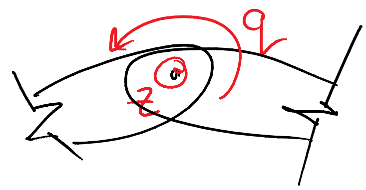
7.1_DescribingRobotArms

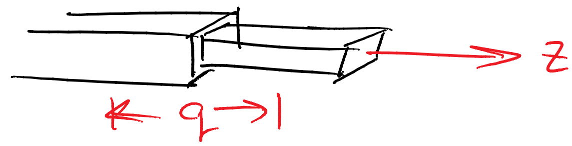
Tuesday, June 12, 2012
12:15 PM

Joint Types:

Revolute –



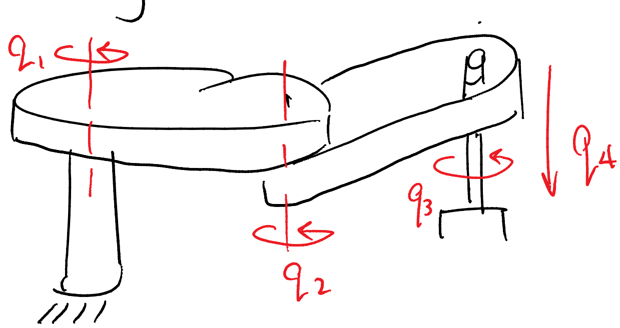
Prismatic –



Arm Types:

Revolute – all joints are revolute

Scara – major links move in a horizontal plane.
Usually used for vertical insertion tasks.



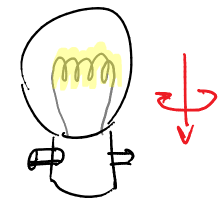
Task requirements:

General tasks – end effector must have 6 dof.
The set of end effector poses must be a 6D subset of $SE(3)$.

Vertical insertion tasks – end effector must cover a 4D subset of $SE(2) \times I^1$.

Pros – simpler, more accurate, faster

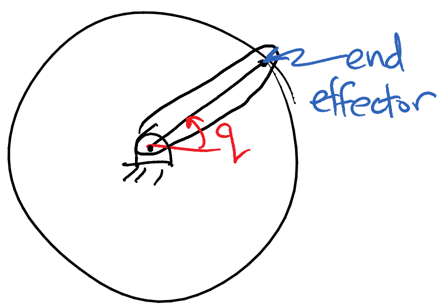
Cons – can't perform arbitrary tasks.



Workspace

The subset of $\mathbb{R}^n$ where the origin of the tool frame can be placed, where $n = 2$ or 3.

1R planar



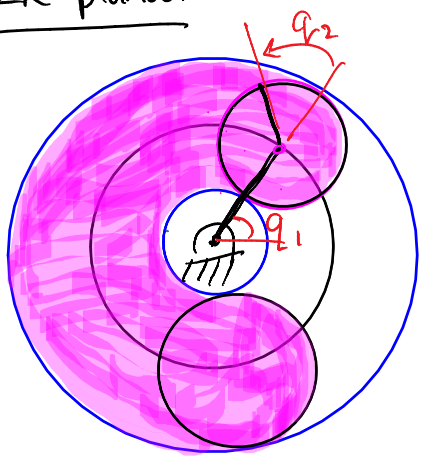
$\mathcal{W} = S^1$

or I^1 with joint limits.

1P-planar

$\mathcal{W} = I^1$

2R-planar



with no joint limits,

$\mathcal{W} =$ disc with a hole

with joint limits

$\mathcal{W} =$ disc

$\mathcal{W}$ of 3R-planar robot is similar.

$\mathcal{W}$ of all planar robots with small # of R and P joints are easy to construct via enumeration.

Dexterous Workspace, $\mathcal{W}_D$

Def: The subset of $\mathcal{W}$ that can be reached with all end effector orientations.

Desirable for $\mathcal{W}_D$ to be 2D for planar robots and 3D for spatial robots.

Necessary conditions:

Planar case: At least one revolute joint w/o limits and at least two other joints to position the end effector.

Spatial case: At least three revolute joints and at least three other joints to position the end effector.

Note: Joint limits will reduce volume of dexterous workspace.

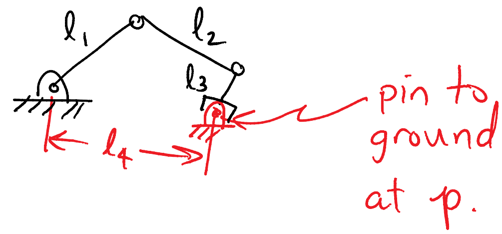
Solutions for boundary of $\mathcal{W}_D$:

3D: Analytical solutions do not exist
Exact Numerical solutions are very expensive.

2D: Analytical solutions should exist
Exact numerical solutions should be easy to compute.

A topological perspective for 3R-planar robots

Pin the end effector to the ground and check if it can be rotated by 2π radians.

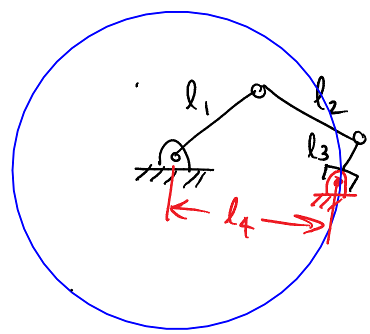


Theorem:

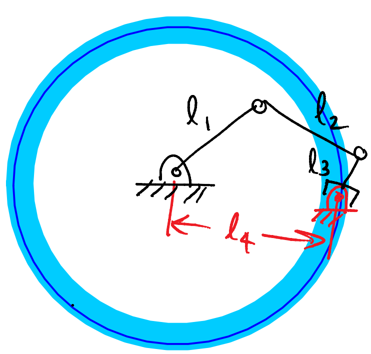
Let $L = \sum_{i=1}^{4} l_i$. Iff $l_i + l_j \geq \frac{1}{2} L \;\; \forall \; i,j \in \{1,2,4\}$,

then l_3 may rotate 2π radian while pinned at p.

If thm. is satisfied at p, the it is satisfied at all points on the circle of radius l_4.



Now vary l_4 to "thicken" the dexterous work space.



Experiment with some numbers.

Let $l_1 = 5$, $l_2 = 4$, $l_3 = 1$.

Also let λ_i be the ith longest link.

From the theorem, l_3 must be the shortest link

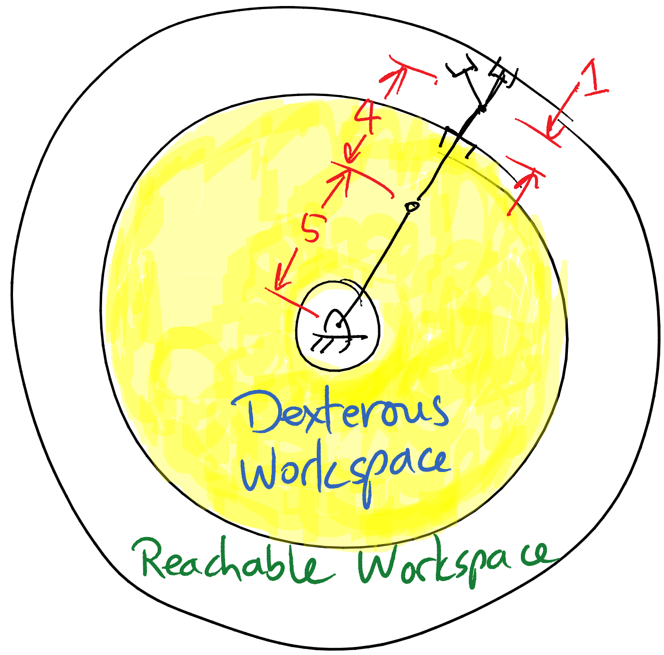
and $\lambda_2 + \lambda_3 > \frac{1}{2} L = \frac{1}{2}(l_1 + l_2 + l_3 + l_4)$

Clearly $0 \le l_4 \le 10$.

How big can l_4 be and still satisfy the conditions?

If $l_4 \ge 5$, then we require $l_1 + l_2 > \frac{1}{2}(l_1 + l_2 + l_3 + l_4)$

l_4	$\lambda_2 + \lambda_3$	$L/2$
10	9	10
9	9	9.5
8	9	9
7	9	8.5
6	9	8
5	9	7.5
4	8	7
3	7	6.5
2	6	6
1.9	5.9	5.95



Note: The theorem is valid for any number of revolute joints and prismatic joints can be incorporated by allowing the link lengths to vary.

Standard Denavit-Hartenberg parameters

This is a minimal representation

Only 4 parameters:

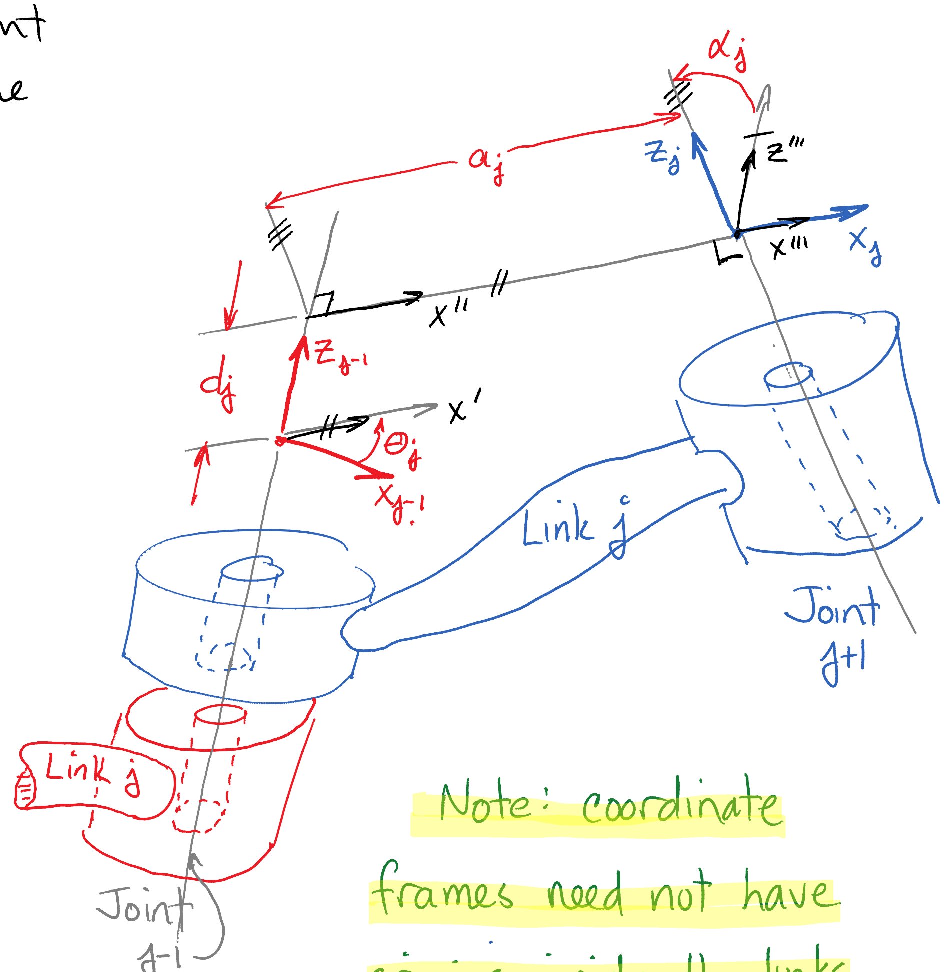
3 are constant

1 is variable

Frame assignment

- Identify joint axes.
  Label them z_j, $j = 1, 2, \ldots, N$
- Identify common normal of z_{j-1} and z_j, $\forall j$
  Label them x_j
- Define parameters

θ_j = angle about z_{j-1} from x_{j-1} to x_j



Note: coordinate frames need not have origins inside the links.

d_j = distance along z_{j-1} from origin $\{j-1\}$ to origin $\{j\}$

a_j = distance along x_j from " " " " "

(i.e., the distance between z_{j-1} and z_j)

α_j = angle about x_j from z_{j-1} to z_j

$$\sigma_j = \begin{cases} 0 \Rightarrow \text{joint } j \text{ is revolute, and } \theta_j \text{ is the joint var.} \\ 1 \Rightarrow \text{joint } j \text{ is prismatic, and } d_j \text{ is the joint var.} \end{cases}$$

Relative pose:

Corke

$$^{j-1}A_j = T_{Rz}(\theta_j)\, T_z(d_j)\, T_x(a_j)\, T_{Rx}(\alpha_j) \qquad (7.2)$$

$$= \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & a C_\theta \\ S_\theta & C_\theta C_\alpha & C_\theta S_\alpha & a S_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}_j$$

where all quantities have subscript, j

Note error in text. "a" is correct, not "α".

Terminology:

Robot configuration: $q \in \mathbb{R}^N$, $q = [q_1, q_2, \dots q_N]$

where $q_i = \begin{cases} \theta_i \text{ if } \sigma_i = 0 \\ d_i \text{ if } \sigma_i = 1 \end{cases}$

L= Link([0, 0.1, 0.2, π/2, 0])