Foundations of Artificial Intelligence

Prof. Dr. W. Burgard, Prof. Dr. B. Nebel, Prof. Dr. M. Riedmiller
J. Aldinger, B. Frank
University of Freiburg
Department of Computer Science
Summer Term 2012

Exercise Sheet 5

Due: Wednesday, July 11, 2012

Exercise 5.1 (Substitutions and Unification)

(a) Compute the substitutions
  (i) $P(x,y)\{\frac{x}{A}, \frac{y}{f(B)}\}$,
  (ii) $P(x,y)\{\frac{x}{f(y)}\}\{\frac{y}{g(B,B)}\}$,
  (iii) $P(x,y)\{\frac{x}{f(y)}, \frac{y}{g(B,B)}\}$ and
  (iv) $P(x,y)\{\frac{z}{f(B)}, \frac{x}{A}\}$

(b) Apply the unification algorithm to the following set of literals: $\{R(h(x), f(h(u), y)), R(y, f(y, h(g(A))))\}$. In each step, give the values of T_k, s_k, D_k, v_k, and t_k.

Exercise 5.2 (Allen's Interval Calculus)

(a) Consider the non-empty intervals *Match*, *GoalShot*, *Cheering* und *FinalWhistle* together with the constraints

  (i) *FinalWhistle f Match*
  (ii) *GoalShot m Cheering*
  (iii) *GoalShot (d,f) Match*
  (iv) *GoalShot (<,m) FinalWhistle*

  Which of the following relations are entailed?

  (a) *GoalShot d Match*
  (b) *Cheering d Match*

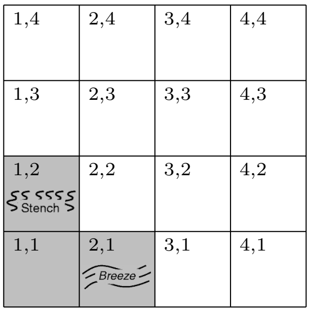
(b) In general, the composition of two binary relations R and S (over X) is defined as

$$R \circ S = \{(x,z) \mid \exists y \in X \text{ such that } (x,y) \in R \text{ and } (y,z) \in S\}.$$

  Allen's interval calculus is *closed under composition* which means that every composition of Allen relations (also for unions of the 13 base relations) can be represented as union of base relations. For example, $f \circ s = d$ because for arbitrary intervals A, B and C with AfB and BsC it must hold that AdC. Note that in general the composition of two base relations needs not to result in a single base relation, as you can see from the example $f^{-1} \circ d = (o, d, s)$. Determine the following compositions:

  (1) $o \circ m$
  (2) $m \circ f$
  (3) $(o, f^{-1}) \circ f$

Exercise 5.3 (Wumpus world and resolution)
Consider the following situation in the wumpus world:

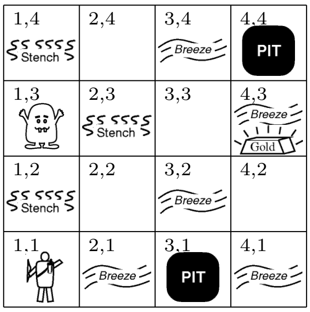


The gray squares have already been visited, the others not. The percepts in the corresponding squares are indicated by *breeze* and *stench* .

(a) Formalize the general connections between breezes and pits using propositional formulae. Use 16 variables $P_{i,j}$ (meaning there is a *pit* in square $[i,j]$) and 16 variables $B_{i,j}$ (*breeze* in square $[i,j]$).

(b) Show, using *resolution*, that square $[3,1]$ contains a pit in the given situation, i.e., show that KB $\models P_{3,1}$. The knowledge base KB consists of the propositions from part (a) as well as the percepts of the agent. Note: squares that already have been visited do not contain pits. If necessary, convert the knowledge base into CNF (conjunctive normal form).

Exercise 5.4 (Planning in the wumpus world)
Consider the following initial state in the wumpus world:



The agent in square $[1,1]$ did not attend the "Action Planning" lecture, thus, he isn't able to solve planning tasks with partial observability. Additionally he is more excited about hunting the wumpus than about finding gold. Therefore, we define the planning problem as[1]:

[1] *stench*,*breeze* and *gold* will not be formalized here and serve only for the purpose of illustration (or confusion?).

Initial state $\mathcal{I}$:

$\{\texttt{connected}([1,1],[2,1]), \texttt{connected}([2,1],[3,1]), \ldots,$
$\texttt{connected}([4,3],[4,4]), \texttt{at}(\texttt{agent},[1,1]), \texttt{at}(\texttt{wumpus},[1,3]),$
$\texttt{at}(\texttt{pit},[3,1]), \texttt{at}(\texttt{pit},[4,4]), \texttt{arrowleft}, \texttt{agent\_alive}\}$

Operators $\mathcal{O}$:

$\texttt{Move}(x,y)$

$$\begin{aligned}\text{PRE}:&\texttt{at}(\texttt{agent},x) \wedge \texttt{connected}(x,y) \wedge \texttt{agent\_alive}\\ \text{EFF}:&\texttt{at}(\texttt{wumpus},y) \rhd \neg\texttt{agent\_alive},\\ &\texttt{at}(\texttt{pit},y) \rhd \neg\texttt{agent\_alive},\\ &\texttt{at}(\texttt{agent},y),\\ &\neg\texttt{at}(\texttt{agent},x)\end{aligned}$$

$\texttt{Shoot}(x,y)$

$$\begin{aligned}\text{PRE}:&\texttt{at}(\texttt{agent},x) \wedge \texttt{connected}(x,y) \wedge \texttt{arrowleft} \wedge \texttt{agent\_alive}\\ \text{EFF}:&\texttt{at}(\texttt{wumpus},y) \rhd \texttt{scream},\\ &\neg\texttt{arrowleft}\end{aligned}$$

Goal $\mathcal{G}$:

$\texttt{scream} \wedge \texttt{agent\_alive}$

(a) Suppose, you want to solve a simplified, monotonic planning problem by ignoring negative effects (aka. the "delete relaxation") in order to calculate a heuristic.
Specify the operators of the relaxed planning task.

(b) Sketch the first two levels of the relaxed planning graph. Facts that do not change in the relaxed problem, e.g. $\texttt{agent\_alive}$, $\texttt{at}(\texttt{pit},x)$ and $\texttt{connected}(x,y)$ can be omitted (In the initial state in layer $F0$ you only have to sketch the fact $\texttt{at}(\texttt{agent},[1,1])$).
To further simplify the problem, you may compile away the conditional effect $\texttt{at}(\texttt{wumpus},y) \rhd \texttt{scream}$ of $\texttt{Shoot}(x,y)$ by moving the effect precondition to the operator precondition[2].

(c) Contrary to the PlanGraph method presented in the lecture, actions cannot be conflicting in a relaxed planning problem since they neither contain negative preconditions nor negative effects. Therefore, relaxed plans can be found more easily and thus be used to derive heuristic estimates.
Specify the relaxed plan. Is this plan also applicable in the original problem?

[2] When compiling away conditional effects, usually two operators (one with the effect condition and one with the negated effect condition) are created. However, $\texttt{Shoot}'(x,y) = \langle \text{PRE}: \texttt{at}(\texttt{agent},x), \neg\texttt{at}(\texttt{wumpus},y), \ldots \text{EFF}: \emptyset\rangle$ does not have any effect and might be excluded here as a result.