Introduction to Mobile Robotics

SLAM: Simultaneous Localization and Mapping

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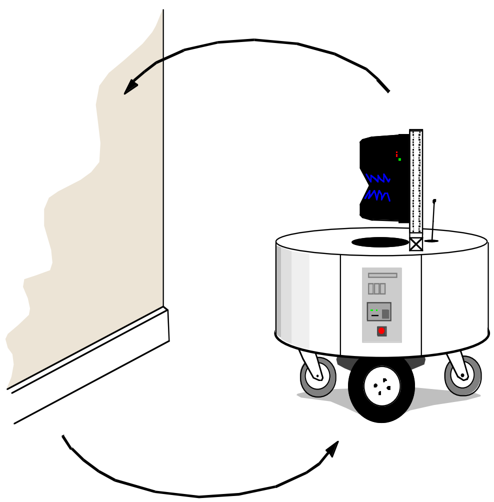
What is SLAM?

- Estimate the pose of a robot and the map of the environment at the same time
- SLAM is hard, because
  - a map is needed for localization and
  - a good pose estimate is needed for mapping

- **Localization:** inferring location given a map
- **Mapping:** inferring a map given locations
- **SLAM:** learning a map and locating the robot simultaneously

The SLAM Problem

- SLAM has long been regarded as a **chicken-or-egg** problem:
  → a map is needed for localization and
  → a pose estimate is needed for mapping



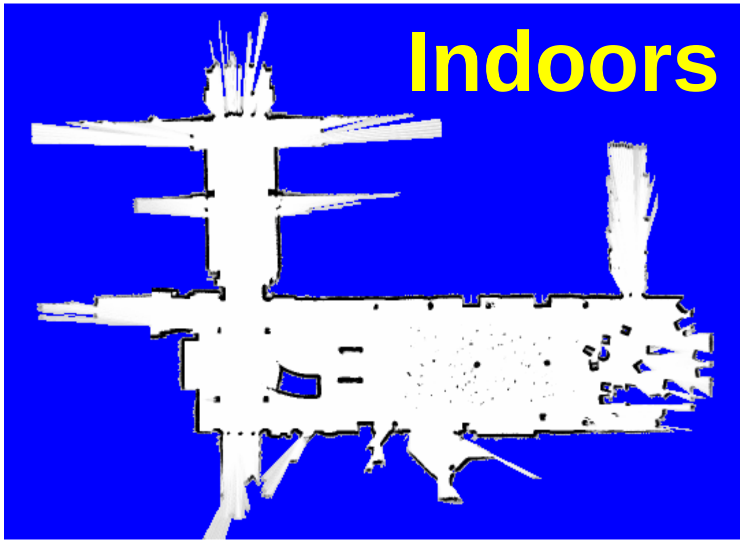
SLAM Applications

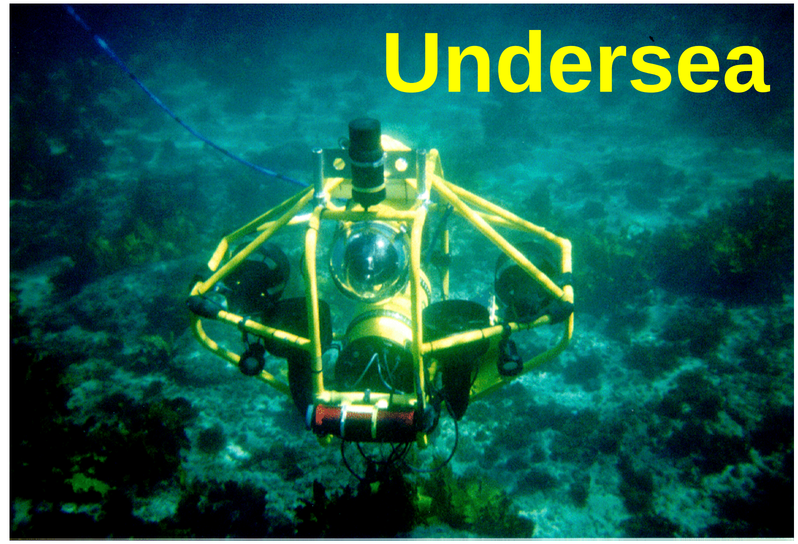
- SLAM is central to a range of indoor, outdoor, in-air and underwater applications for both manned and autonomous vehicles.

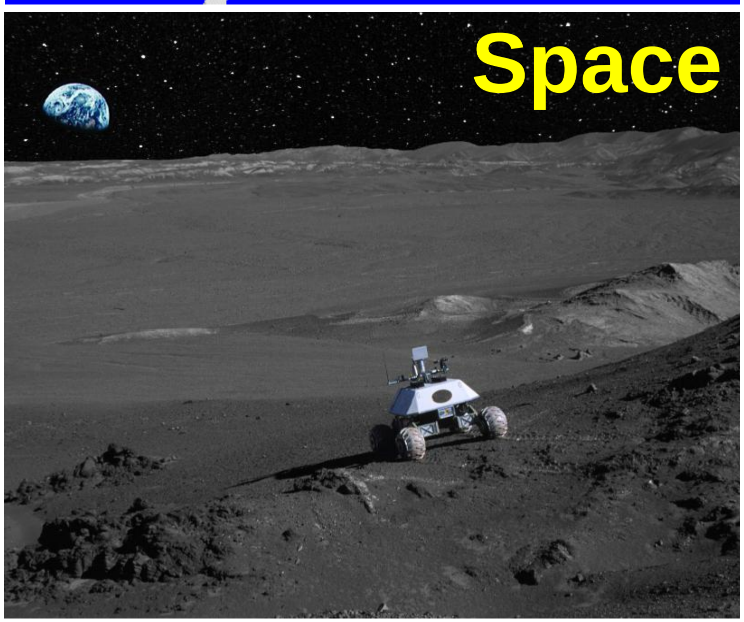
Examples:

- At home: vacuum cleaner, lawn mower
- Air: surveillance with unmanned air vehicles
- Underwater: reef monitoring
- Underground: exploration of mines
- Space: terrain mapping for localization

SLAM Applications

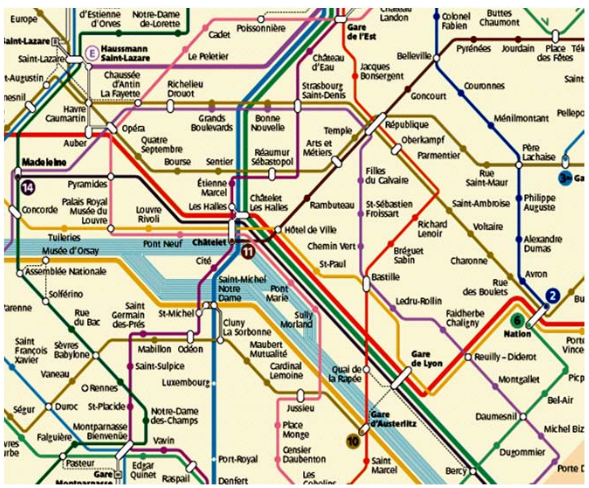


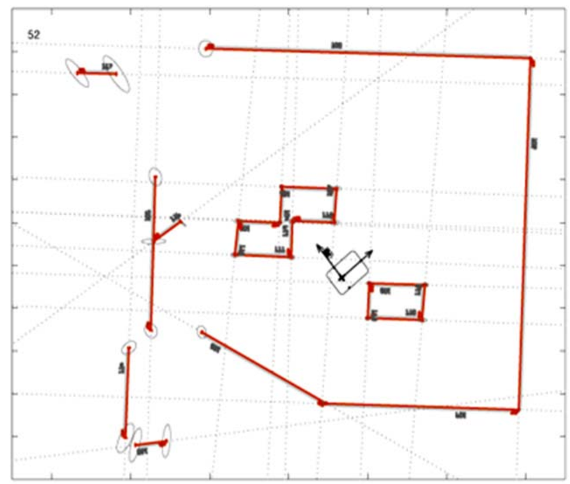




Map Representations

Examples: Subway map, city map, landmark-based map

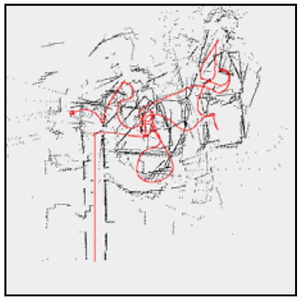


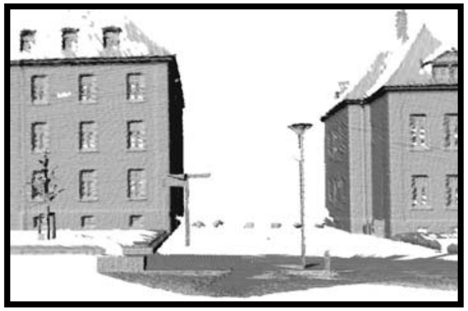


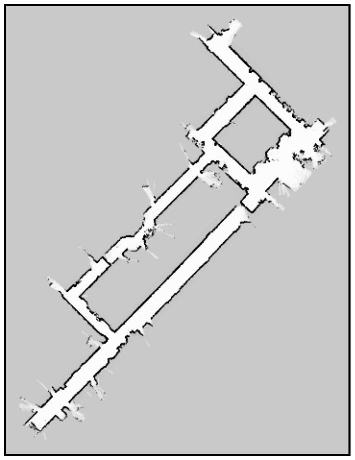
Maps are **topological** and/or **metric models** of the environment

Map Representations in Robotics

- Grid maps or scans, 2d, 3d

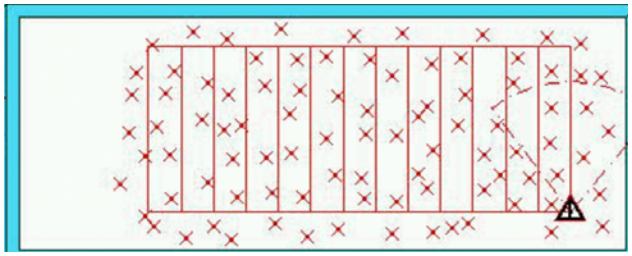


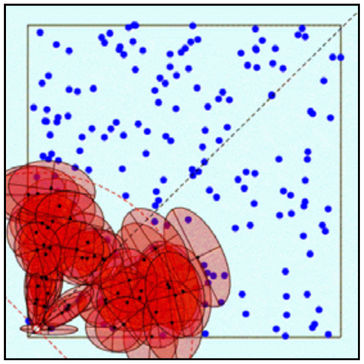




[Lu & Milios, 97; Gutmann, 98: Thrun 98; Burgard, 99; Konolige & Gutmann, 00; Thrun, 00; Arras, 99; Haehnel, 01; Grisetti et al., 05; ...]

- Landmark-based





[Leonard et al., 98; Castelanos et al., 99: Dissanayake et al., 2001; Montemerlo et al., 2002;...

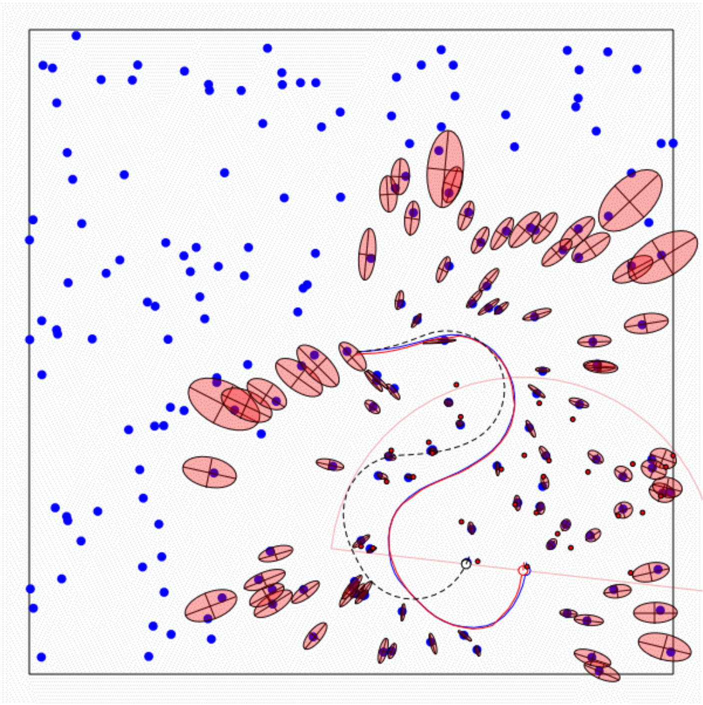
The SLAM Problem

- SLAM is considered a fundamental problems for robots to become truly autonomous
- Large variety of different SLAM approaches have been developed
- The majority uses probabilistic concepts
- History of SLAM dates back to the mid-eighties

Feature-Based SLAM

Given:

- The robot's controls
  $$U_{1:k} = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_k\}$$
- Relative observations
  $$Z_{1:k} = \{\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_k\}$$

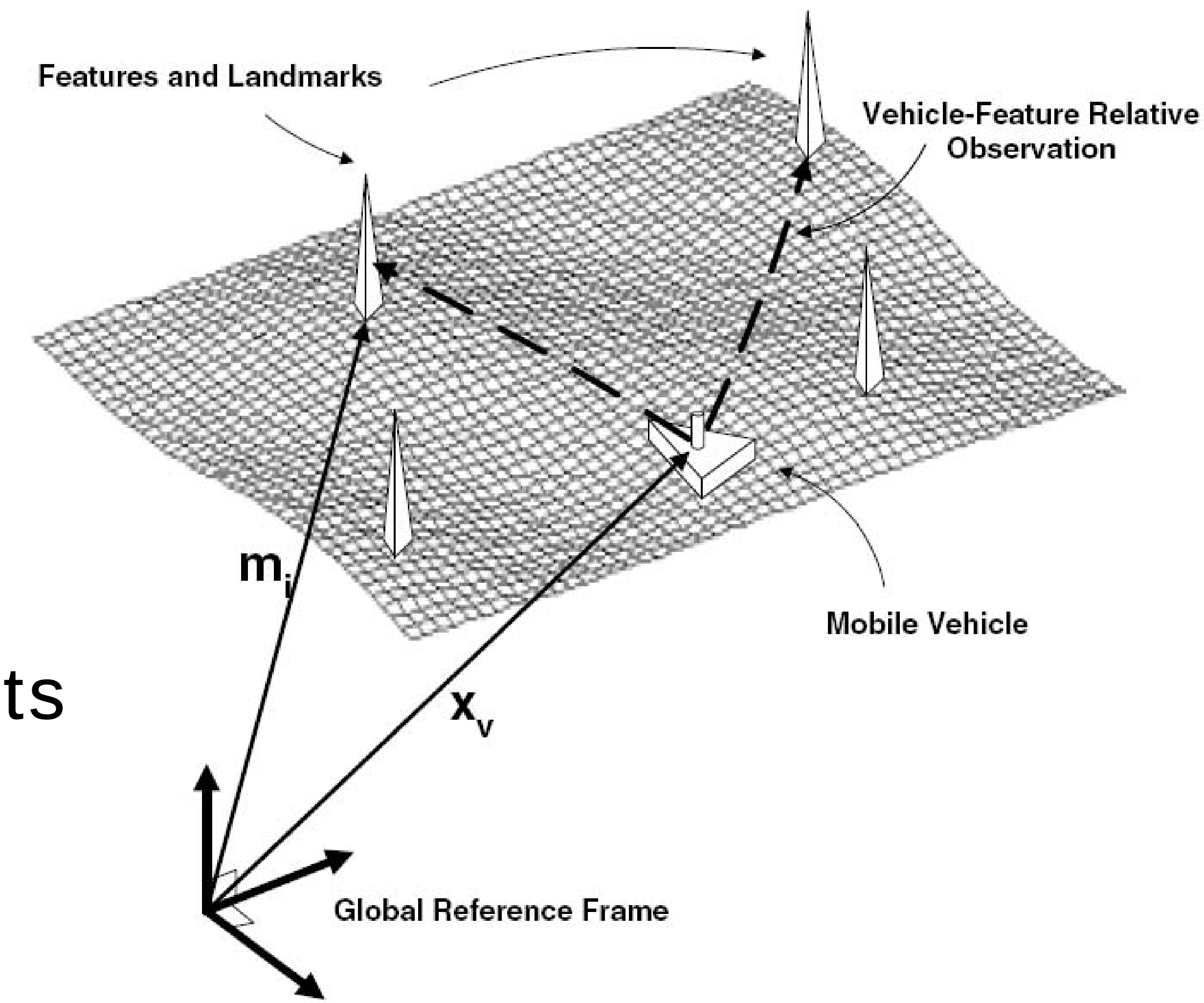


Wanted:

- Map of features
  $$\boldsymbol{m} = \{\boldsymbol{m}_1, \boldsymbol{m}_2, \ldots, \boldsymbol{m}_n\}$$
- Path of the robot
  $$\boldsymbol{X}_{1:k} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_k\}$$

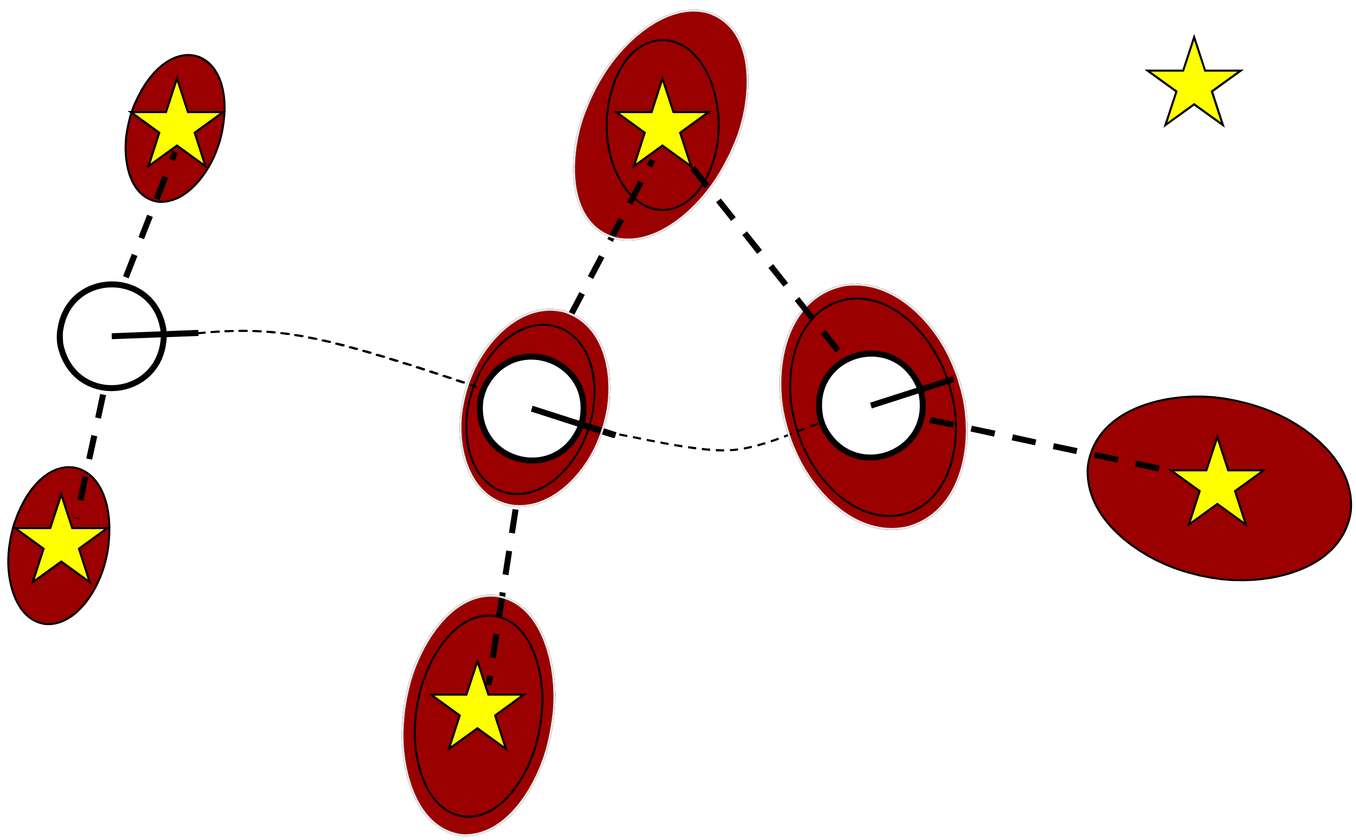
Feature-Based SLAM

- **Absolute** robot poses
- **Absolute** landmark positions
- But only **relative** measurements of landmarks



Why is SLAM a hard problem?

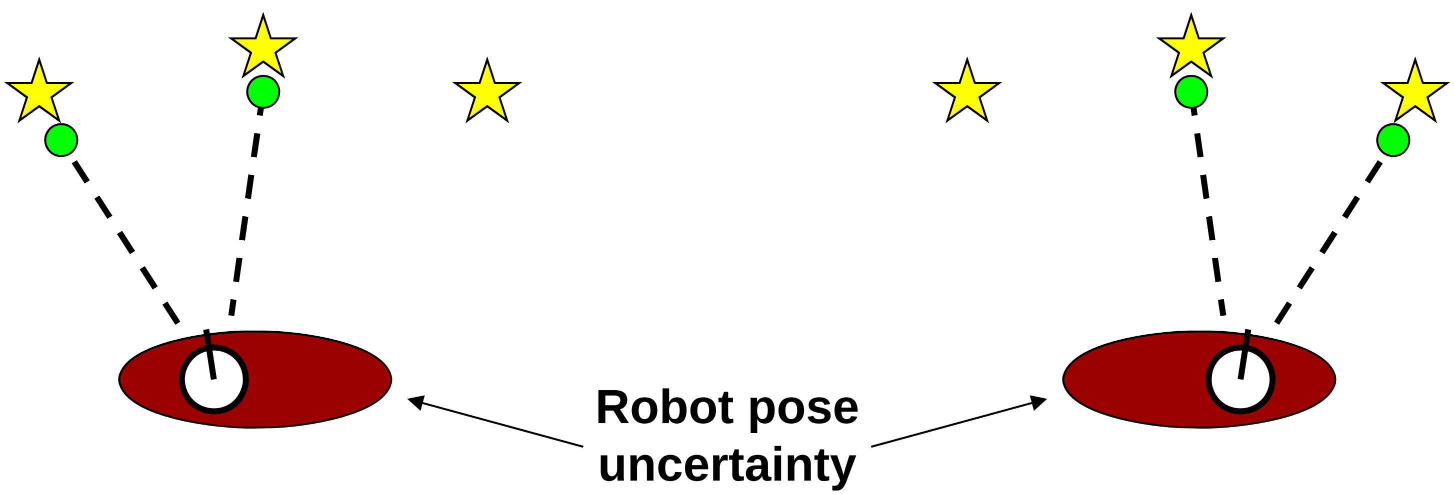
1. Robot path and map are both **unknown**



2. Errors in map and pose estimates correlated

Why is SLAM a hard problem?

- The **mapping between observations and landmarks is unknown**
- Picking **wrong** data associations can have **catastrophic** consequences (divergence)



SLAM: Simultaneous Localization And Mapping

- Full SLAM:

$$p(x_{0:t}, m \mid z_{1:t}, u_{1:t})$$

Estimates entire path and map!

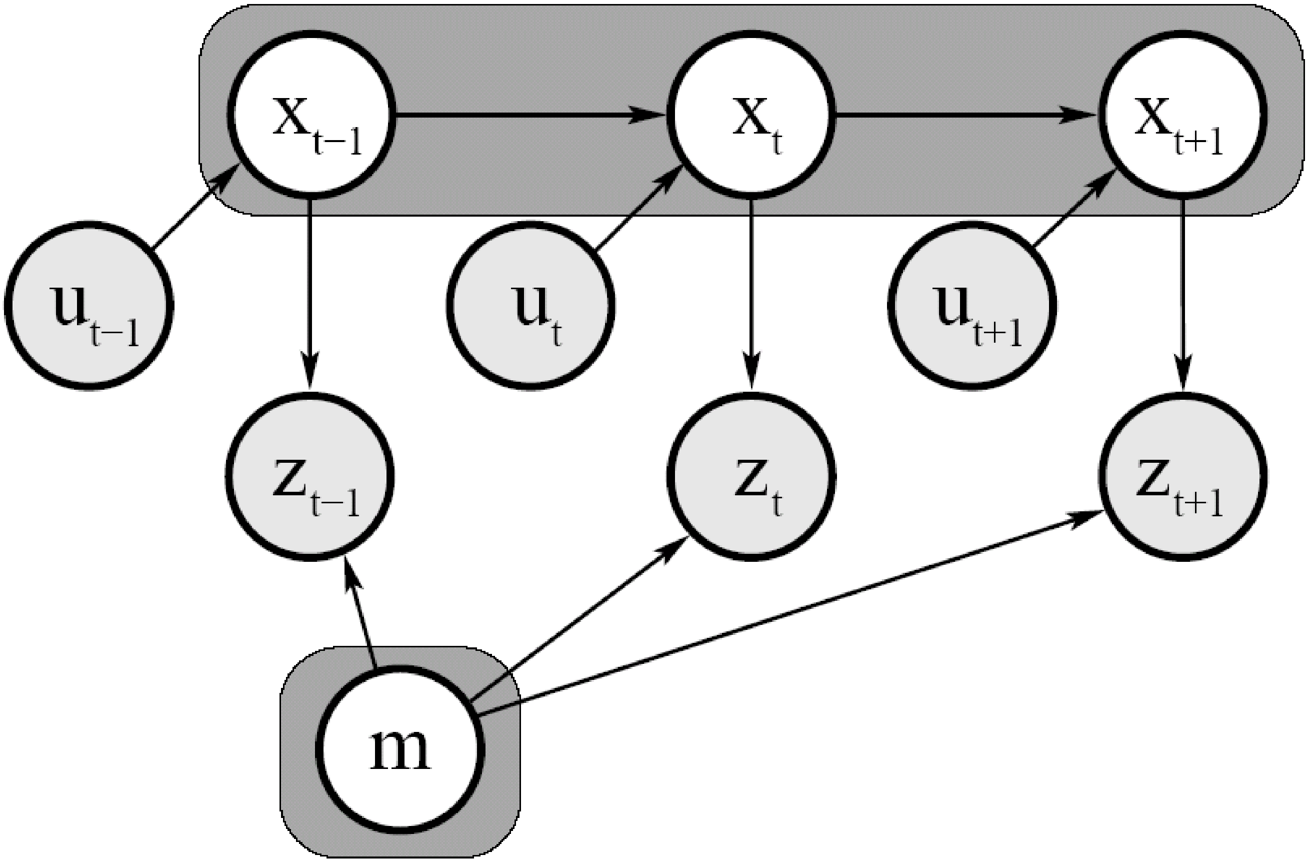
- Online SLAM:

$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \ldots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) dx_1 dx_2 \ldots dx_{t-1}$$

Estimates most recent pose and map!

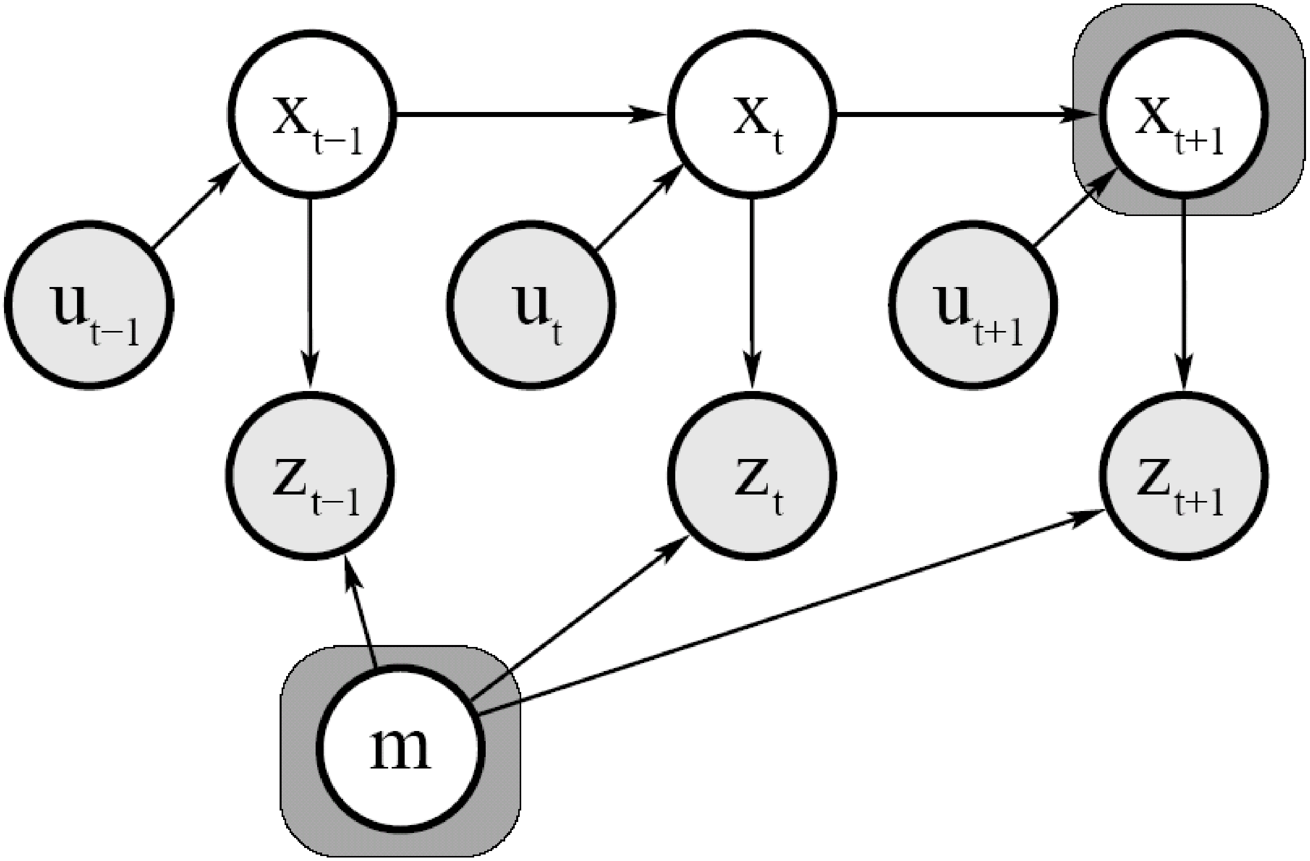
- Integrations (marginalization) typically done recursively, one at a time

Graphical Model of Full SLAM



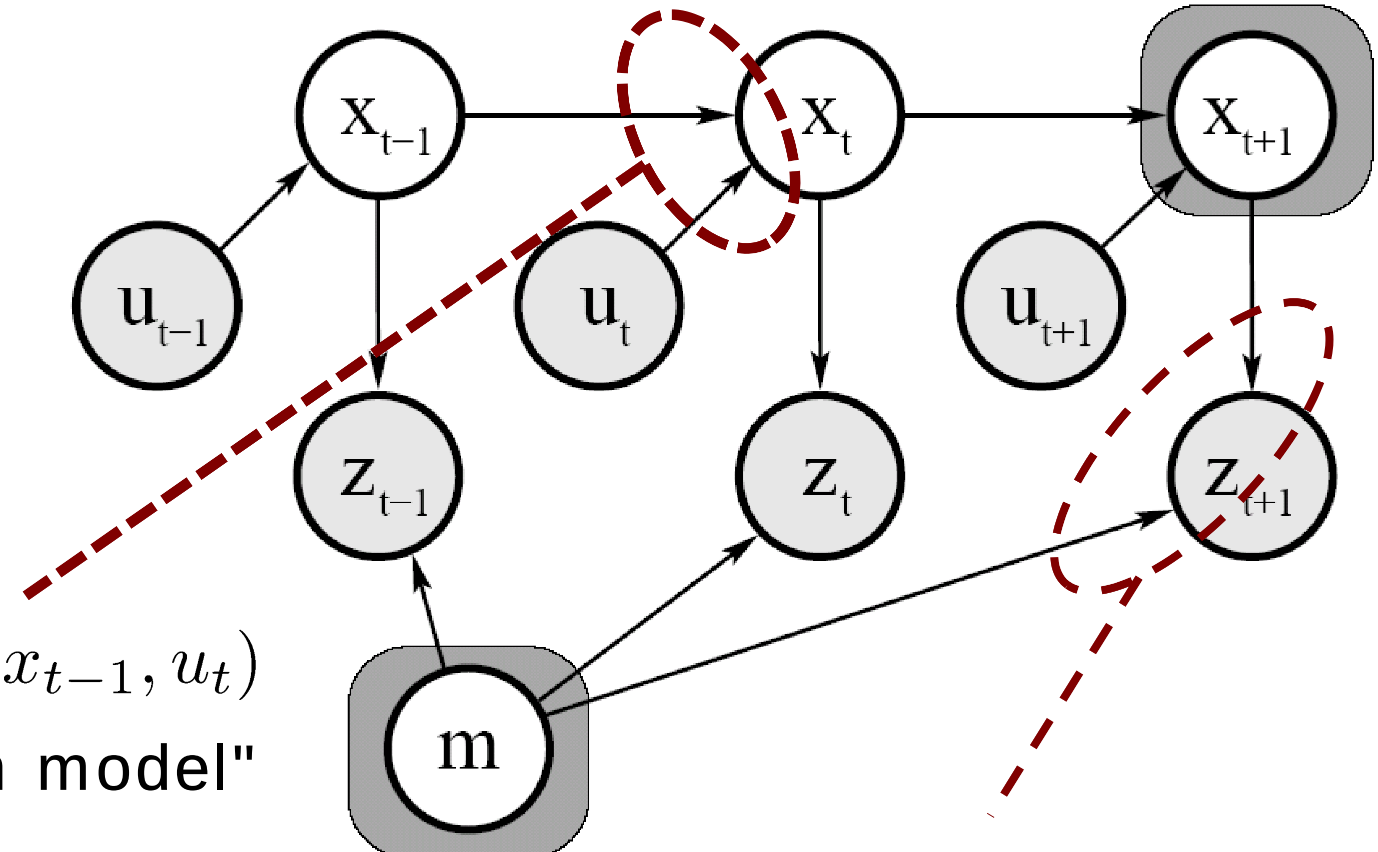
$$p(x_{1:t+1}, m \mid z_{1:t+1}, u_{1:t+1})$$

Graphical Model of Online SLAM



$$p(x_{t+1}, m \mid z_{1:t+1}, u_{1:t+1}) = \int\int \ldots \int p(x_{1:t+1}, m \mid z_{1:t+1}, u_{1:t+1}) dx_1 dx_2 \ldots dx_t$$

Motion and Observation Model



$$x_t = f(x_{t-1}, u_t)$$

"Motion model"

$$z_t = h(x_t, m)$$

"Observation model"

Remember the KF Algorithm

1. Algorithm **Kalman_filter**(μ_{t-1}, Σ_{t-1}, u_t, z_t):
2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$
5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$
9. Return μ_t, Σ_t

EKF SLAM: State representation

- **Localization**

  3x1 pose vector

  3x3 cov. matrix

$$\mathbf{x}_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} \qquad \Sigma_k = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 & \sigma_{x\theta}^2 \\ \sigma_{yx}^2 & \sigma_y^2 & \sigma_{y\theta}^2 \\ \sigma_{\theta x}^2 & \sigma_{\theta y}^2 & \sigma_\theta^2 \end{bmatrix}$$

- **SLAM**

  Landmarks **simply extend** the state.

  Growing state vector and covariance matrix!

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{x}_R \\ \mathbf{m}_1 \\ \mathbf{m}_2 \\ \vdots \\ \mathbf{m}_n \end{bmatrix} \qquad \Sigma_k = \begin{bmatrix} \Sigma_R & \Sigma_{RM_1} & \Sigma_{RM_2} & \cdots & \Sigma_{RM_n} \\ \Sigma_{M_1 R} & \Sigma_{M_1} & \Sigma_{M_1 M_2} & \cdots & \Sigma_{M_1 M_n} \\ \Sigma_{M_2 R} & \Sigma_{M_2 M_1} & \Sigma_{M_2} & \cdots & \Sigma_{M_2 M_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Sigma_{M_n R} & \Sigma_{M_n M_1} & \Sigma_{M_n M_2} & \cdots & \Sigma_{M_n} \end{bmatrix}$$

EKF SLAM: State representation

- Map with *n* landmarks: (3+2*n*)-dimensional Gaussian

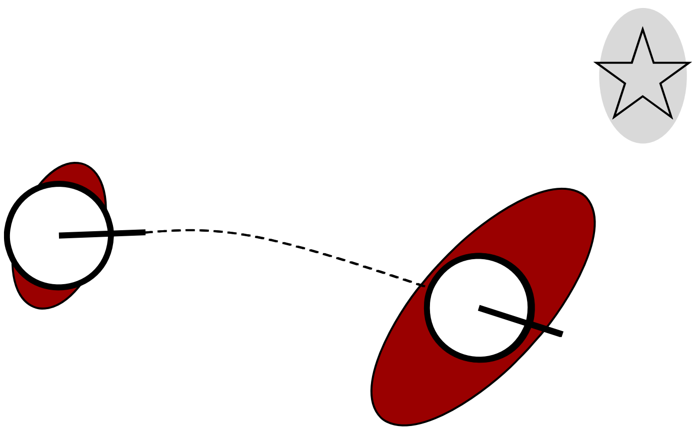
$$\underbrace{\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix}}_{\mu} \underbrace{\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{m_{n,x}} & \sigma_{m_{n,y}} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \\ \sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{\theta} & \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \cdots & \sigma_{m_{1,x}m_{n,x}} & \sigma_{m_{1,x}m_{n,y}} \\ \sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{\theta} & \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \cdots & \sigma_{m_{1,y}m_{n,x}} & \sigma_{m_{1,y}m_{n,y}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{m_{n,x}x} & \sigma_{m_{n,x}y} & \sigma_{\theta} & \sigma_{m_{n,x}m_{1,x}} & \sigma_{m_{n,x}m_{1,y}} & \cdots & \sigma_{m_{n,x}m_{n,x}} & \sigma_{m_{n,x}m_{n,y}} \\ \sigma_{m_{n,y}x} & \sigma_{m_{n,y}y} & \sigma_{\theta} & \sigma_{m_{n,y}m_{1,x}} & \sigma_{m_{n,y}m_{1,y}} & \cdots & \sigma_{m_{n,y}m_{n,x}} & \sigma_{m_{n,y}m_{n,y}} \end{pmatrix}}_{\Sigma}$$

- Can handle hundreds of dimensions

EKF SLAM: Filter Cycle

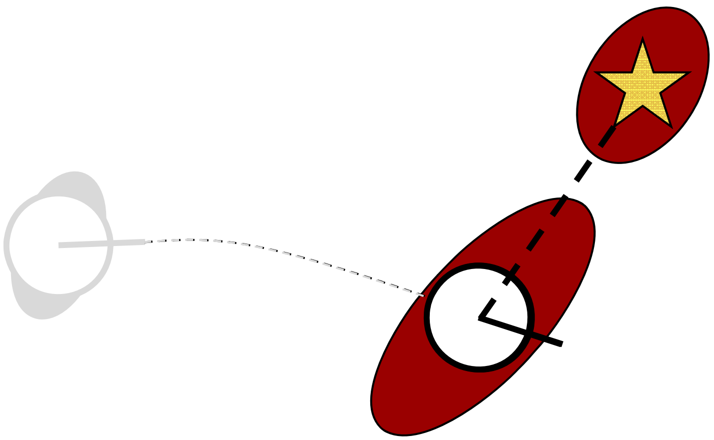
1. State prediction
2. Measurement prediction
3. Measurement
4. Data association
5. Update

EKF SLAM: State Prediction



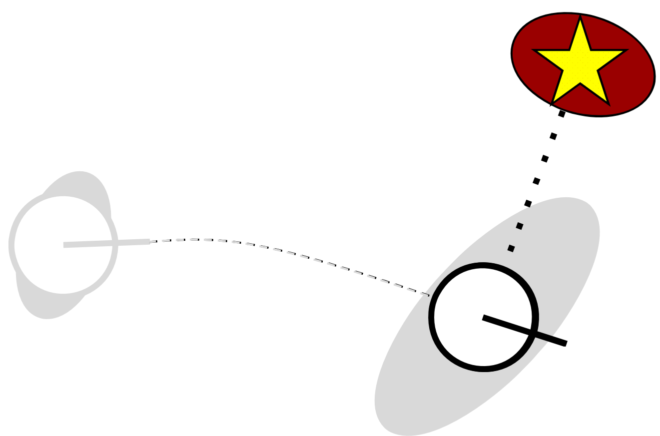
$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

EKF SLAM: Measurement Prediction



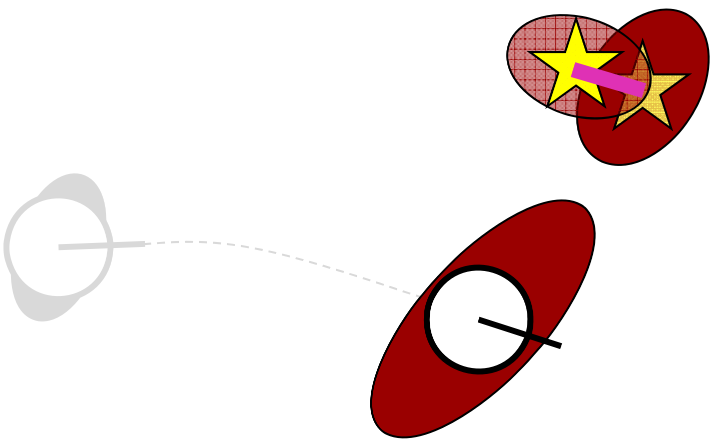
$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

EKF SLAM: Obtained Measurement



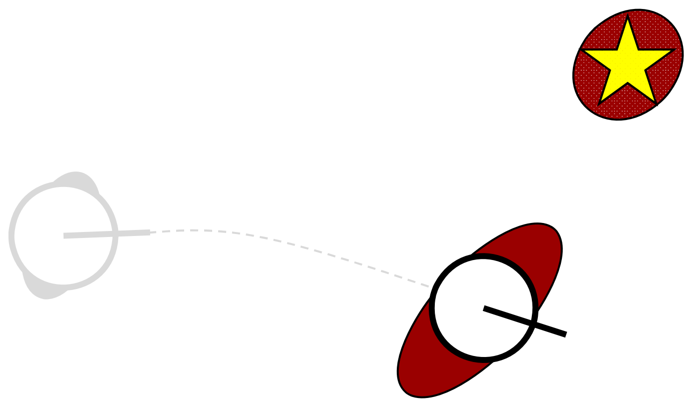
$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

EKF SLAM: Data Association



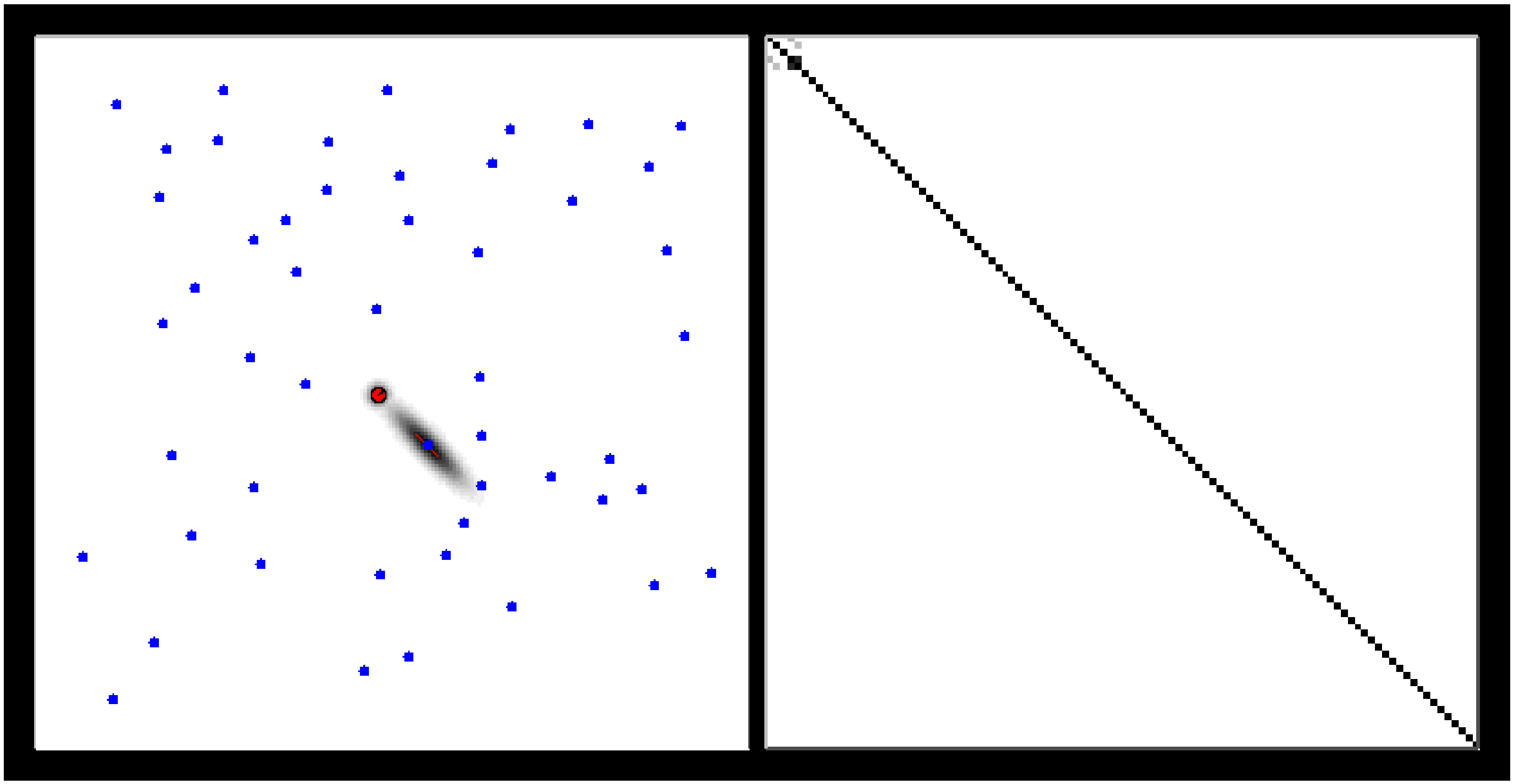
$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

EKF SLAM: Update Step



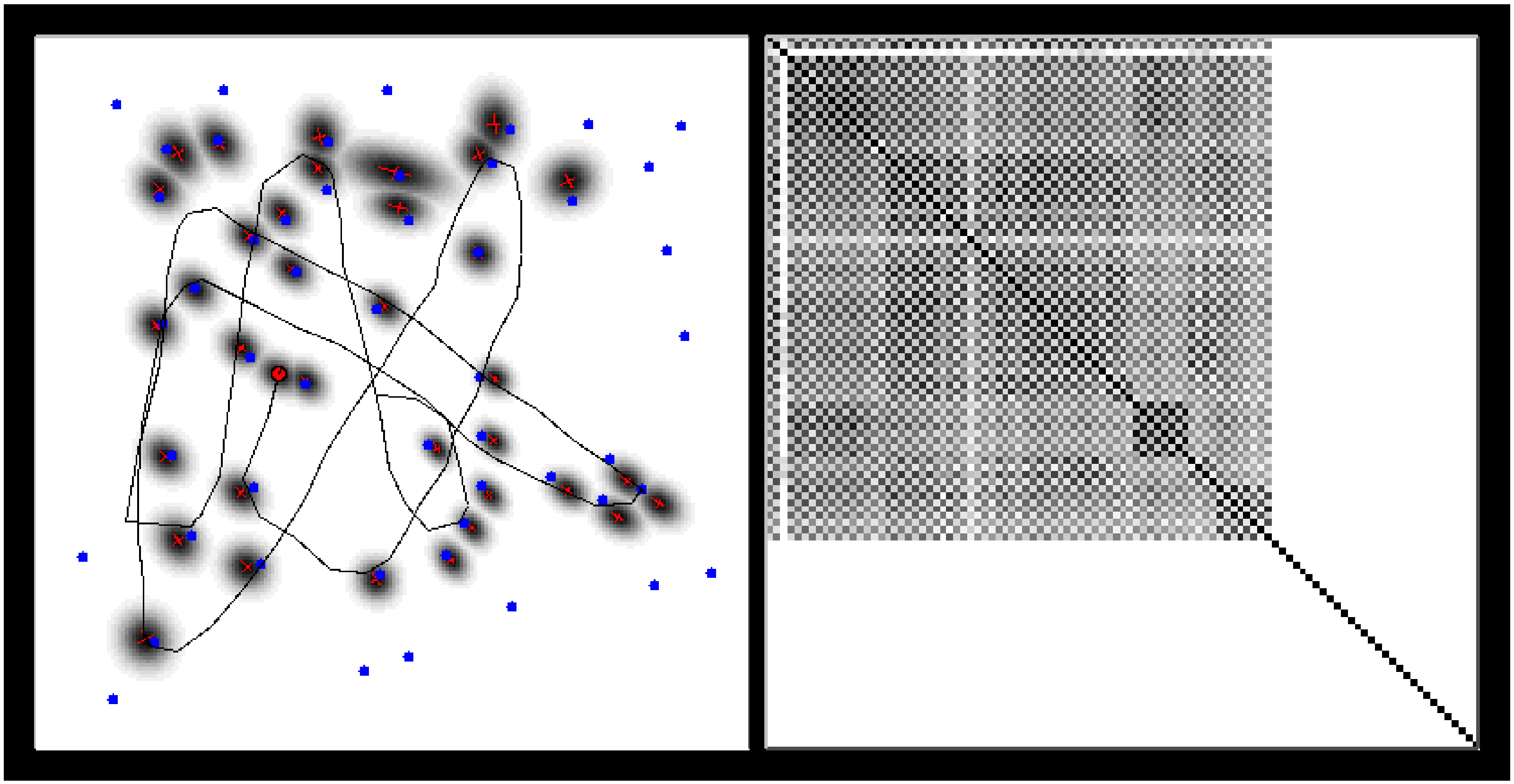
$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

EKF SLAM



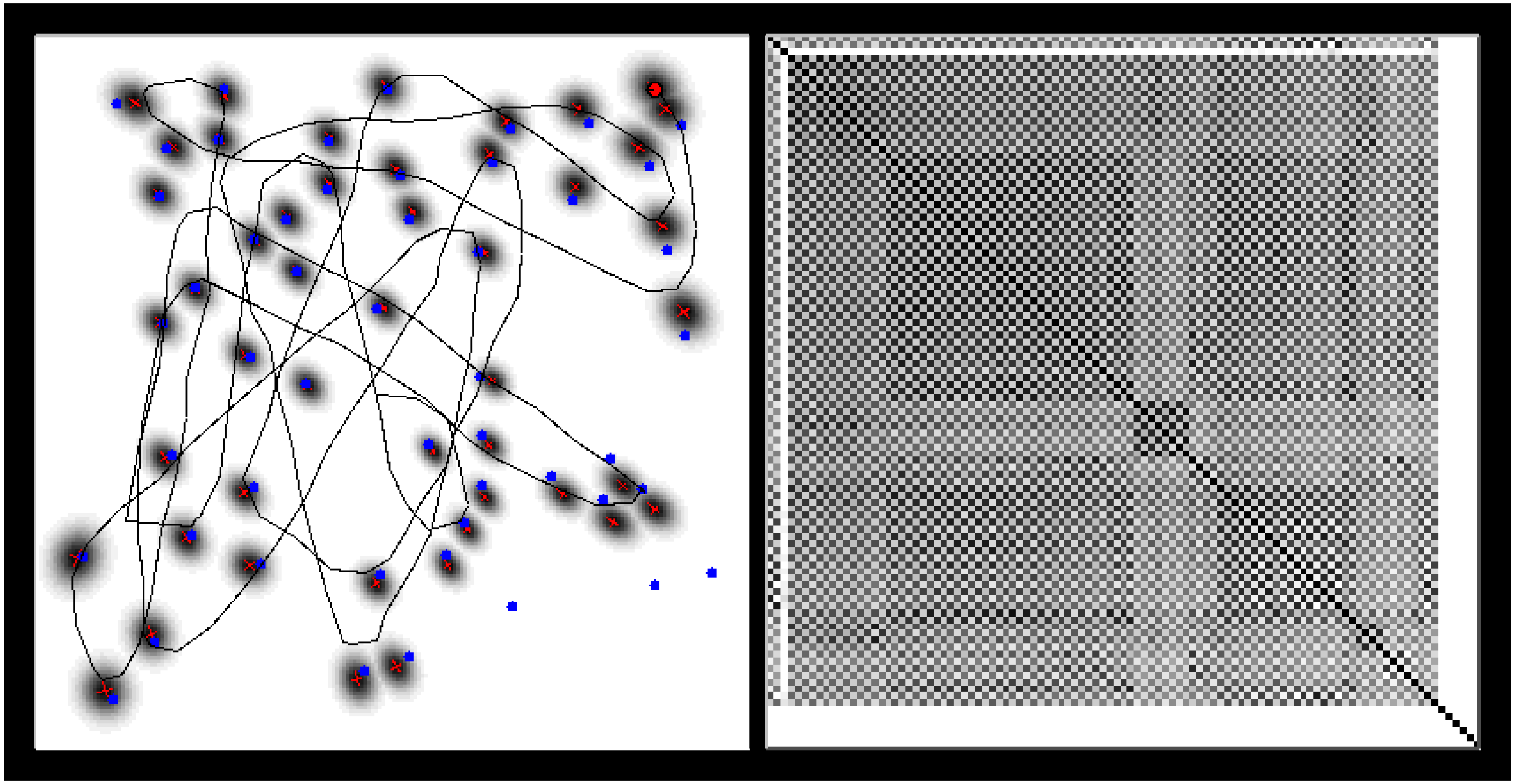
Map Correlation matrix

EKF SLAM



Map Correlation matrix

EKF SLAM



Map Correlation matrix

EKF SLAM: Correlations Matter

- What if we neglected cross-correlations?

$$\Sigma_k = \begin{bmatrix} \Sigma_R & 0 & \cdots & 0 \\ 0 & \Sigma_{M_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_{M_n} \end{bmatrix}$$

$$\Sigma_{RM_i} = \mathbf{0}_{3\times 2}$$

$$\Sigma_{M_i M_{i+1}} = \mathbf{0}_{2\times 2}$$

EKF SLAM: Correlations Matter

- What if we neglected cross-correlations?

$$\Sigma_k = \begin{bmatrix} \Sigma_R & 0 & \cdots & 0 \\ 0 & \Sigma_{M_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_{M_n} \end{bmatrix}$$

$$\Sigma_{RM_i} = \mathbf{0}_{3\times 2}$$

$$\Sigma_{M_i M_{i+1}} = \mathbf{0}_{2\times 2}$$

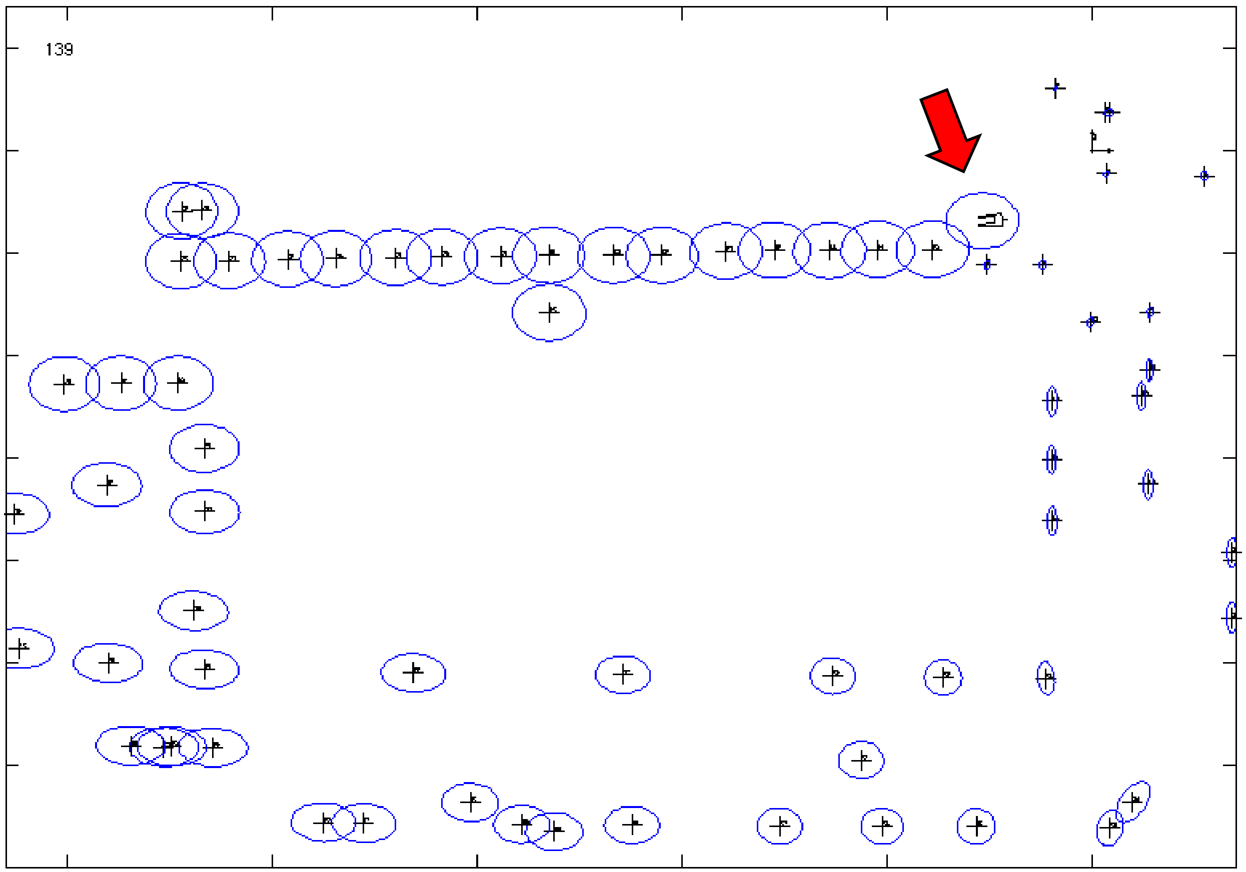
- Landmark and robot uncertainties would become overly optimistic
- Data association would fail
- Multiple map entries of the same landmark
- Inconsistent map

SLAM: Loop Closure

- **Recognizing an already mapped area**, typically after a long exploration path (the robot "closes a loop")
- Structurally identical to data association, but
  - high levels of ambiguity
  - possibly useless validation gates
  - environment symmetries
- Uncertainties **collapse** after a loop closure (whether the closure was correct or not)

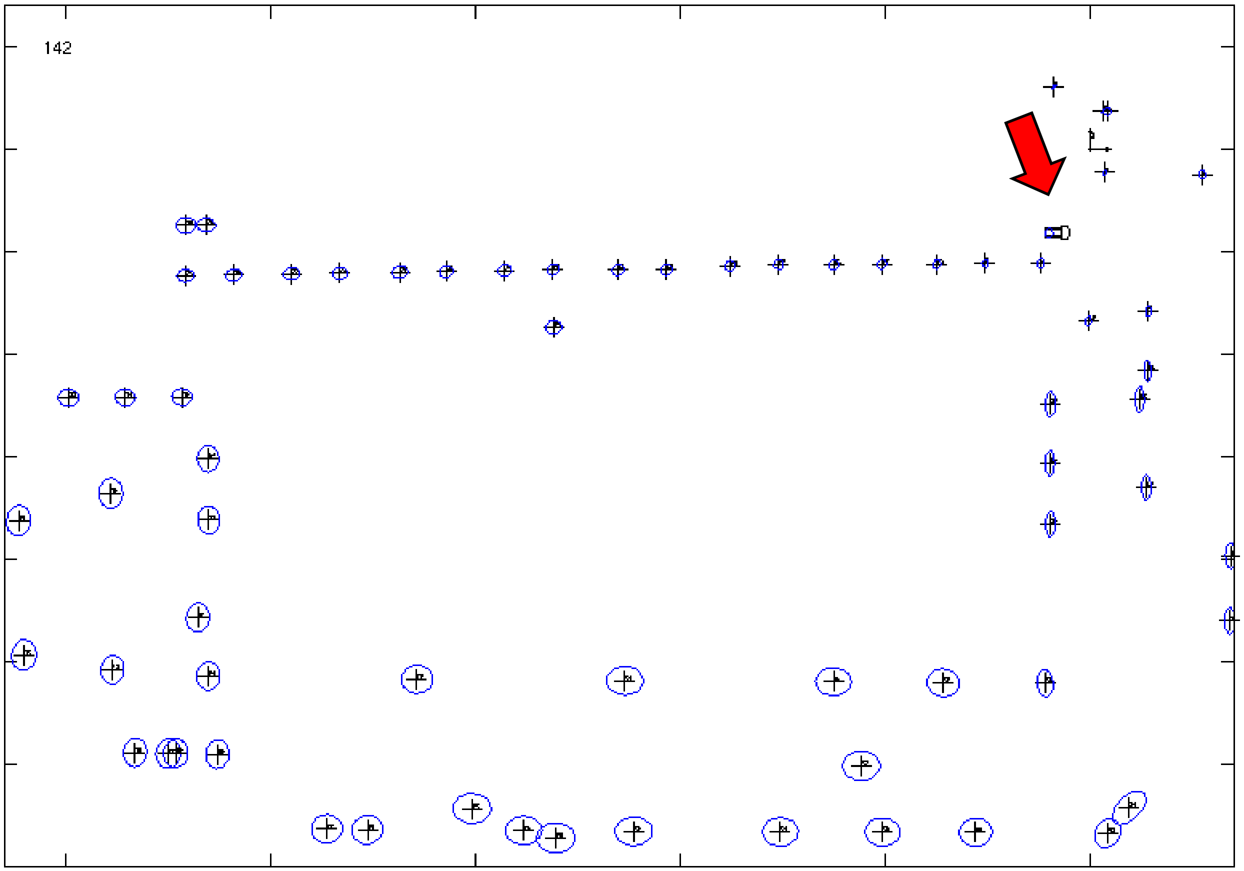
SLAM: Loop Closure

- Before loop closure



SLAM: Loop Closure

- After loop closure



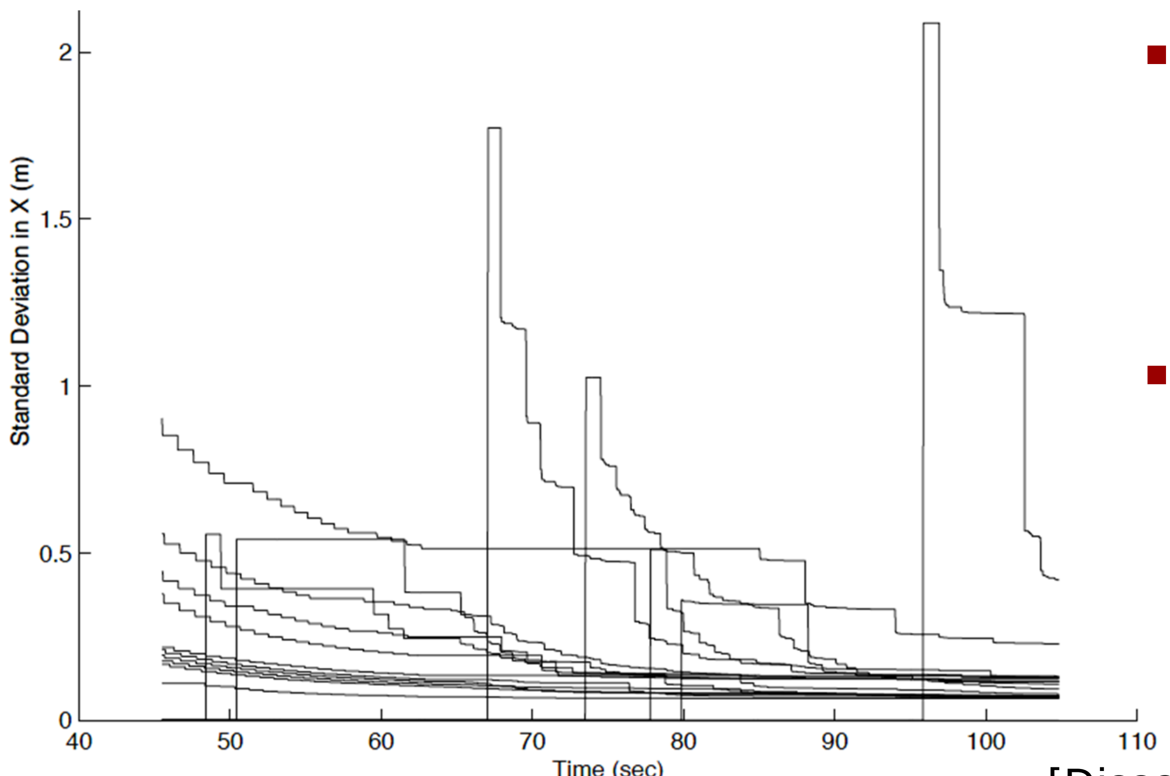
SLAM: Loop Closure

- By revisiting already mapped areas, uncertainties in robot and landmark estimates can be **reduced**
- This can be exploited when **exploring** an environment for the sake of better (e.g. more accurate) maps
- Exploration: the problem of ***where* to acquire new information**

→ See separate chapter on exploration

KF-SLAM Properties (Linear Case)

- The **determinant** of any sub-matrix of the map covariance matrix **decreases monotonically** as successive observations are made

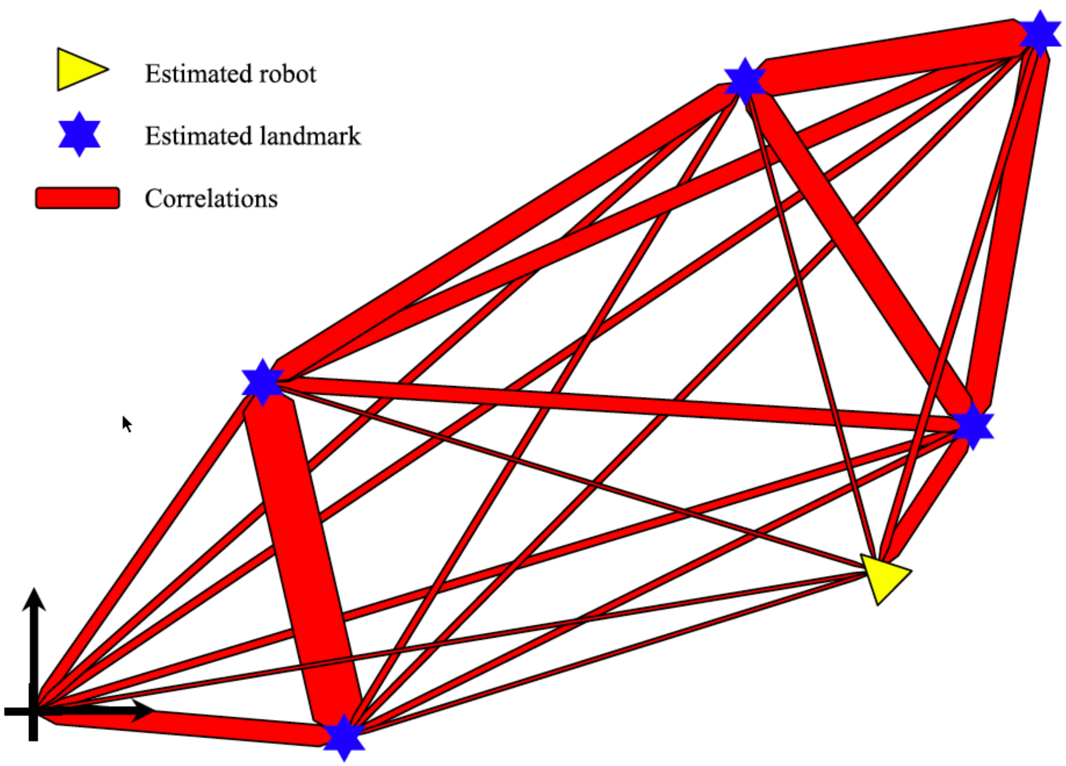


- When a new land-mark is initialized, its **uncertainty** is **maximal**
- Landmark uncertainty **decreases monotonically** with each new observation

[Dissanayake et al., 2001]

KF-SLAM Properties (Linear Case)

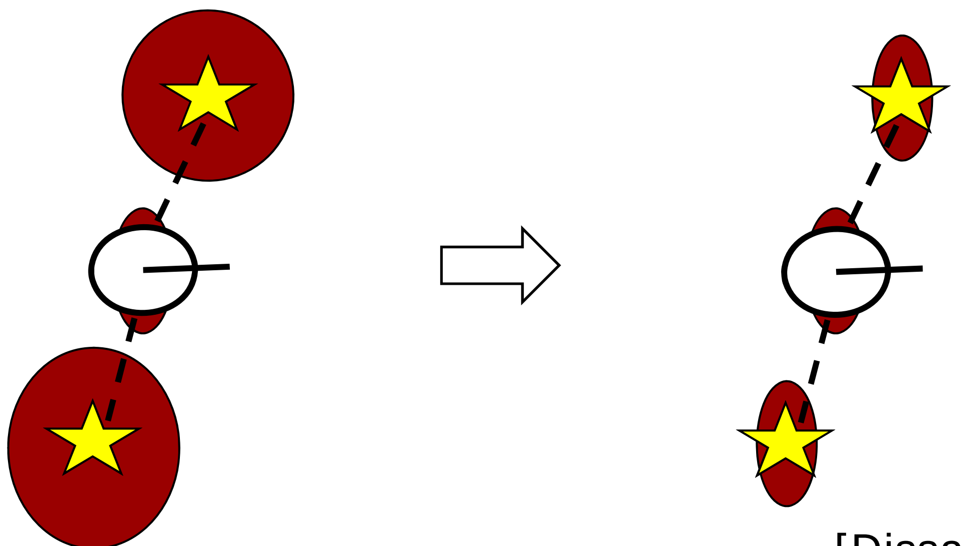
- In the limit, the landmark estimates become **fully correlated**



[Dissanayake et al., 2001]

KF-SLAM Properties (Linear Case)

- In the limit, the **covariance** associated with any single landmark location estimate is determined only by the **initial covariance in the vehicle location estimate**.



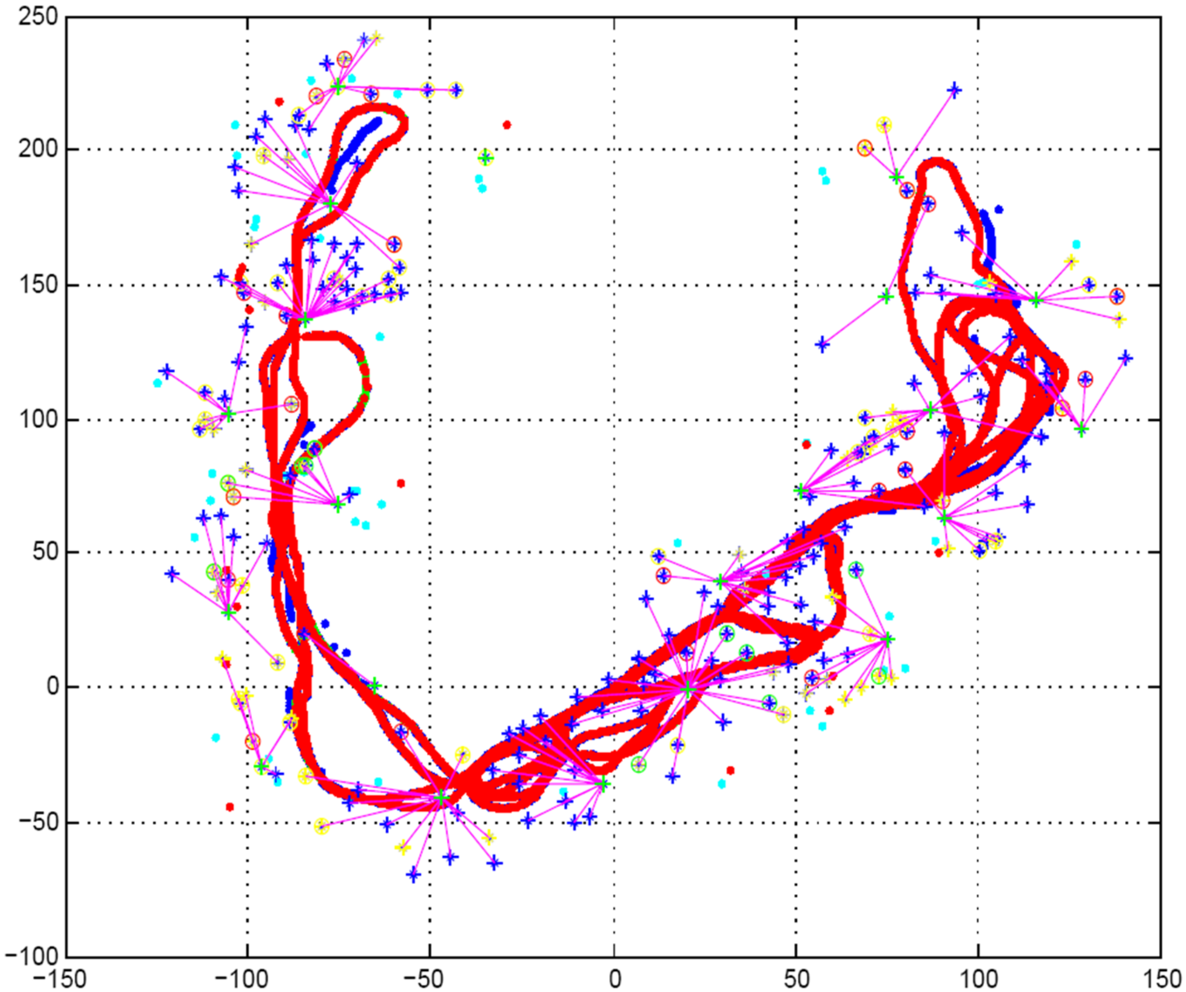
[Dissanayake et al., 2001]

EKF SLAM Example: Victoria Park Dataset

Victoria Park: Data Acquisition

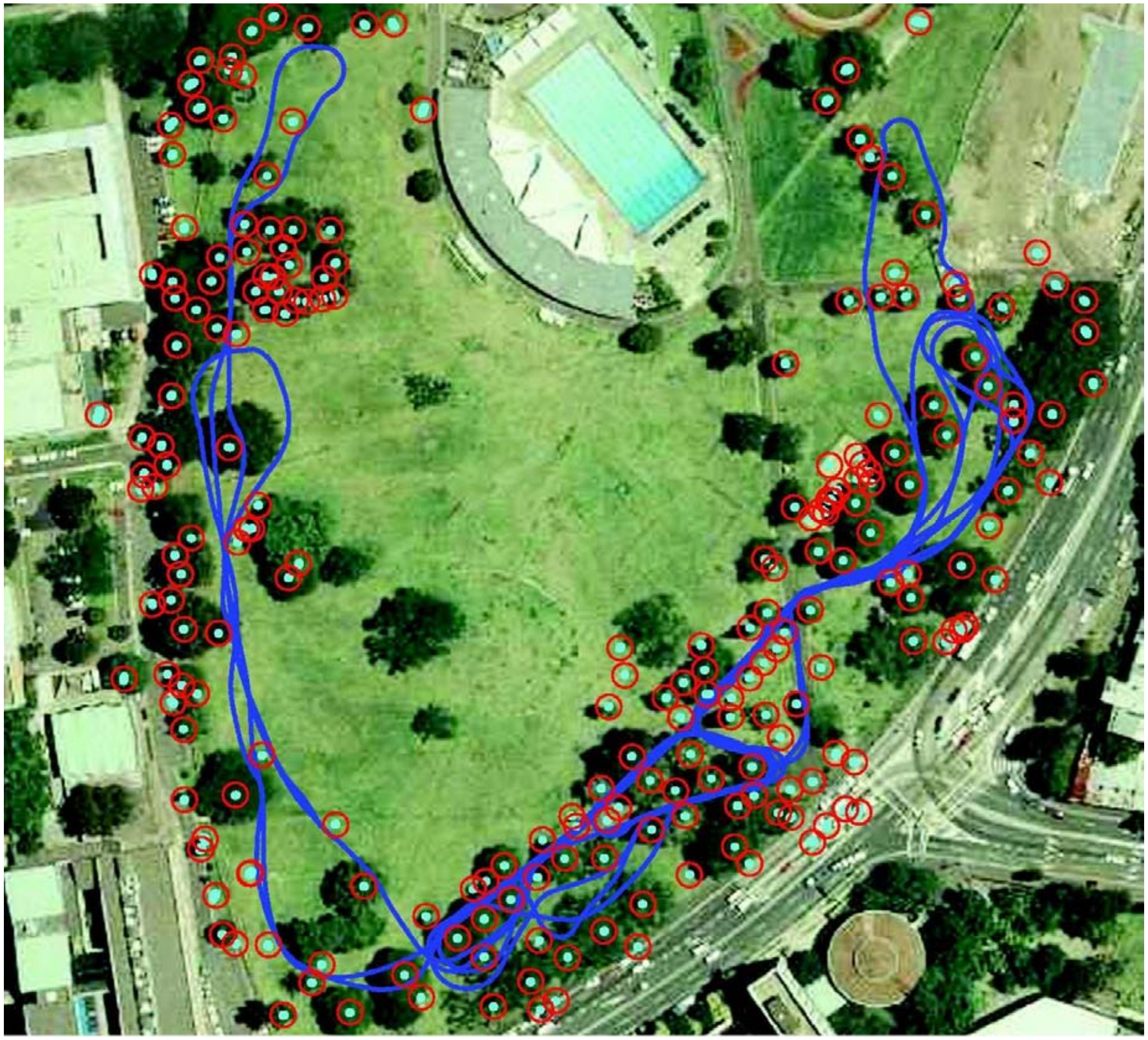
[courtesy by E. Nebot]

Victoria Park: Estimated Trajectory



[courtesy by E. Nebot]

Victoria Park: Landmarks

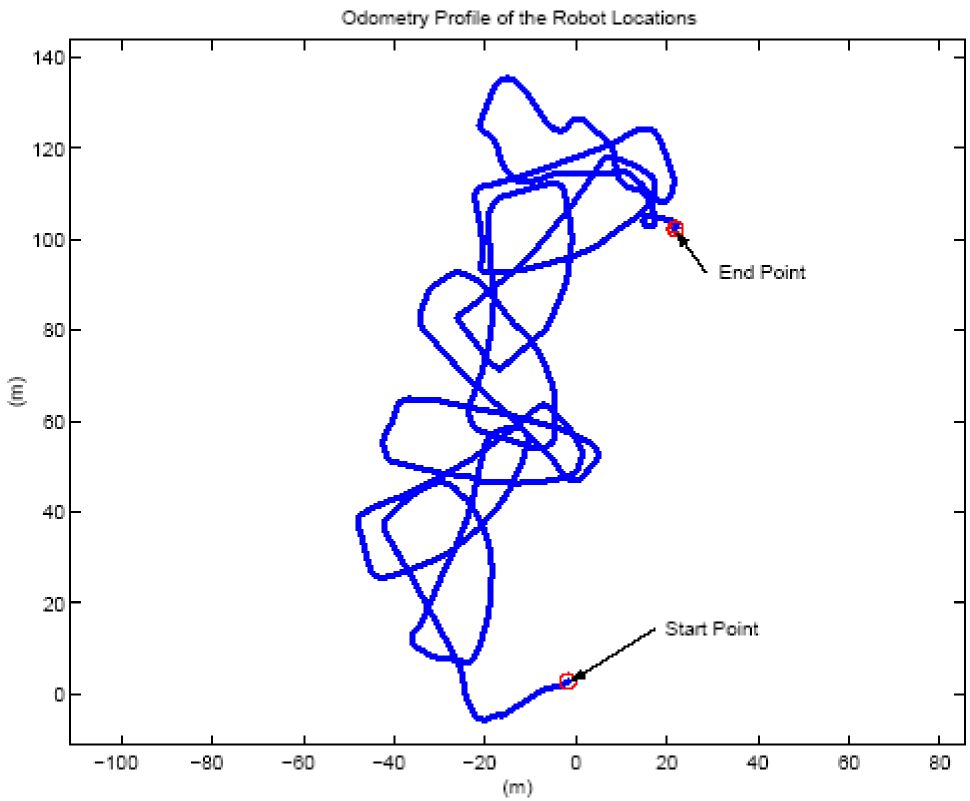


[courtesy by E. Nebot]

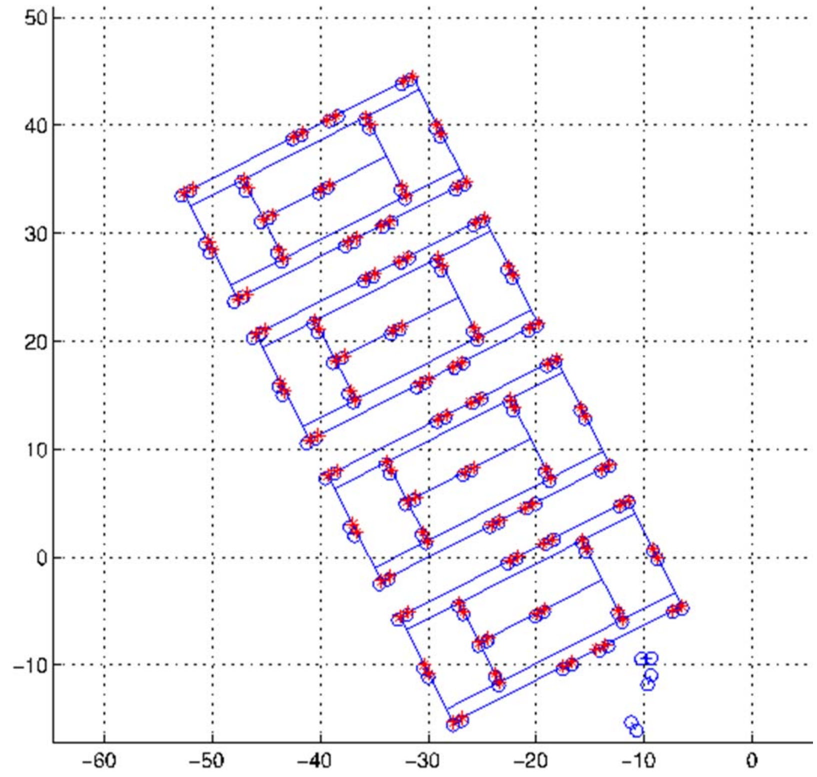
EKF SLAM Example: Tennis Court

[courtesy by J. Leonard]

EKF SLAM Example: Tennis Court



odometry

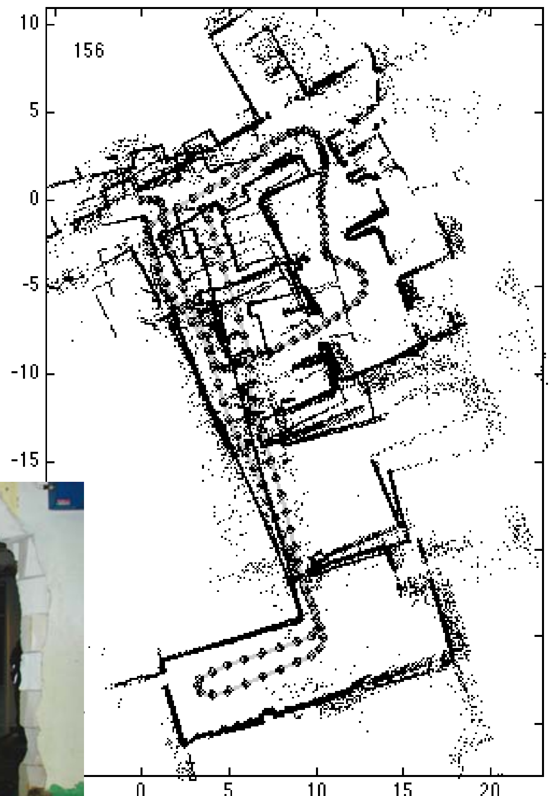


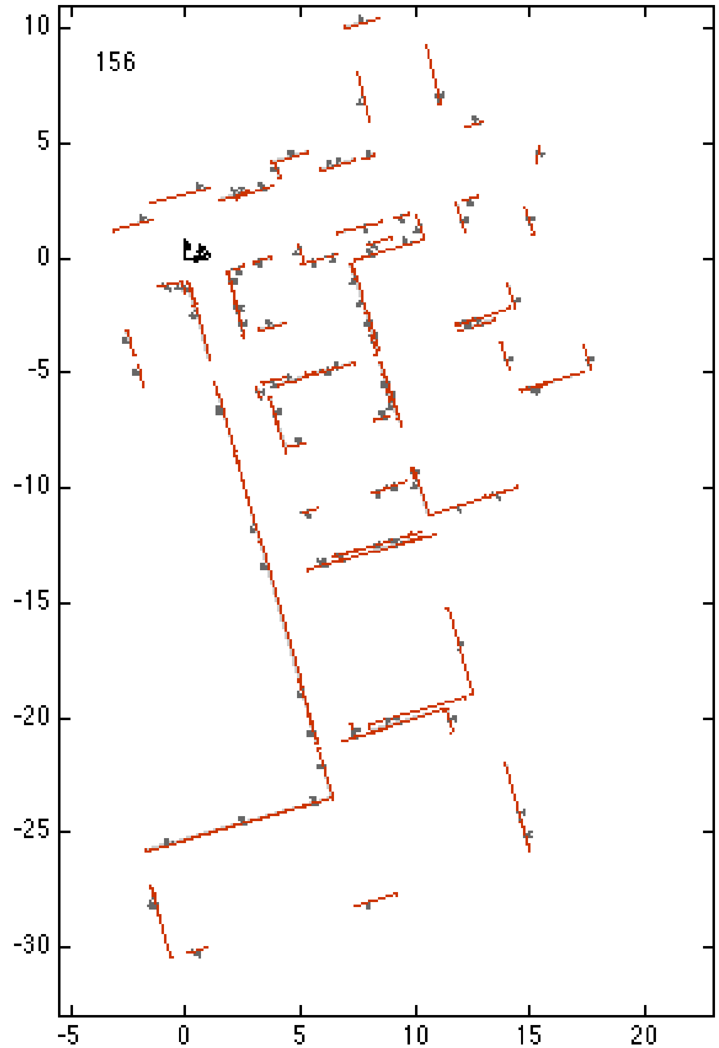
estimated trajectory

[courtesy by John Leonard]

EKF SLAM Example: Line Features

- KTH Bakery Data Set





[Wulf et al., ICRA 04]

EKF-SLAM: Complexity

- Cost per step: quadratic in n, the number of landmarks: $O(n^2)$
- Total cost to build a map with n landmarks: $O(n^3)$
- Memory consumption: $O(n^2)$
- Problem: becomes computationally intractable for large maps!
- There exists variants to circumvent these problems

SLAM Techniques

- EKF SLAM
- FastSLAM
- Graph-based SLAM
- Topological SLAM
  (mainly place recognition)
- Scan Matching / Visual Odometry
  (only locally consistent maps)
- Approximations for SLAM: Local submaps, Sparse extended information filters, Sparse links, Thin junction tree filters, etc.
- …

EKF-SLAM: Summary

- The first SLAM solution
- Convergence proof for linear Gaussian case
- Can diverge if nonlinearities are large (and the real world is nonlinear ...)
- Can deal only with a single mode
- Successful in medium-scale scenes
- Approximations exists to reduce the computational complexity