2.0_RepresentingPositionAndOrientation

Saturday, January 28, 2012
9:29 AM

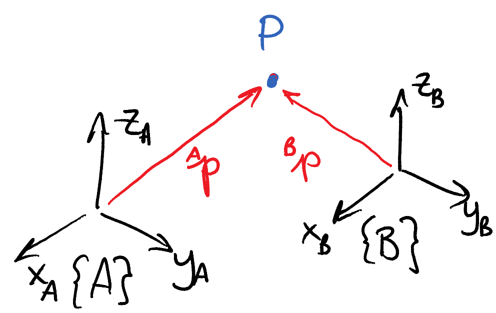
2. Representing Position and Orientation
    a. Point = p = 3-vector
    b. Frame = {A} = three mutually orthogonal axes that meet at a point (the origin).
    c. xi = transformation describing one frame relative to another. ^A xi _B is the pose of {B} expressed in {A}. xi is also called a relative pose.
    d. A point can be represented in any frame
    e. ^A p is a point expressed in {A}
    f. ^A p = ^A xi_B * ^B p   eq.(2.1)
    g. A rigid body is a collection of points that maintain a fixed distance relative to each other.
    h. A frame is fixed to the world and a rigid body. Then the relative transform   xi   describes the position and orientation of every point on the rigid body.
    i. Think of {A} as a rigid body defined by four points: origin, unit x-vector, unit y-vector, and unit z-vector
    j. A relative transform also can be thought of as describing a rigid body displacement
    k. Relative transforms are composable: ^A xi_B (+) ^B_xi_C = ^A xi_C
    l. ^A p = ^A xi_C * ^C p   expand this
    m. Show picture with multiple loops and write the t-form equations
    n. Relative tforms are elements of the group SE(3). Groups have inverse, identity, composition operation that yields another element of the group (i.e., closure).
    o. Algebraic rules in orange box on page18. (+) and (-).

Some notation:

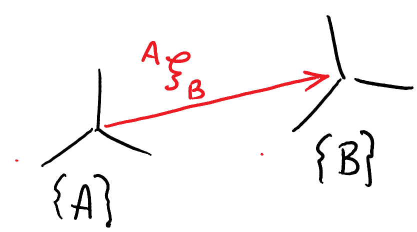
$\{A\}$ – coordinate frame

p – a point in space

$^A p$ – vector representation of P expressed in $\{A\}$



$^A \xi_B$ – relative transformation defining $\{B\}$ with respect to $\{A\}$



ξ – also called "relative pose"

$^A \xi_A$ = null relative pose, i.e. no transformation = "0"

Points may be transformed from one frame to another

$${}^{A}p = {}^{A}\xi_{B} \circ {}^{B}p$$

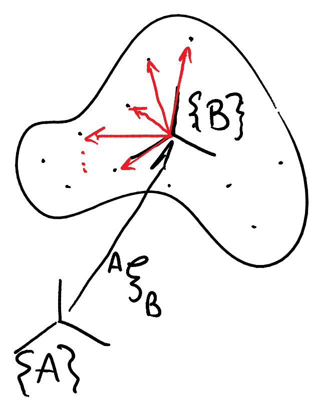
an operator - will be matrix-vector multiplication operator.

We will see later that ξ is an element of

a mathematical group,

called the Special Euclidean group

Rigid body - a set of points with fixed distances between each pair.



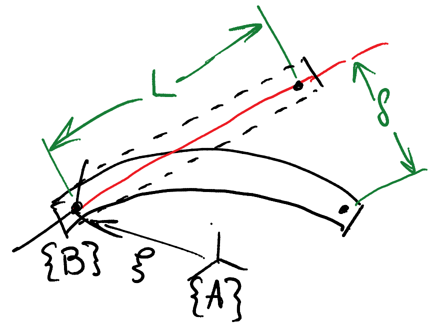
if ${}^{A}\xi_{B}$ is known and the positions of all point are known w.r.t. {B}, then the position of every point in the rigid body is known in {A}

Deformable body:

Distances between points can change.

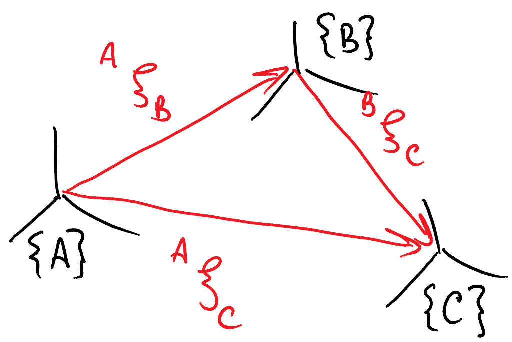
For special deformable bodies, are simple functions of a small # of parameters.

Example: inextensible elastic rod



If L, δ, and ξ are known, then the position of all points on the rod are known w.r.t. $\{A\}$

Composition of relative poses



$$ {}^A\xi_C = {}^A\xi_B \oplus {}^B\xi_C $$

$$ {}^Ap = {}^A\xi_C \, {}^Cp = \left({}^A\xi_B \oplus {}^B\xi_C\right) \cdot {}^Cp $$

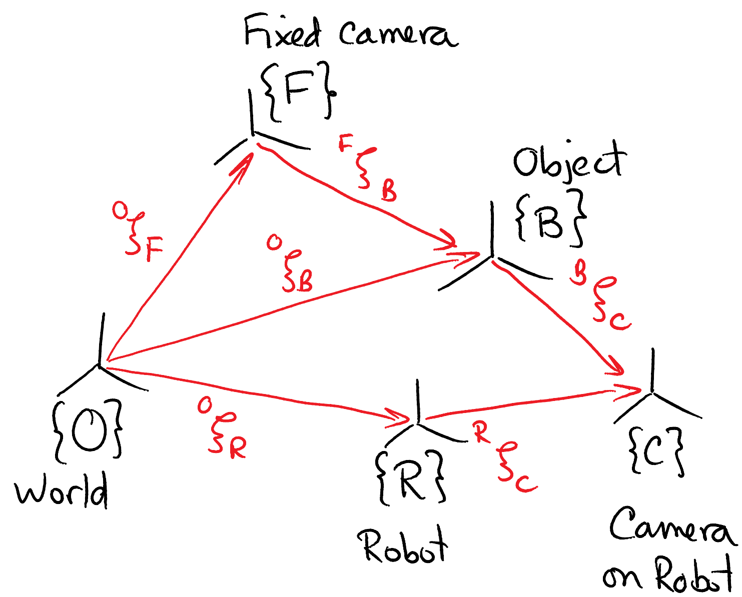
We also know that:

$${}^{B}p = {}^{B}\xi_{C} \cdot {}^{C}p \quad \& \quad {}^{A}p = {}^{A}\xi_{B} \cdot {}^{B}p \,.$$

Therefore, we have:

$${}^{A}p = {}^{A}\xi_{B} \cdot \left({}^{B}\xi_{C} \cdot {}^{C}p \right)$$

Graph Representation



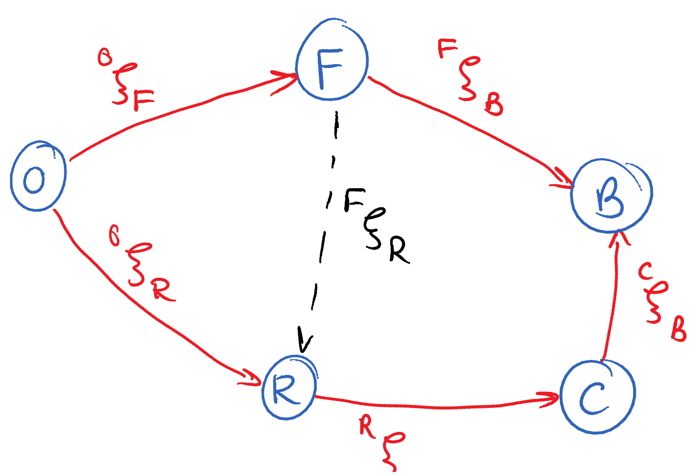
Constant relative poses:

${}^{O}\xi_{F}$ and ${}^{R}\xi_{C}$

Time varying rel. poses:

${}^{F}\xi_{B}$, ${}^{B}\xi_{C}$, ${}^{O}\xi_{R}$, ${}^{O}\xi_{B}$

Each loop of the graph defines an equation.



$$R \xrightarrow{{}^R\xi_C} C$$

$${}^0\xi_B = {}^0\xi_F \oplus {}^F\xi_B = {}^0\xi_R \oplus {}^R\xi_C \oplus {}^C\xi_B = {}^0\xi_B$$

Algebraic Rules:

$$\xi \oplus 0 = \xi \qquad \xi \ominus 0 = \xi$$

$$\xi \ominus \xi = 0 \qquad \ominus\xi \oplus \xi = 0$$

$$\xi_1 \oplus \xi_2 = \xi_3 \quad \leftarrow \text{set closure}$$

where 0 is the null relative pose.

The rules imply $\exists$ an inverse:

$$\ominus {}^X\xi_Y = {}^Y\xi_X \quad \Longrightarrow \quad {}^X\xi_Y \ominus {}^X\xi_Y = {}^X\xi_Y \oplus {}^Y\xi_X = {}^X\xi_X = 0$$

Composition is <u>NOT</u> commutative:

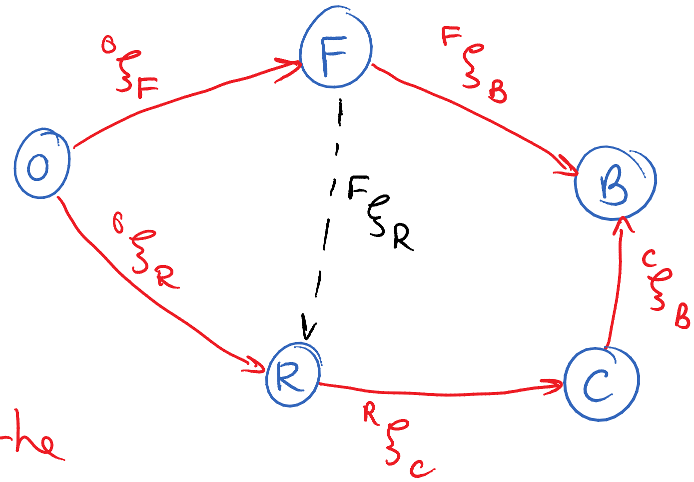
$${}^X\xi_Y \oplus {}^Y\xi_Z \neq {}^Y\xi_Z \oplus {}^X\xi_Y$$

exception: $X = Z \Longrightarrow {}^X\xi_Y \oplus {}^Y\xi_Z = 0$

Return to the graph.

$${}^F\xi_R = \ominus\, {}^O\xi_F \oplus {}^O\xi_R$$

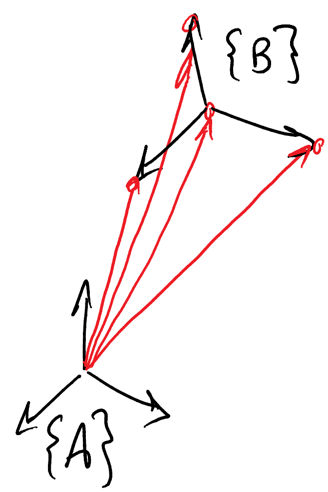
$$= {}^F\xi_B \ominus {}^C\xi_B \ominus {}^R\xi_C$$



The $\ominus$ operator switches the superscript and the subscript.

One way to think of the relative pose:

It defines the origin and unit vectors of the new frame in the base frame.



$${}^B p_x = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad {}^B p_y = \begin{bmatrix}0\\1\\0\end{bmatrix} \quad {}^B p_z = \begin{bmatrix}0\\0\\1\end{bmatrix} \quad {}^B p_o = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$${}^A\xi_B \cdot \left({}^B\begin{bmatrix}1\\0\\0\end{bmatrix}, {}^B\begin{bmatrix}0\\1\\0\end{bmatrix}, {}^B\begin{bmatrix}0\\0\\1\end{bmatrix}, {}^B\begin{bmatrix}0\\0\\0\end{bmatrix} \right) \triangleq \{B\}$$

operate on 4 points in the way described above.