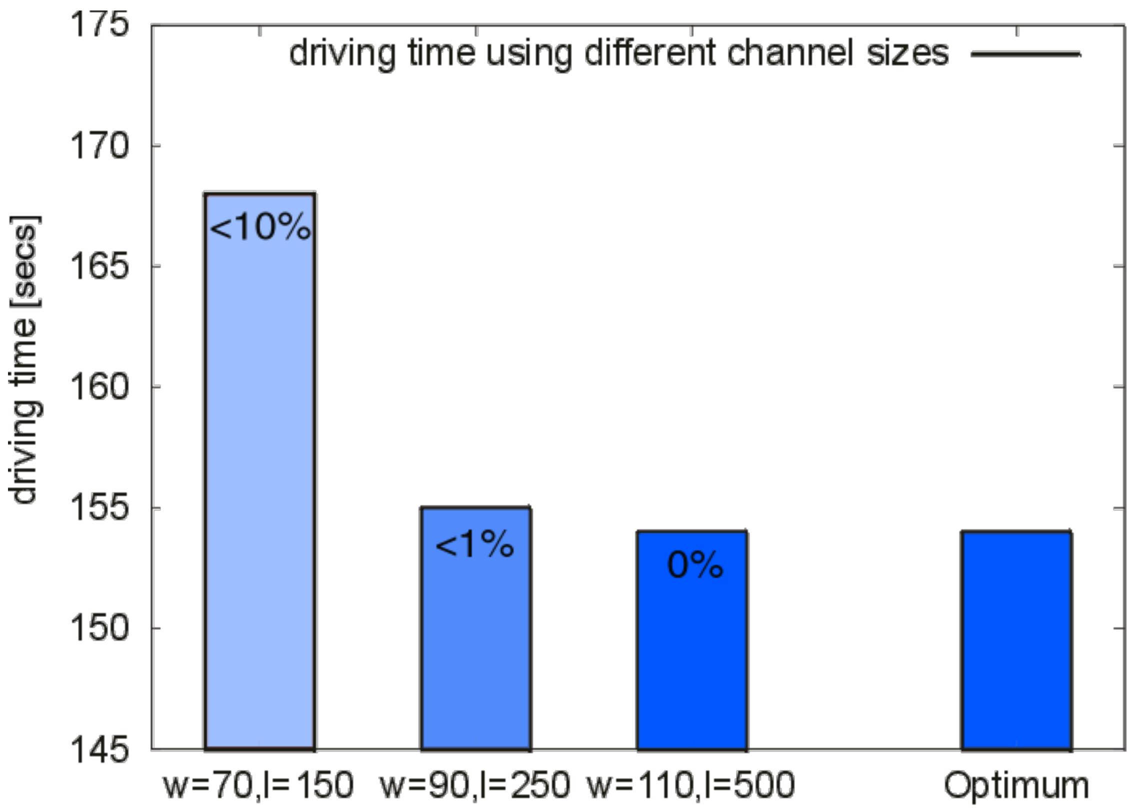
5D approach.

Comparison to the DWA (2)



The presented approach results in significantly faster motion when driving through narrow passages!

Comparison to the Optimum



Channel: with length=5m, width=1.1m

Resulting actions are close to the optimal solution

Summary

- Robust navigation requires combined path planning & collision avoidance
- Approaches need to consider robot's kinematic constraints and plans in the velocity space
- Combination of search and reactive techniques show better results than the pure DWA in a variety of situations
- Using the 5D-approach the quality of the trajectory scales with the computational resources available
- The resulting paths are often close to the optimal ones

What' s Missing?

- More complex vehicles (e.g., cars).
- Moving obstacles, motion prediction.
- Path planning
- …