8.3_ForceMappings

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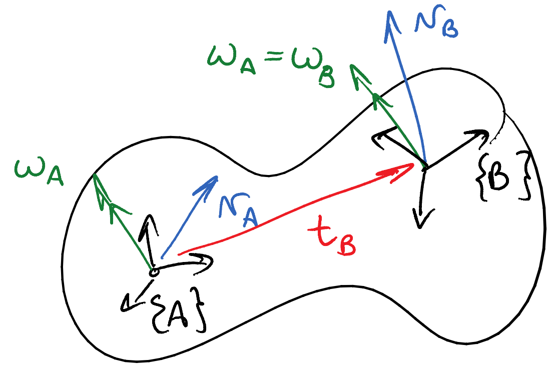
We desire the analog to:

$$ {}^{B}\nu_B = J_v({}^{A}T_B)\, {}^{A}\nu_A $$

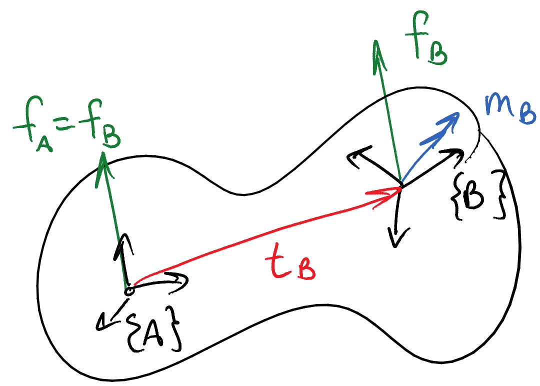
in the force domain.



$$ \nu_B = \nu_A + \omega_A \times t_B $$
$$ \omega_B = \omega_A $$

$$ {}^{A}g_A = \left(J_v({}^{A}T_B)\right)^T \, {}^{B}g_B $$

$$ g = \begin{bmatrix} f \\ m \end{bmatrix}_{6\times 1} $$



$$ f_A = f_B $$
$$ \underbrace{m_A = m_B + t_B \times f_B} $$

Proof: via basic relationships between force & moment.

matrix form

$$ g_A = \begin{bmatrix} f_A \\ m_A \end{bmatrix} = \begin{bmatrix} I & 0 \\ S(t_B) & I \end{bmatrix} \begin{bmatrix} f_B \\ m_B \end{bmatrix} = g_B $$

We must be very clear about the frame of representation of t_B. Assume {A}, i.e. we have ${}^{A}t_B$

Now consider frames of representation, choosing to represent f_A, m_A in $\{A\}$ and f_B, m_B in $\{B\}$

$$\underbrace{\begin{bmatrix} I & 0 \\ S({}^At_B) & I \end{bmatrix}}_{\text{quantity in } \{A\}} \underbrace{\begin{bmatrix} {}^Bf_B \\ {}^Bm_B \end{bmatrix}}_{\text{quantity in } \{B\}.} = \begin{bmatrix} {}^Bf_A \\ {}^Bm_A \end{bmatrix}$$

Express ${}^B\nu_B$ in $\{A\}$ so matrix multiply is valid!

$$\underbrace{\begin{bmatrix} I & 0 \\ S({}^At_B) & I \end{bmatrix}}_{\bar{S}} \underbrace{\begin{bmatrix} {}^AR_B & 0 \\ 0 & {}^AR_B \end{bmatrix}}_{\bar{R}} \begin{bmatrix} {}^Bf_B \\ {}^Bm_B \end{bmatrix} = \begin{bmatrix} {}^Af_A \\ {}^Am_A \end{bmatrix}$$

$$\boxed{\bar{S}\bar{R}\,{}^Bg_B = {}^Ag_A}$$

Compare to velocity transform.

$${}^Av_B = {}^Av_A + {}^A\omega_A \times {}^At_B = {}^Av_A - {}^At_B \times {}^A\omega_A = {}^Av_A - S({}^At_B)\,{}^A\omega_A$$

$${}^A\omega_B = {}^A\omega_A$$

$$\begin{bmatrix} {}^Av_B \\ {}^A\omega_B \end{bmatrix} = \begin{bmatrix} I & -S({}^At_B) \\ 0 & I \end{bmatrix} \begin{bmatrix} {}^Av_A \\ {}^A\omega_A \end{bmatrix}$$

$$\begin{bmatrix} {}^B v_B \\ {}^B \omega_B \end{bmatrix} = \underbrace{\begin{bmatrix} {}^B R_A & 0 \\ 0 & {}^B R_A \end{bmatrix}}_{\bar{R}^T} \underbrace{\begin{bmatrix} I & -S({}^A t_B) \\ 0 & I \end{bmatrix}}_{\bar{S}^T} \begin{bmatrix} {}^A v_A \\ {}^A \omega_A \end{bmatrix}$$

Recall that $S^T = -S$.

$${}^B \nu_B = \bar{R}^T \bar{S}^T \, {}^A \nu_A$$

$$\boxed{{}^B \nu_B = (\bar{S} \bar{R})^T \, {}^A \nu_A = J_\nu({}^A T_B) \, {}^A \nu_A}$$

where $\bar{S}\bar{R} = {}^B J_A = J_\nu({}^A T_B)$ Corke's text eq. (8.4).

Now the easy way!

Virtual Work or Conservation of Energy

$$\delta W = f^T v + m^T \omega = g^T \nu$$

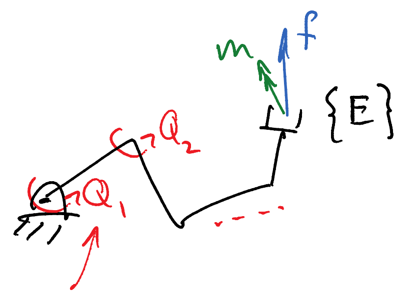
δW is the same regardless of the frame in which it measured.

$${}^A g_A^T \, {}^A \nu_A = {}^B g_B^T \, {}^B \nu_B = {}^B g_B^T \, J_\nu({}^A T_B) \, {}^A \nu_A$$

$$\Rightarrow {}^B g_B^T \, J_\nu({}^A T_B) = {}^A g_A^T$$

$$\therefore \quad \boxed{J_v^T({}^A T_B)\, {}^B g_B = {}^A g_A}$$

What about mapping wrenches from $\{E\}$ to the robot's joints?



$$\delta W = {}^E g_E^T \, {}^E \nu_E = Q^T \dot{q}$$

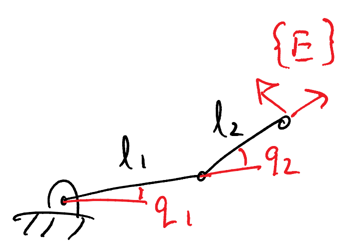
$$= {}^E g_E^T \, {}^E J_E \, \dot{q}$$

Q_i = actuator force at joint i.

$$\Rightarrow \quad {}^E g_E^T \, {}^E J_E = Q^T$$

$$\therefore \quad \boxed{Q = {}^E J_E^T \, {}^E g_E}$$ = Q required to exert g onto environment when robot is not moving!

Example: 2R-Planar Manipulator.



$$p_x = l_1 c_1 + l_2 c_{12}$$

$$p_y = l_1 s_1 + l_2 s_{12}$$

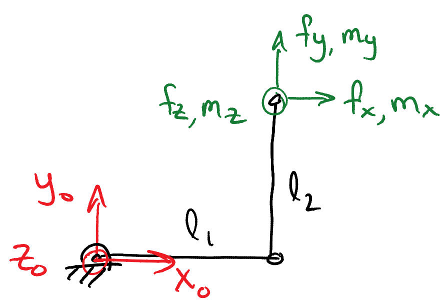
$\phi = q_1 + q_2$

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{\phi} \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \\ 1 & 1 \end{array}\right]}_{{}^0J_E \ (3\times 2)} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Let the robot apply a wrench g onto the environment

Assume $q_1 = 0$, $q_2 = \pi/2$.

$l_1 = l_2 = 1$.

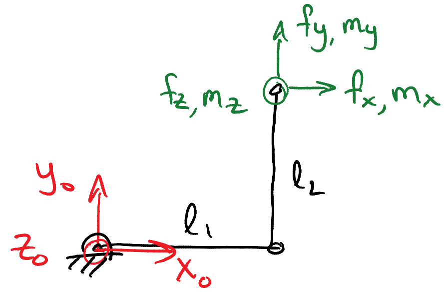


$${}^0J_E = \begin{bmatrix} -l_2 & -l_2 \\ l_1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$Q = {}^0J_E^T \, {}^0g_E = \begin{bmatrix} -l_2 & l_1 & 0 & 0 & 0 & 1 \\ -l_2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^0f_x \\ {}^0f_y \\ {}^0f_z \\ {}^0m_x \\ {}^0m_y \\ {}^0m_z \end{bmatrix}$$

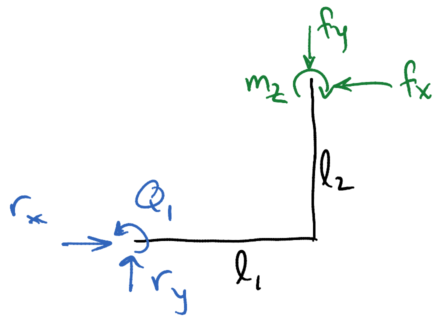
$$Q = \begin{bmatrix} -l_2 {}^0f_x + l_1 {}^0f_y + {}^0m_z \\ -l_2 {}^0f_x + {}^0m_z \end{bmatrix}$$

Physical interpretation $\Rightarrow$



Use free-body diagrams

Negative wrench is applied by env. onto robot.

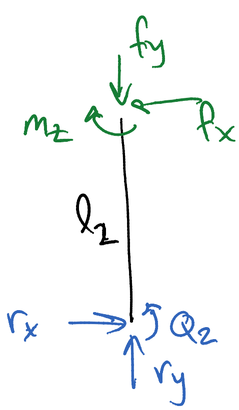


Equilibrium Equations

$$\left.\begin{array}{l} -f_x + r_x = 0 \Rightarrow r_x = f_x \\ -f_y + r_y = 0 \Rightarrow r_y = f_y \end{array}\right\}$$ reaction forces by unactuated structure

$$-m_z + Q_1 + f_x l_2 - f_y l_1 = 0$$

$$\Rightarrow Q_1 = m_z + f_y l_1 - f_x l_2 \; \}$$ reaction by actuator.



Similarly, we see that

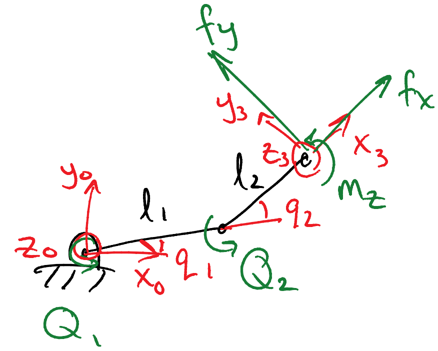
$r_x = f_x$

$r_y = f_y$

$Q_2 = m_z - l_2 f_x$

How would this work if we had ${}^E J_E$ and ${}^E g_E$?

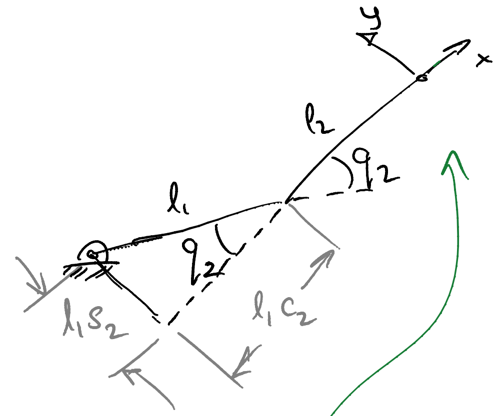
$${}^E J_E = \begin{bmatrix} 0 & l_1 s_1 \\ l_2 & l_2 + l_1 c_2 \\ 1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 0 & l_1 s_2 & 1 \\ l_1 s_2 & l_2 + l_1 c_2 & 1 \end{bmatrix} \begin{bmatrix} {}^E f_x \\ {}^E f_y \\ {}^E m_z \end{bmatrix}$$

$$Q_1 = l_1 s_2 \, {}^E f_y + {}^E m_z$$

$$Q_2 = l_1 s_2 \, {}^E f_x + (l_2 + l_1 c_2) \, {}^E f_y + {}^E m_z$$



wrench applied to environment.