(10) Derive formulas for the minimum time required to move from s_0 to s_T using a linear segment with parabolic blends. Initial and final speeds, v_0 and v_T, and the maximum speed and acceleration are given, such that:

$$V_{max} > 0 \qquad a_{max} > 0$$

$$-V_{max} < V_0 < V_{max}$$

$$-V_{max} < V_T < V_{max}$$

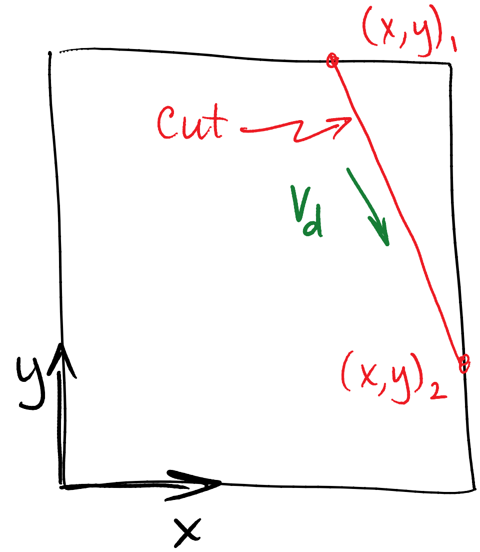
$$V_0 = V_T$$

(11) Derive a cubic spline to move from S_0 to S_T with $\dot{s}_0 = \dot{s}_T = 0$. Then answer the following questions:

i. Given T, what is the maximum speed, $\dot{s}(t)$, along the trajectory?

ii. Given the maximum allowable acceleration, a_{max}, what is the minimum T possible?

(12) A robot with a laser cutter will make a straight cut across a piece of sheet metal as shown in the figure. To make the cleanest cut, the cutter should move at the desired speed, V_d. The maximum allowable acceleration is a_{max}.



Let $V_d = 1$, $a_{max} = 2$, $(x,y)_1 = (5,10)$, $(x,y)_2 = (8,3)$.

Make the following plots of the cutting trajectory

i. $y(t)$ vs. $x(t)$.

ii. speed along the path vs. time.

iii. acceleration along the path vs. time.

(13) Derive the Jacobian, B, relating the angular velocity of a frame to the rates of the Euler angles:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = B(\phi, \theta, \psi) \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Identify the singular configurations and describe them physically.