Basic Probability Rules

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1 Basic Axioms

$$0 \leq p(x) \leq 1 \tag{1}$$

$$p(\mathit{true}) = 1 \text{ and } p(\mathit{false}) = 0 \tag{2}$$

$$p(x \vee y) = p(x) + p(y) - p(x \wedge y) \tag{3}$$

2 Complement Probability

$$p(\neg x) = 1 - p(x). \tag{4}$$

3 Product Rule

The following equation is called the product rule

$$p(x, y) = p(x \mid y) \cdot p(y) \tag{5}$$

$$= p(y \mid x) \cdot p(x). \tag{6}$$

4 Independence

If x and y are independent, we have

$$p(x, y) = p(x) \cdot p(y). \tag{7}$$

5 Bayes' Rule

The Bayes' rule, which is frequently used in this thesis, is given by

$$p(x \mid y) = \frac{p(y \mid x) \cdot p(x)}{p(y)}. \tag{8}$$

The denominator is a normalizing constant that ensures that the posterior of the left hand side adds up to 1 over all possible values. Thus, we often write

$$p(x \mid y) = \eta \cdot p(y \mid x) \cdot p(x). \tag{9}$$

In case the background knowledge e is given, Bayes' rule turns into

$$p(x \mid y, e) = \frac{p(y \mid x, e) \cdot p(x \mid e)}{p(y \mid e)}. \tag{10}$$

6 Marginalization

The marginalization rule is the following equation

$$p(x) \quad = \quad \int_y p(x, y)\, dy. \tag{11}$$

In the discrete case, the integral turns into a sum

$$p(x) \quad = \quad \sum_y p(x, y). \tag{12}$$

7 Law of Total Probability

The law of total probability is a variant of the marginalization rule, which can be derived using the product rule

$$p(x) \quad = \quad \int_y p(x \mid y) \cdot p(y)\, dy, \tag{13}$$

and the corresponding sum for the discrete case

$$p(x) \quad = \quad \sum_y p(x \mid y) \cdot p(y). \tag{14}$$

8 Markov Assumption

The Markov assumption (also called Markov property) characterizes the fact that a variable x_t depends only on its direct predecessor state x_{t-1} and not on $x_{t'}$ with $t' < t - 1$

$$p(x_t \mid x_{1:t-1}) \quad = \quad p(x_t \mid x_{t-1}). \tag{15}$$