Introduction to Mobile Robotics

EKF Localization

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UNI FREIBURG

Slides by Kai Arras and Wolfram Burgard
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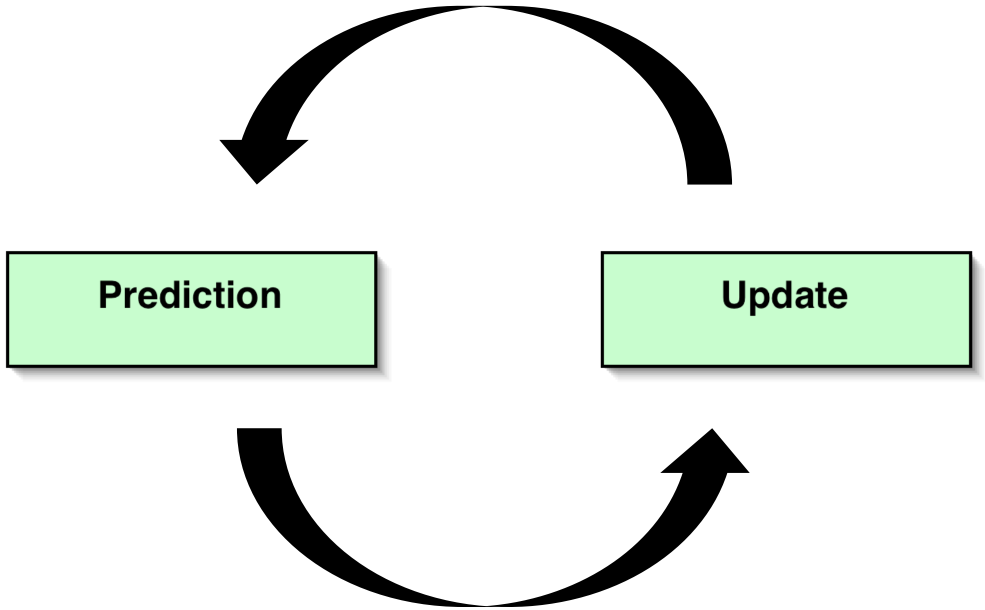
Localization

> "Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities." [Cox '91]

- **Given**
  - Map of the environment.
  - Sequence of sensor measurements.
- **Wanted**
  - Estimate of the robot's position.
- **Problem classes**
  - Position tracking
  - Global localization
  - Kidnapped robot problem (recovery)

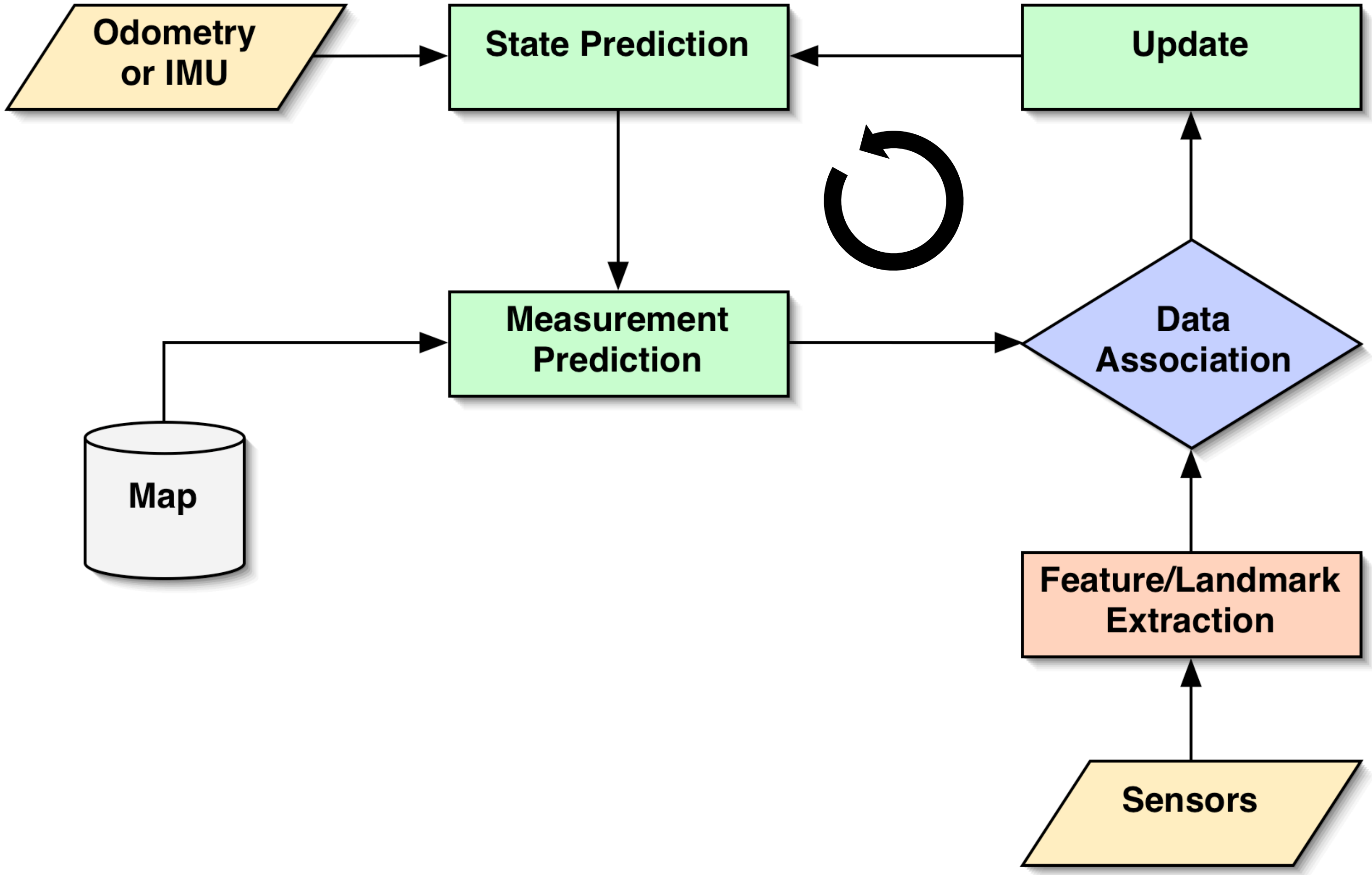
Landmark-based Localization

EKF Localization: Basic Cycle



Landmark-based Localization

EKF Localization: Basic Cycle



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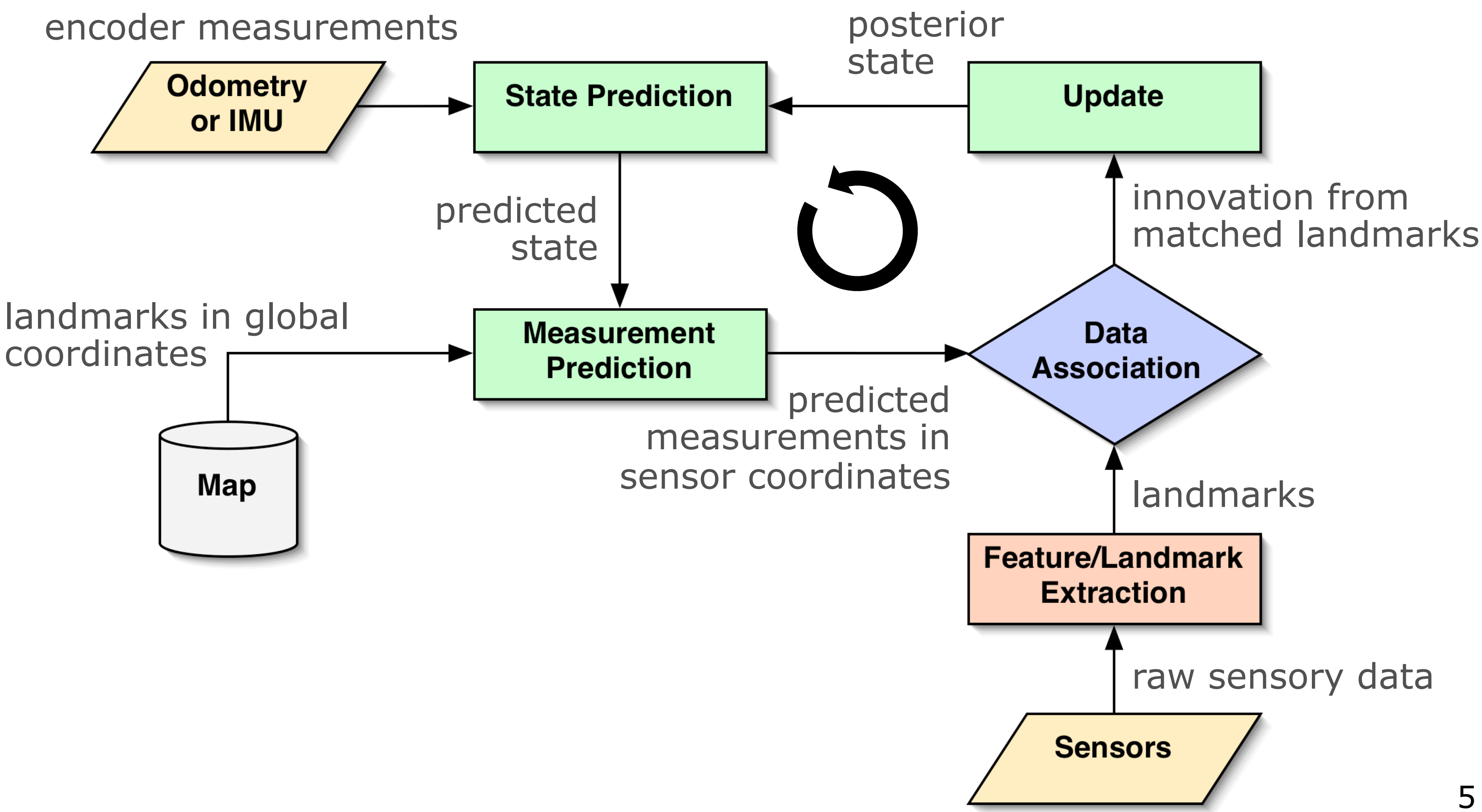
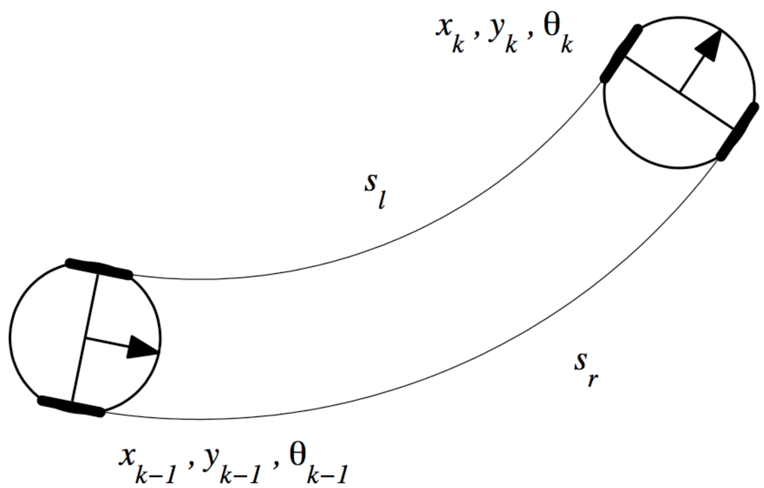
EKF Localization: Basic Cycle



Landmark-based Localization

State Prediction (Odometry)

$$\hat{\mathbf{x}}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k)$$
$$\hat{C}_k = F_x \, C_k \, F_x^T + F_u \, U_k \, F_u^T$$



Control $\mathbf{u}_k$: wheel displacements s_l, s_r

$$\mathbf{u}_k = (s_l \;\; s_r)^T \qquad U_k = \begin{bmatrix} \sigma_l^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}$$

Error model: linear growth

$$\begin{aligned} \sigma_l &= k_l \, |s_l| \\ \sigma_r &= k_r \, |s_r| \end{aligned}$$

Nonlinear process model f :

$$\mathbf{x}_k = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{b}{2} \, \frac{s_l + s_r}{s_r - s_l} \left(-\sin \theta_{k-1} + \sin(\theta_{k-1} + \frac{s_r - s_l}{b}) \right) \\ \frac{b}{2} \, \frac{s_l + s_r}{s_r - s_l} \left(\cos \theta_{k-1} - \cos(\theta_{k-1} + \frac{s_r - s_l}{b}) \right) \\ \frac{s_r - s_l}{b} \end{bmatrix}$$

Landmark-based Localization

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$$\hat{\mathbf{x}}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k)$$
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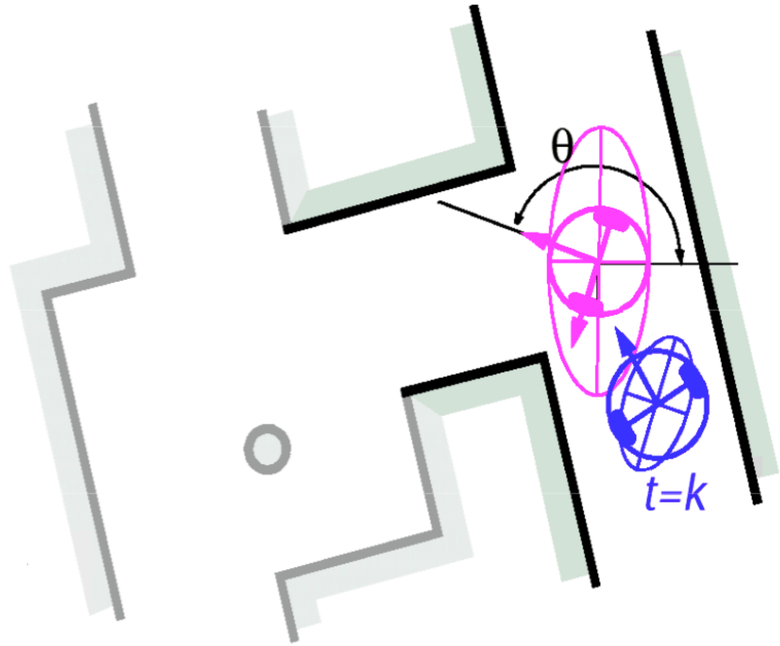

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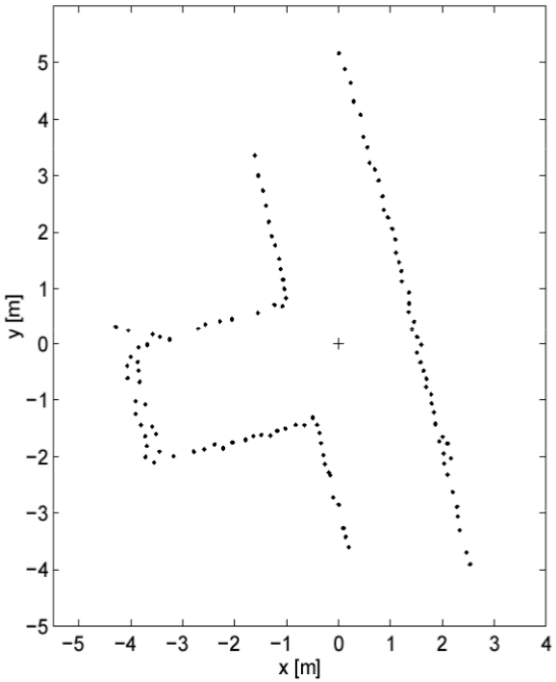
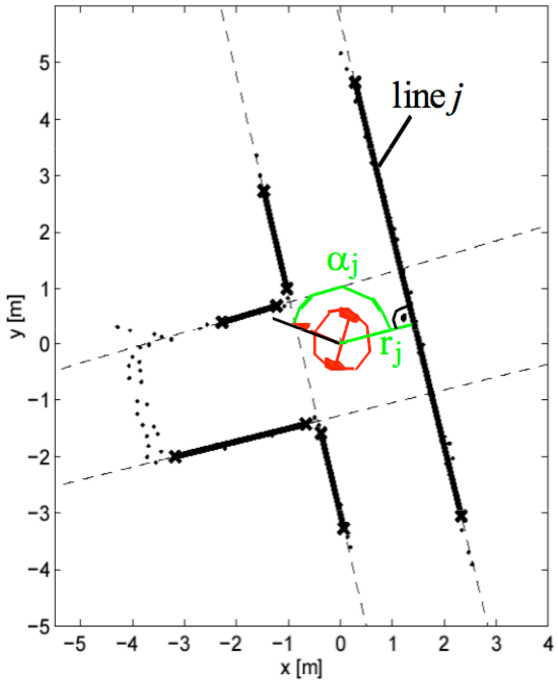
Landmark-based Localization

Landmark Extraction (Observation)

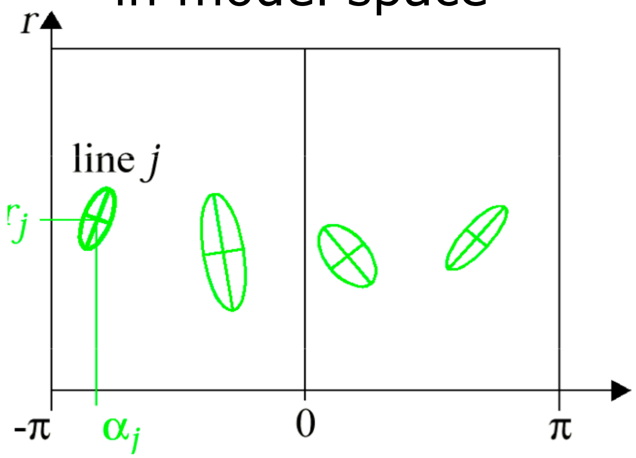
Raw laser range data



Extracted lines



Extracted lines in model space



$$\mathbf{z}_k = \begin{bmatrix} \alpha \\ r \end{bmatrix}$$

$$R_k = \begin{bmatrix} \sigma_\alpha^2 & \sigma_{\alpha r} \\ \sigma_{r\alpha} & \sigma_r^2 \end{bmatrix}$$

Hessian line model

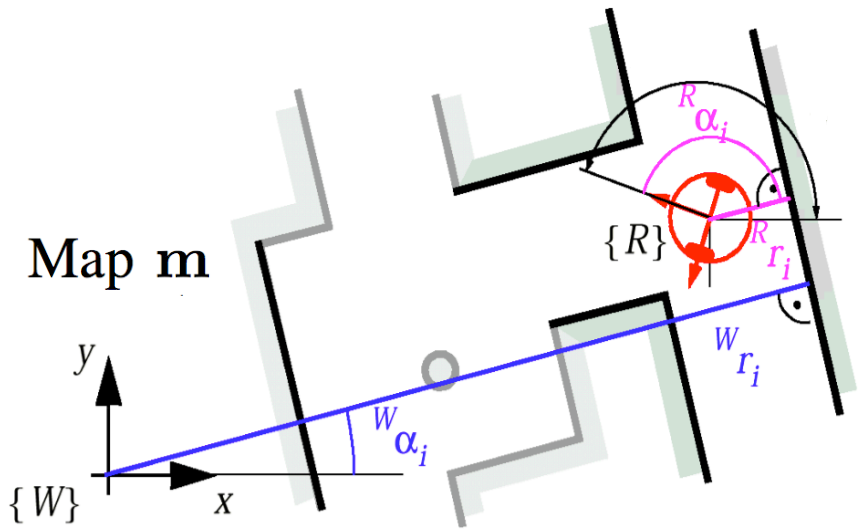
$$x\ cos(\alpha) + y\ sin(\alpha) - r = 0$$

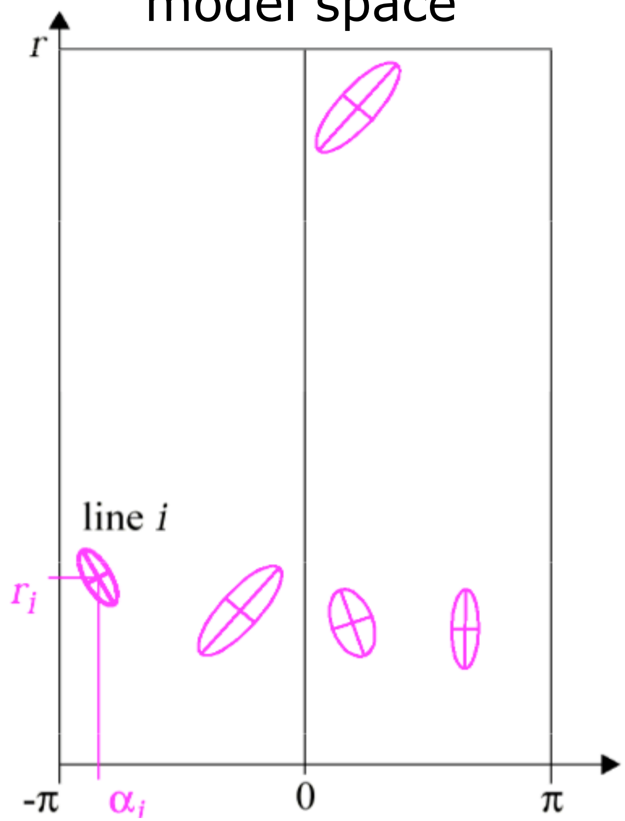
Landmark-based Localization

Measurement Prediction

- ...is a coordinate frame transform world-to-sensor
- Given the predicted state (robot pose), predicts the location $\hat{\mathbf{z}}_k$ and location uncertainty $H\,\hat{C}_k\,H^T$ of expected observations in sensor coordinates

$$\hat{\mathbf{z}}_k = h(\hat{\mathbf{x}}_k, \mathbf{m})$$





Landmark-based Localization

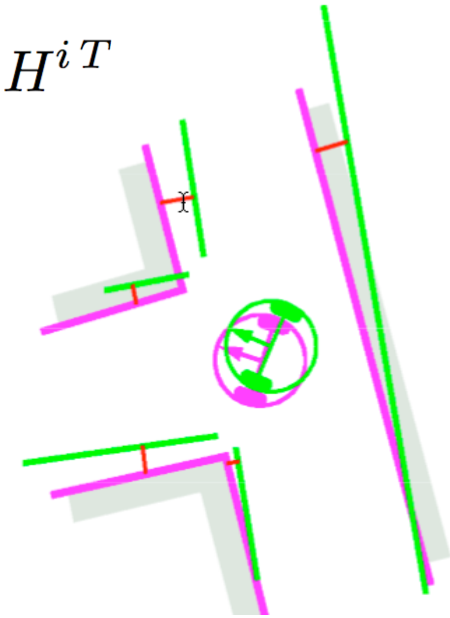
Data Association (Matching)

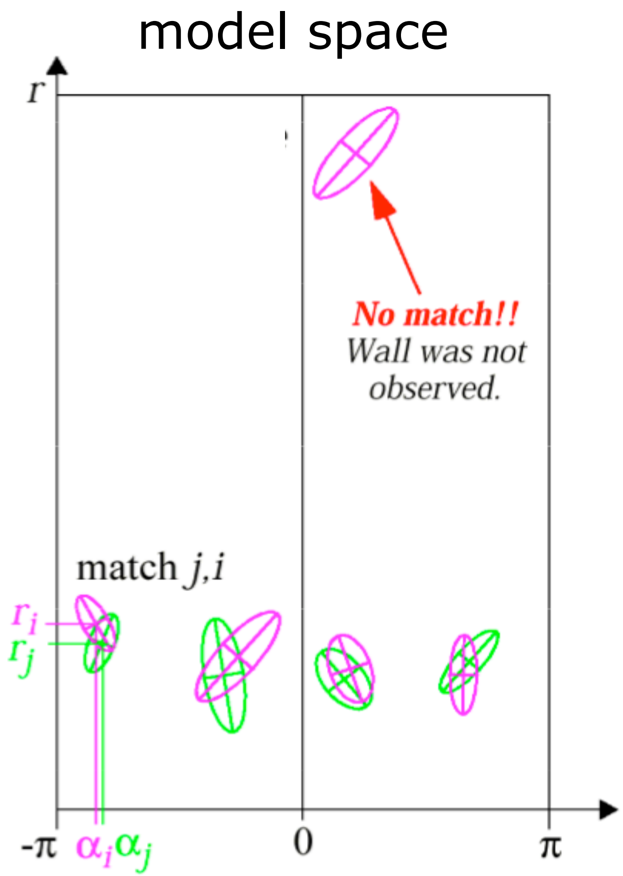
- Associates predicted measurements $\hat{\mathbf{z}}_k^i$ with observations $\mathbf{z}_k^j$

$$\nu_k^{ij} = \mathbf{z}_k^j - \hat{\mathbf{z}}_k^i$$
$$S_k^{ij} = R_k^j + H^i \, \hat{C}_k \, H^{i\,T}$$

- Innovation ν_k^{ij} and innovation covariance S_k^{ij}

- Matching on significance level alpha





Green: observation
Magenta: measurement prediction

Landmark-based Localization

Update

- Kalman gain

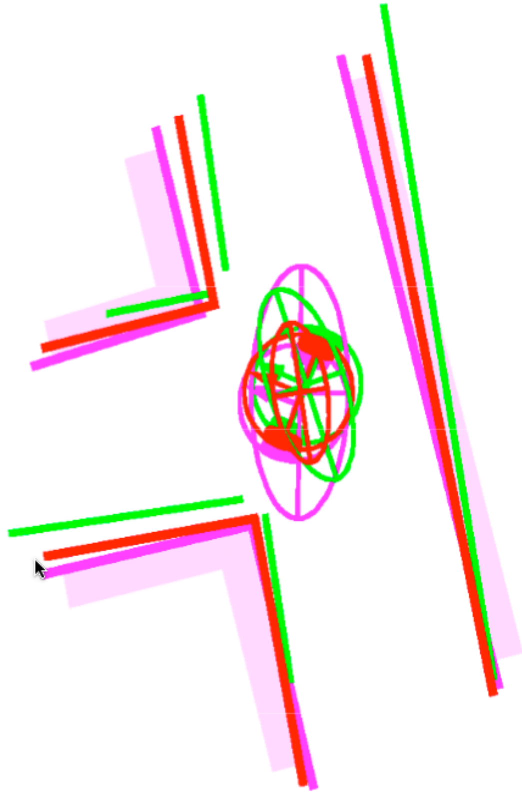
$$K_k = \hat{C}_k \, H^T \, S_k^{-1}$$

- State update (robot pose)

$$\mathbf{x}_k = \hat{\mathbf{x}}_k + K_k \, \nu_k$$

- State covariance update

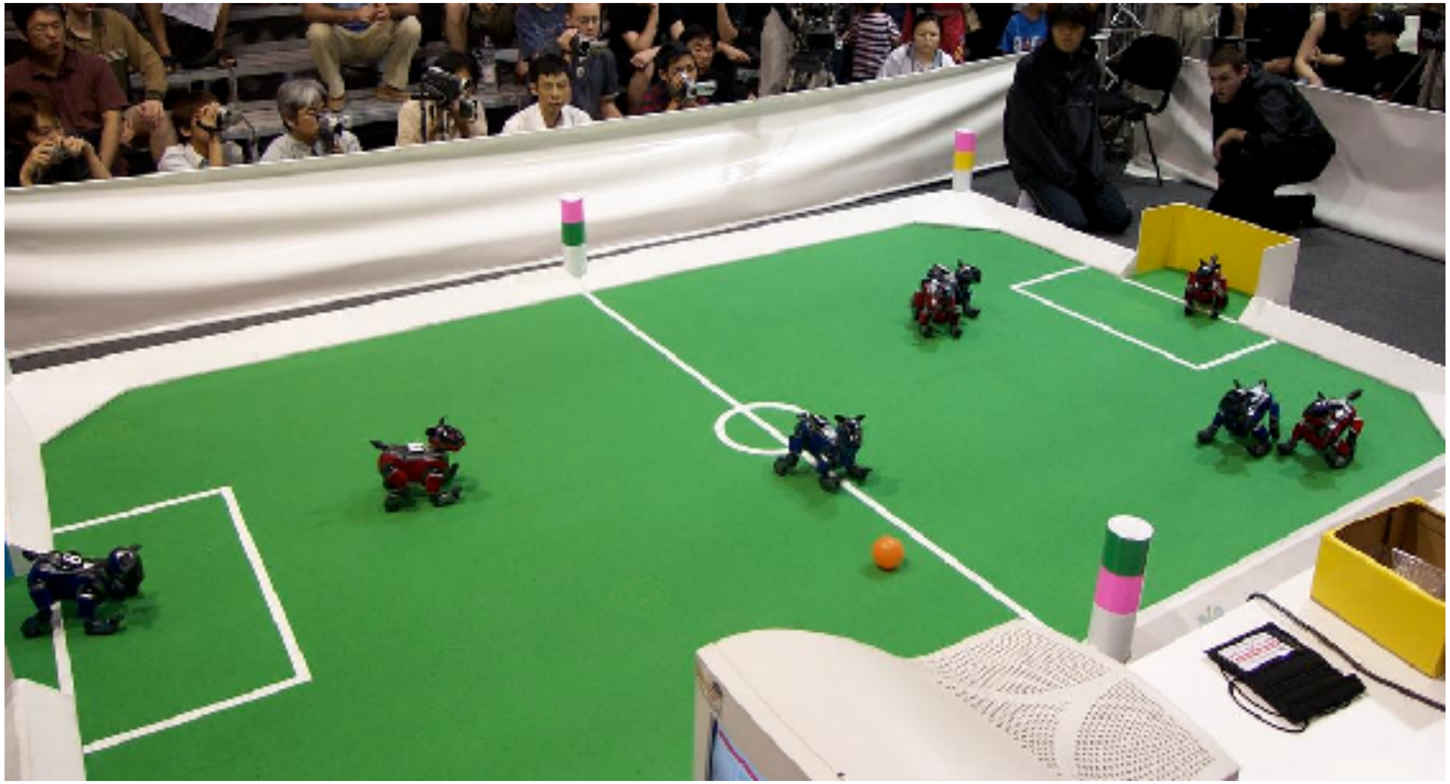
$$C_k = (I - K_k \, H) \, \hat{C}_k$$



Red: posterior estimate

Landmark-based Localization

- **EKF Localization with Point Features**



1. **EKF_localization** (μ_{t-1}, Σ_{t-1}, u_t, z_t, m):

Prediction:

2. $$G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} = \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix}$$ Jacobian of g w.r.t location

3. $$B_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial u_t} = \begin{pmatrix} \frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\ \frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\ \frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t} \end{pmatrix}$$ Jacobian of g w.r.t control

4. $$Q_t = \begin{pmatrix} (\alpha_1 |v_t| + \alpha_2 |\omega_t|)^2 & 0 \\ 0 & (\alpha_3 |v_t| + \alpha_4 |\omega_t|)^2 \end{pmatrix}$$ Motion noise

5. $$\bar{\mu}_t = g(u_t, \mu_{t-1})$$ Predicted mean

6. $$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + B_t Q_t B_t^T$$ Predicted covariance

1. **EKF_localization** (μ_{t-1}, Σ_{t-1}, u_t, z_t, m):

Correction:

2. $$\hat{z}_t = \begin{pmatrix} \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2} \\ \operatorname{atan2}(m_y - \bar{\mu}_{t,y}, m_x - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix}$$ Predicted measurement mean

3. $$H_t = \frac{\partial h(\bar{\mu}_t, m)}{\partial x_t} = \begin{pmatrix} \frac{\partial r_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix}$$ Jacobian of *h* w.r.t location

4. $$R_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{pmatrix}$$

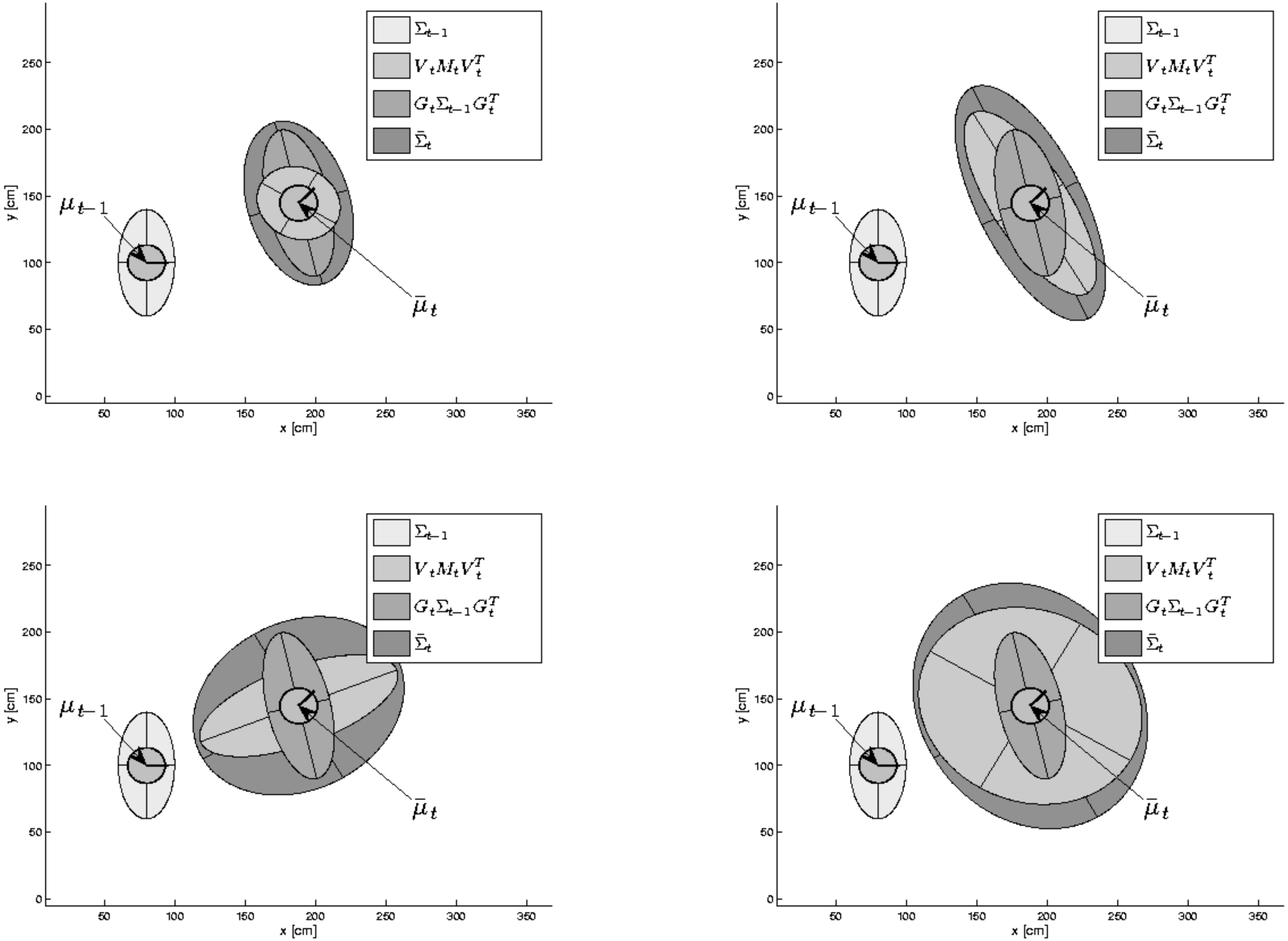
5. $$S_t = H_t \bar{\Sigma}_t H_t^T + R_t$$ Innovation covariance

6. $$K_t = \bar{\Sigma}_t H_t^T S_t^{-1}$$ Kalman gain

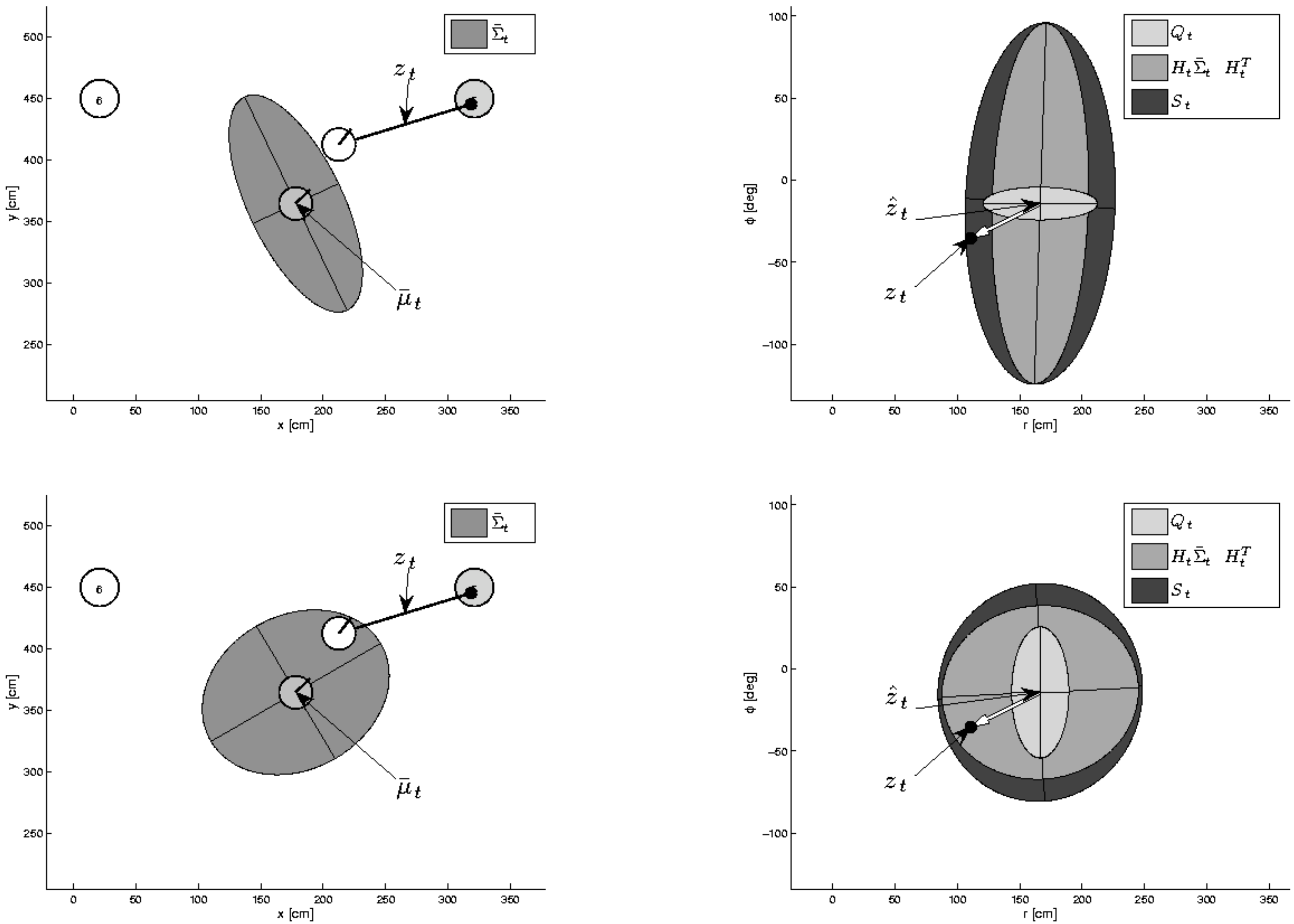
7. $$\mu_t = \bar{\mu}_t + K_t (z_t - \hat{z}_t)$$ Updated mean

8. $$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$$ Updated covariance

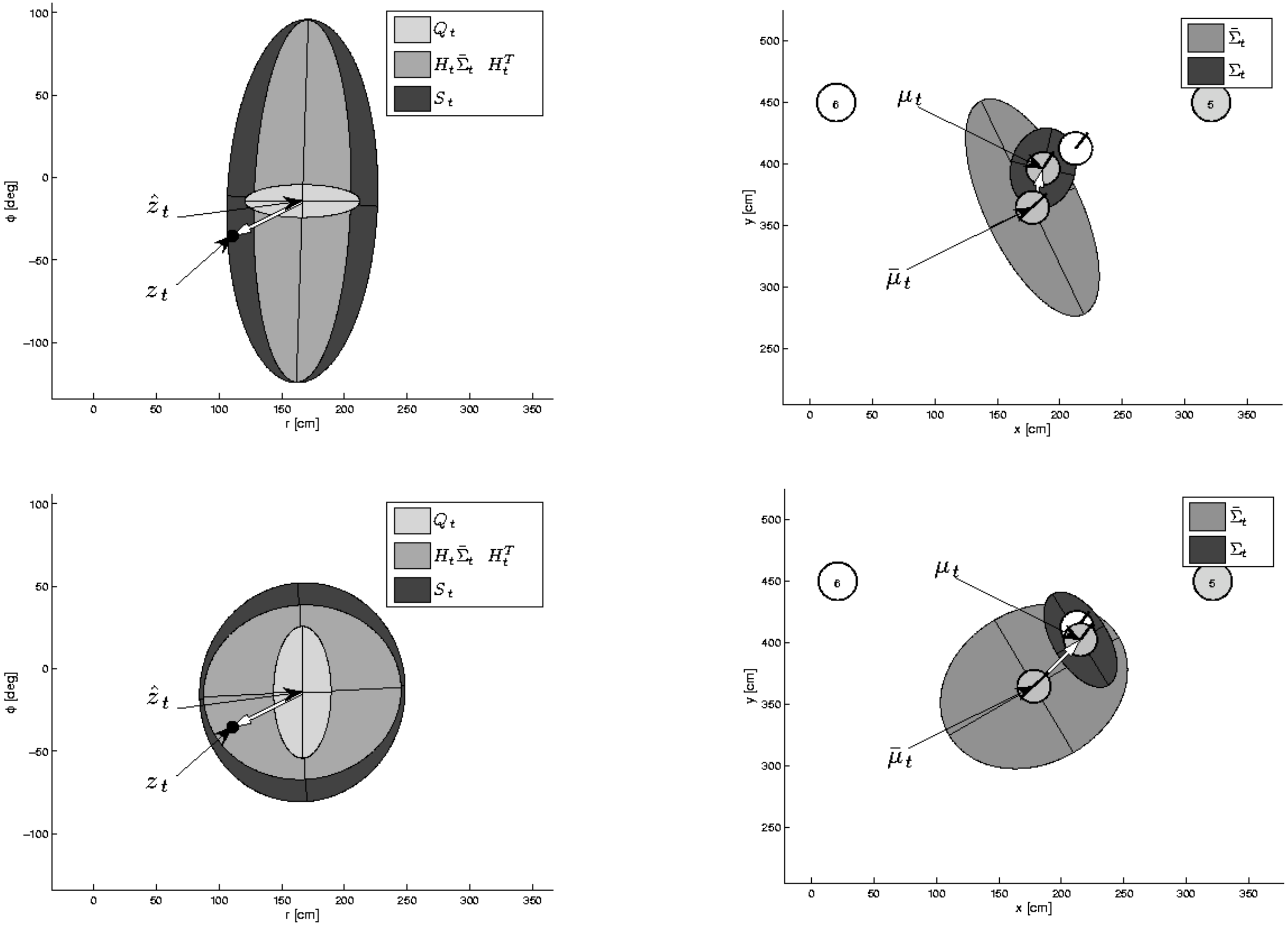
EKF Prediction Step



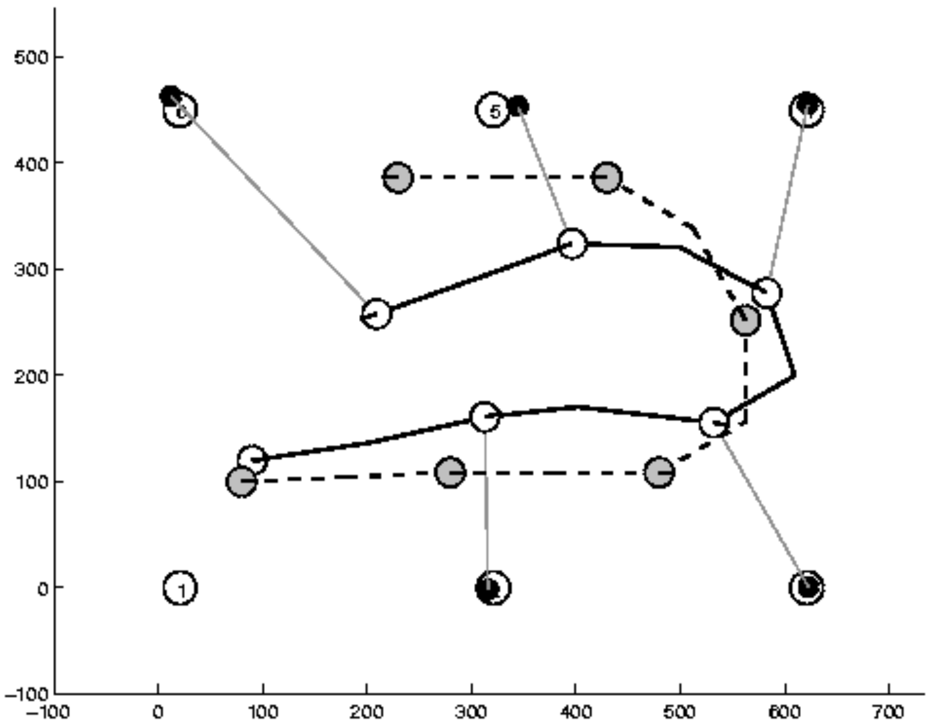
EKF Observation Prediction Step

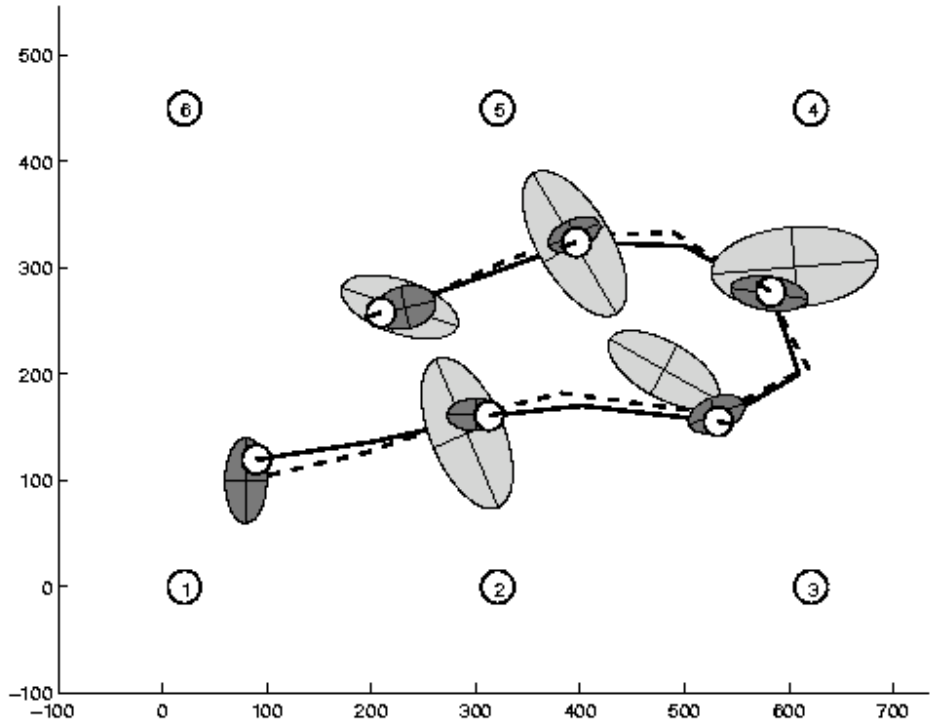


EKF Correction Step

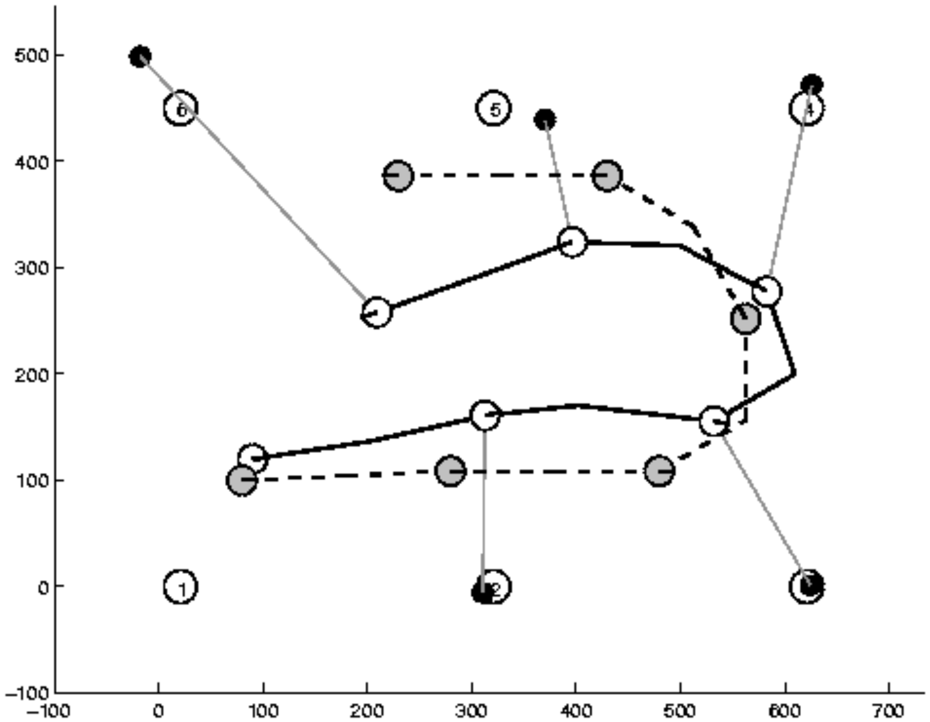


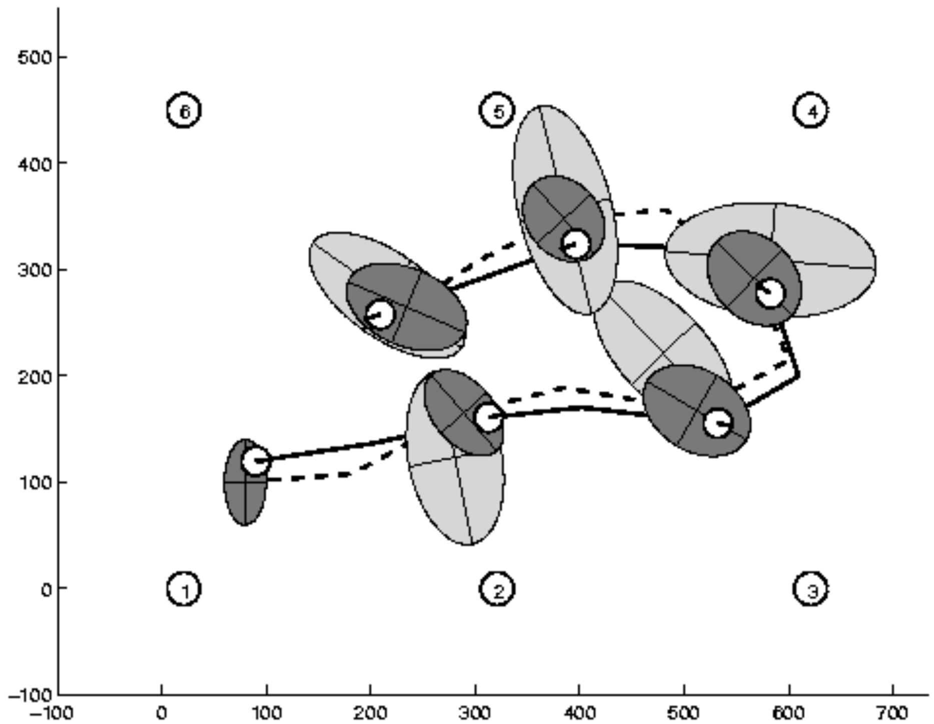
Estimation Sequence (1)



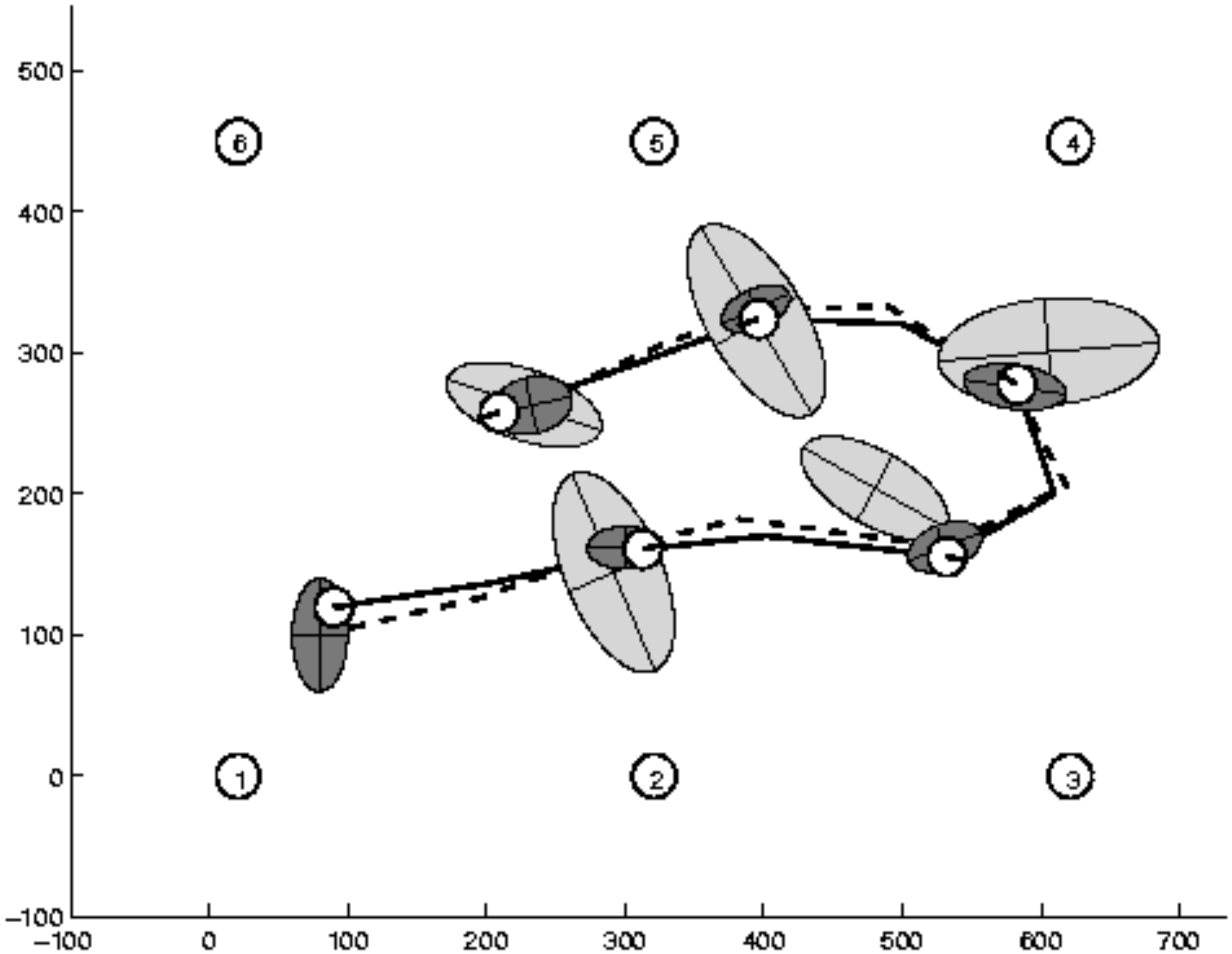


Estimation Sequence (2)





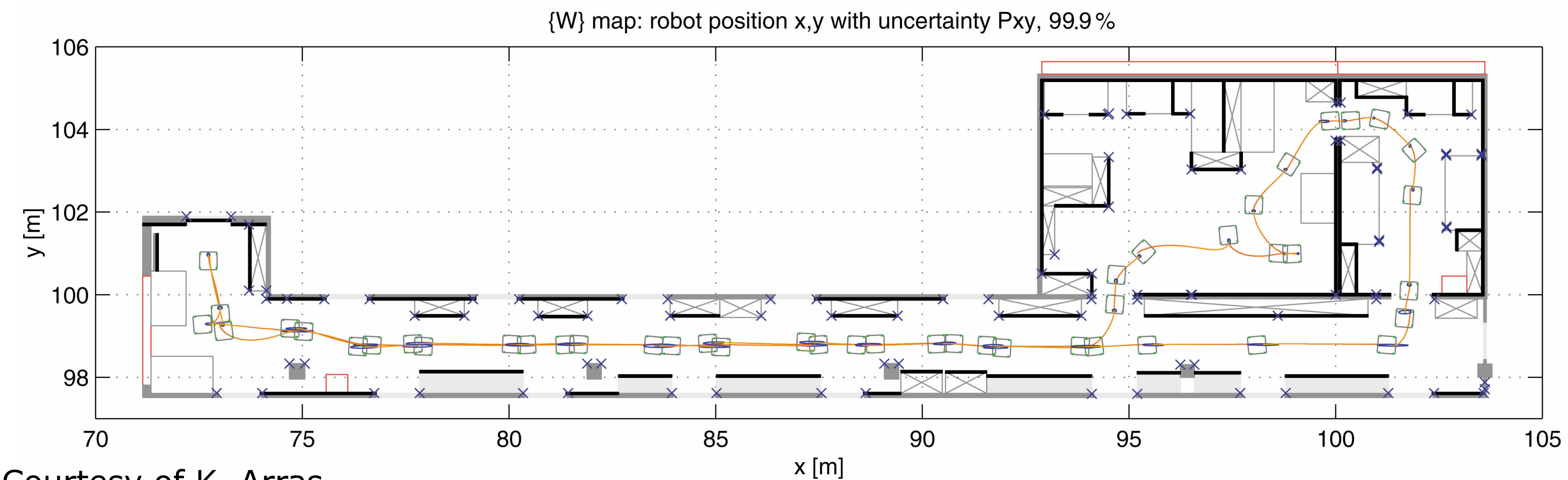
Comparison to GroundTruth



EKF Localization Example

- [Arras et al. 98]:
  - Laser range-finder and vision

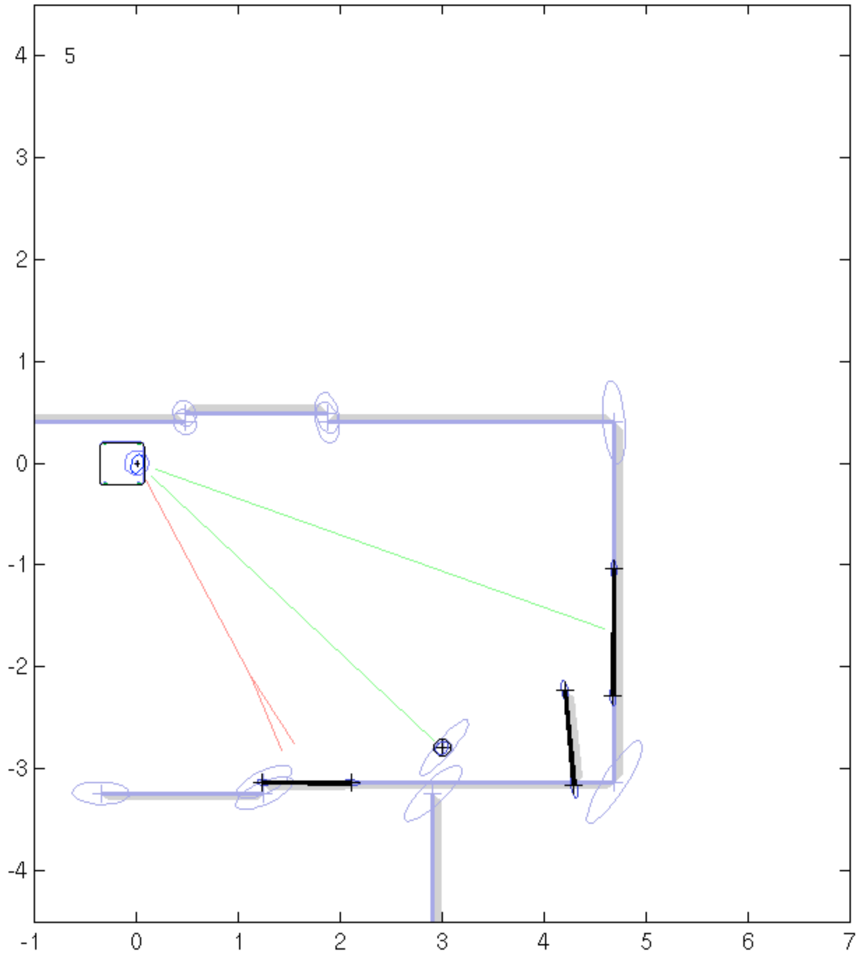
  - High precision (<1cm accuracy)



Courtesy of K. Arras

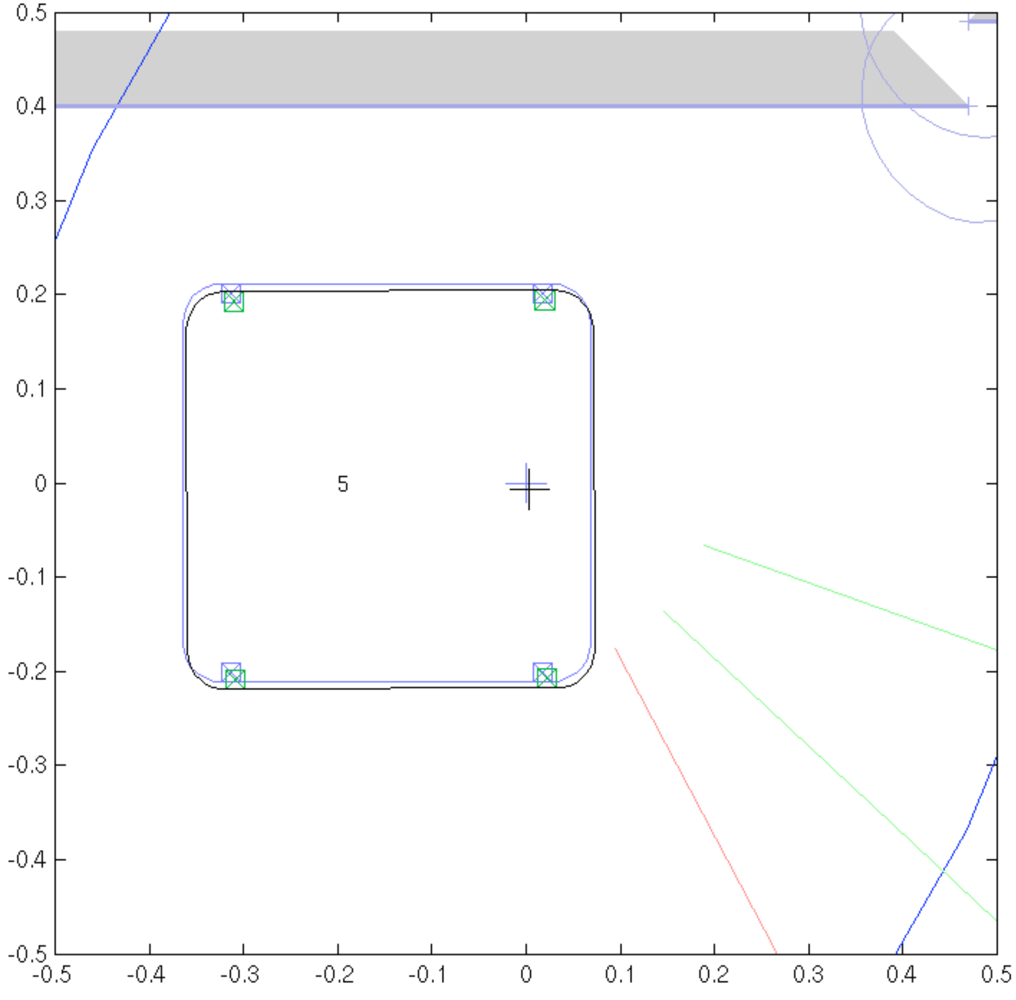
EKF Localization Example

- Line and point landmarks



EKF Localization Example

- Line and point landmarks



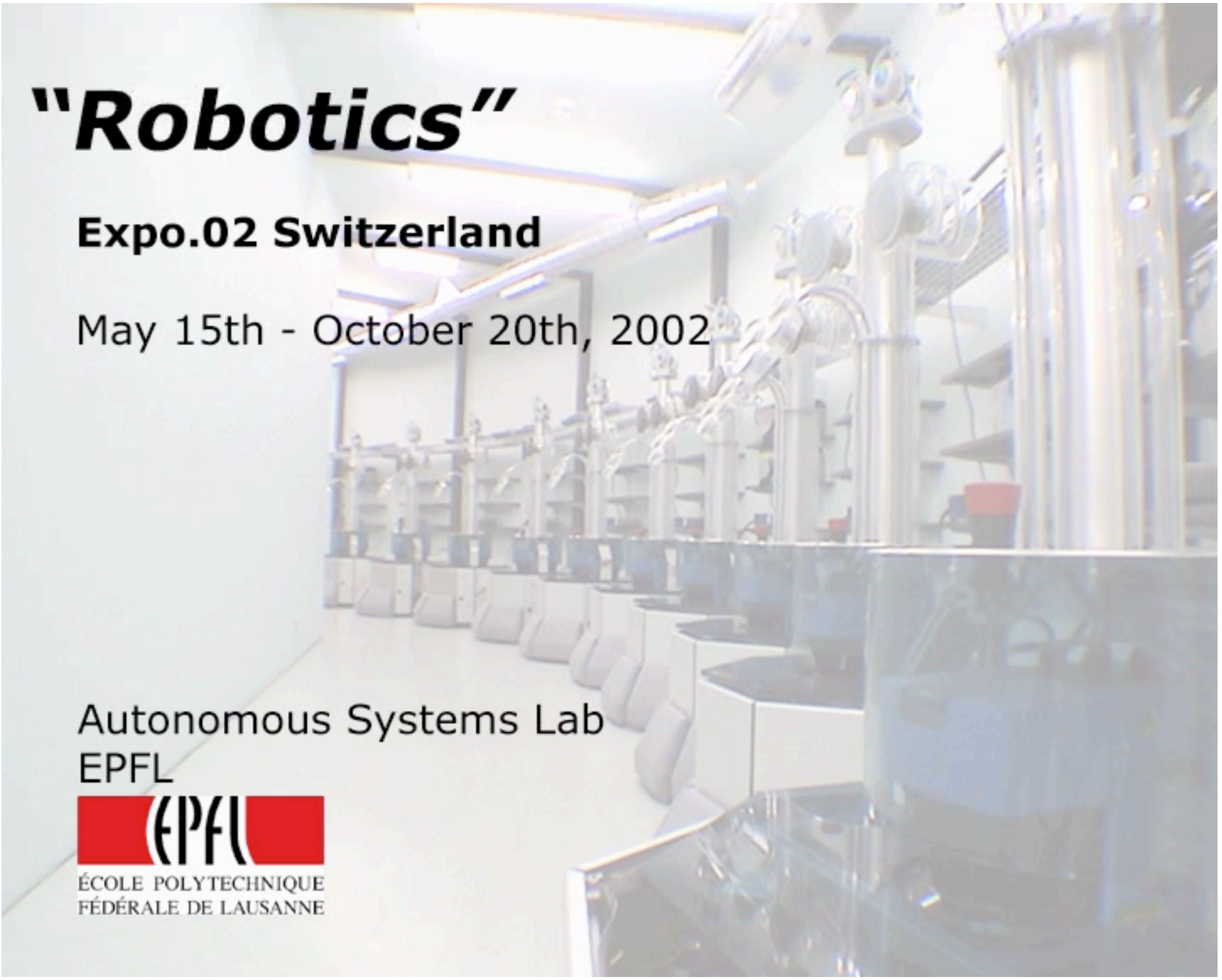
EKF Localization Example

- **Expo.02:** Swiss National Exhibition 2002
- Pavilion "Robotics"
- 11 fully autonomous robots
- tour guides, entertainer, photographer
- 12 hours per day
- 7 days per week
- 5 months

- **3,316** km travel distance
- almost **700,000** visitors
- 400 visitors per hour

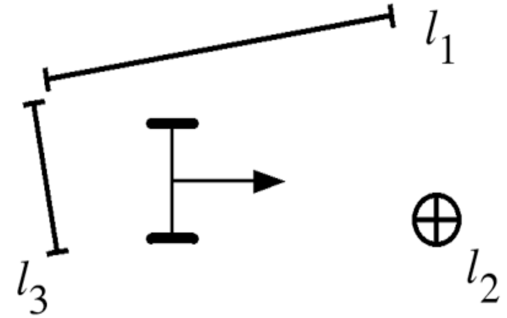
- Localization method: **Line-Based EKF**

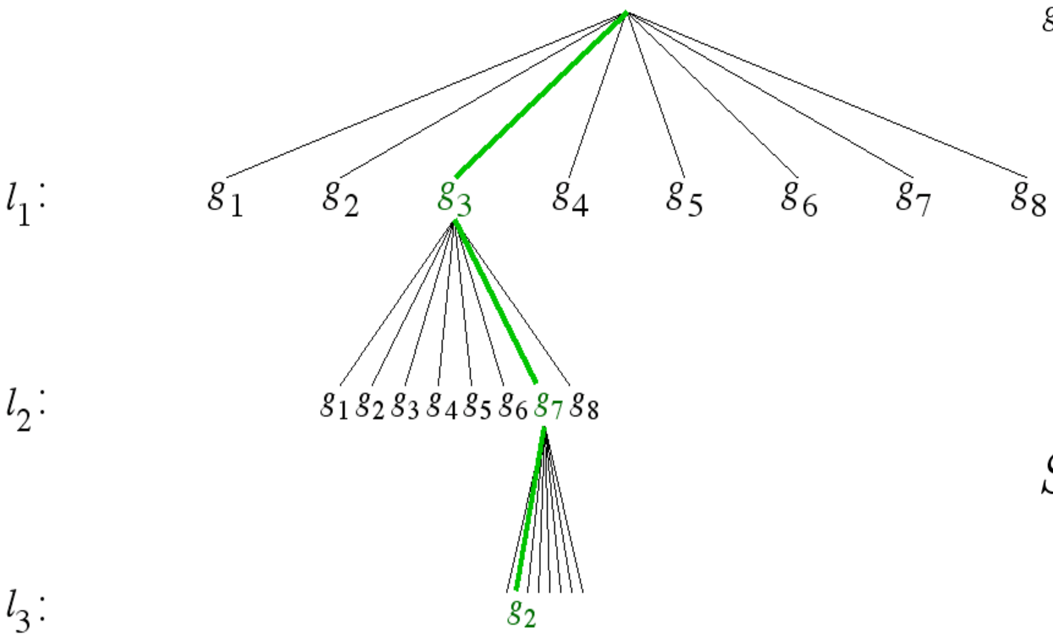
EKF Localization Example

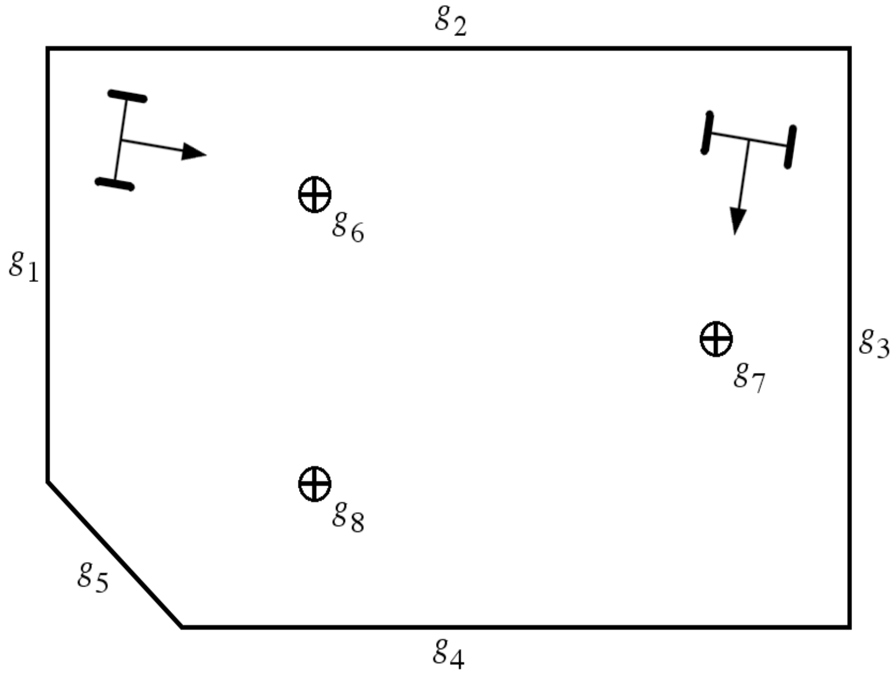


Global EKF Localization

Interpretation tree



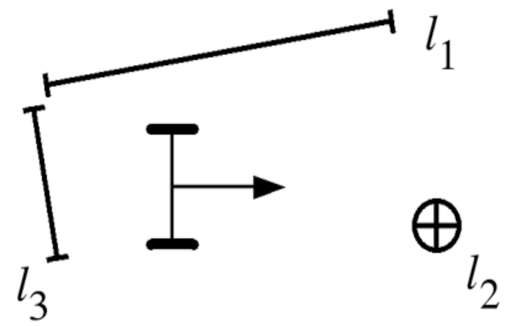


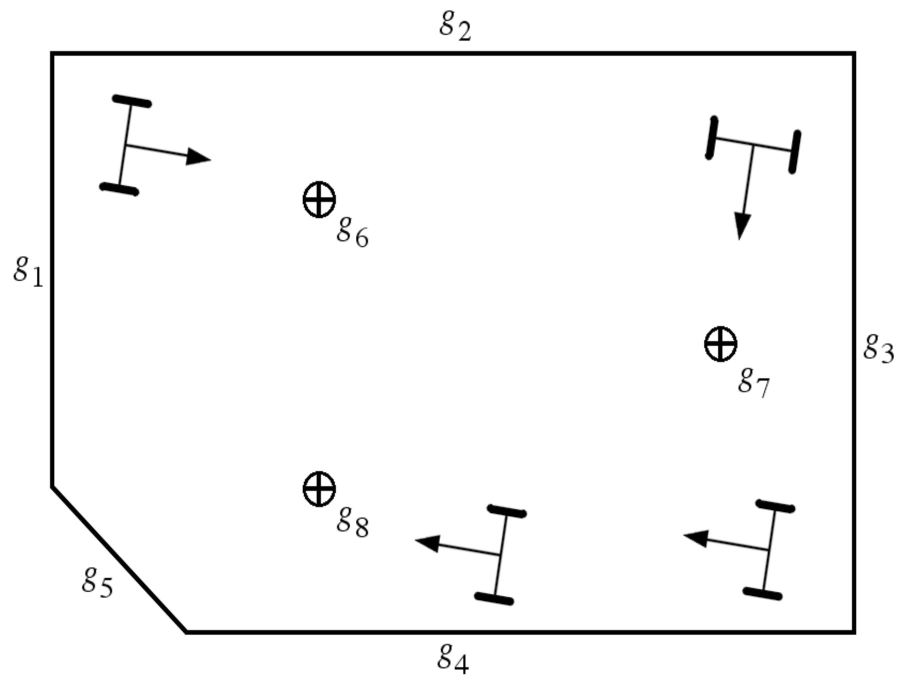


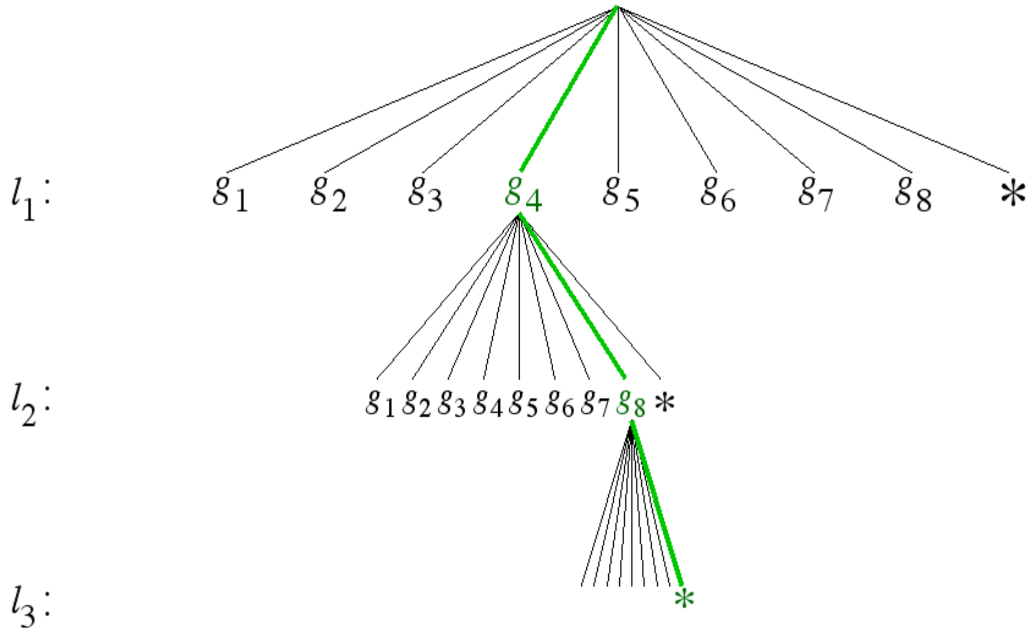
$$S_{h_2} = \{\{l_1, g_3\}, \{l_2, g_7\}, \{l_3, g_2\}\}$$

Global EKF Localization

Env. Dynamics







$$S_h = \{\{l_1, g_4\}, \{l_2, g_8\}, \{l_3, *\}\}$$

Global EKF Localization

Geometric constraints we can exploit

Location independent constraints

Unary constraint:
intrinsic property of feature
e.g. type, color, size

Binary constraint:
relative measure between features
e.g. relative position, angle

Location dependent constraints

Rigidity constraint:
"is the feature where I expect it given my position?"

Visibility constraint:
"is the feature visible from my position?"

Extension constraint:
"do the features overlap at my position?"

All decisions on a significance level α

Global EKF Localization

Interpretation Tree

[Grimson 1987], [Drumheller 1987], [Castellanos 1996], [Lim 2000]

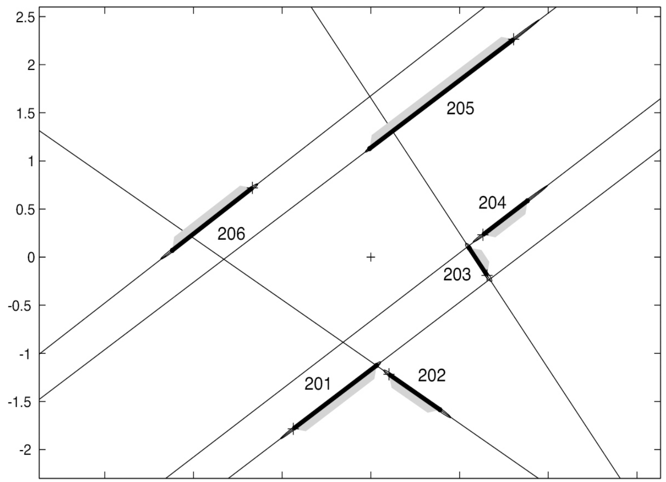
Algorithm

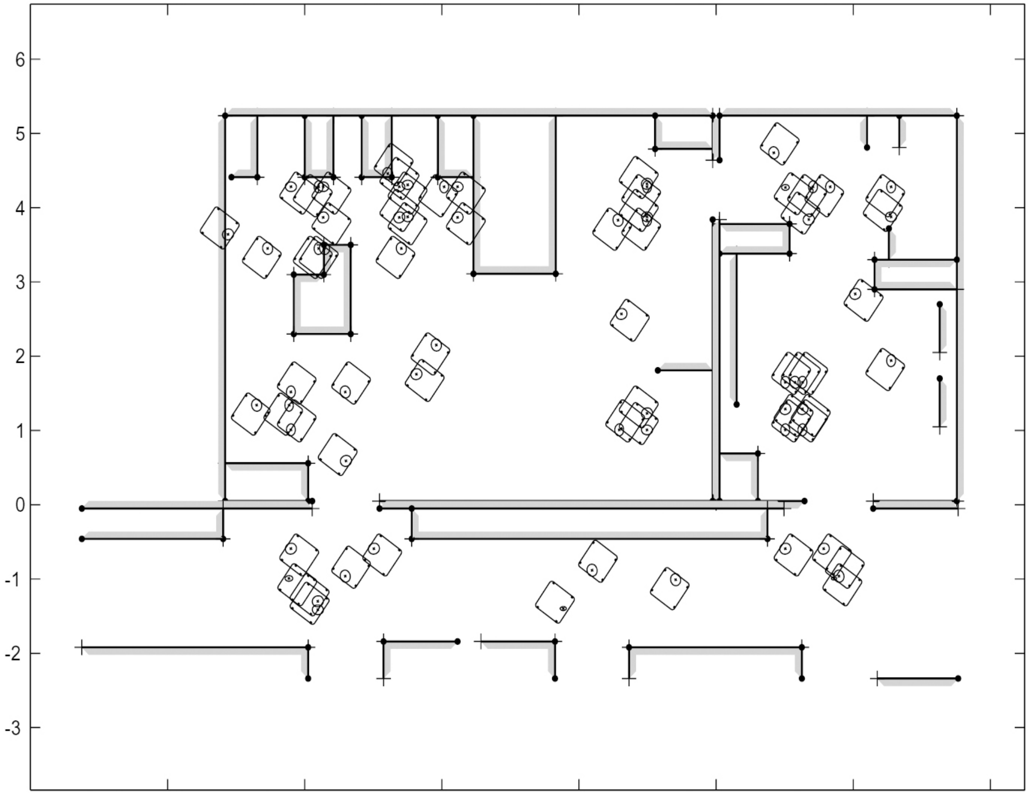
- backtracking
- depth-first
- recursive
- uses geometric constraints
- worst-case exponential complexity

```
function generate_hypotheses( h, L, G )

  H ← {}
  if L = {} then
    H ← H ∪ {h}
  else
    l ← select_observation( L )
    for g ∈ G do
      p ← {l, g}
      if satisfy_unary_constraints( p ) then
        if location_available( h ) then
          // A
          accept ← satisfy_location_dependent_cnstr( L_h, p )
          if accept then
            h' ← h
            S_h' ← S_h ∪ {p}
            L_h' ← estimate_robot_location( S_h' )
          end
        else
          // B
          accept ← true
          for p_p ∈ S_h while accept
            accept ← satisfy_binary_constraints( p_p , p )
          end
          if accept then
            h' ← h
            S_h' ← S_h ∪ {p}
            L_h' ← estimate_robot_location( S_h' )
            if location_available( h' ) then
              for p_p ∈ S_h while accept
                accept ← satisfy_location_dependent_cnstr( L_h' , p)
              end
            end
          end
        end
        if accept then
          generate_hypotheses( h', L\{l}, G )
        end
      end
    end
    generate_hypotheses( h, L\{l}, G )
  end

return H
end
```

Global EKF Localization

Pygmalion

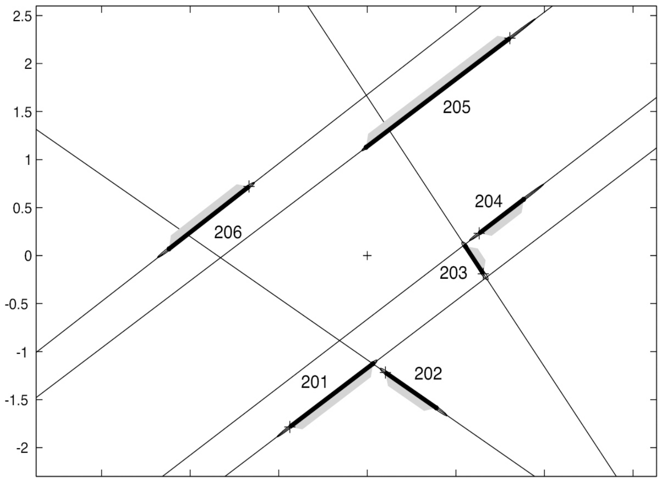


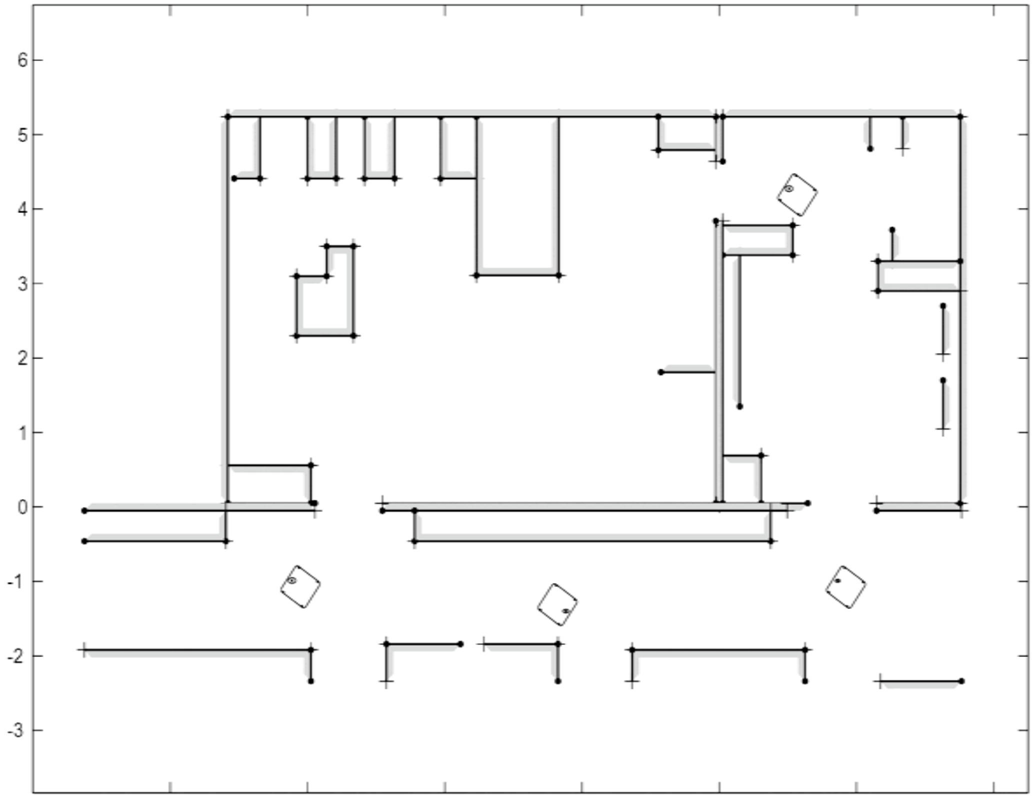


$\alpha = 0.95\ ,\quad p = 2$

Global EKF Localization

Pygmalion

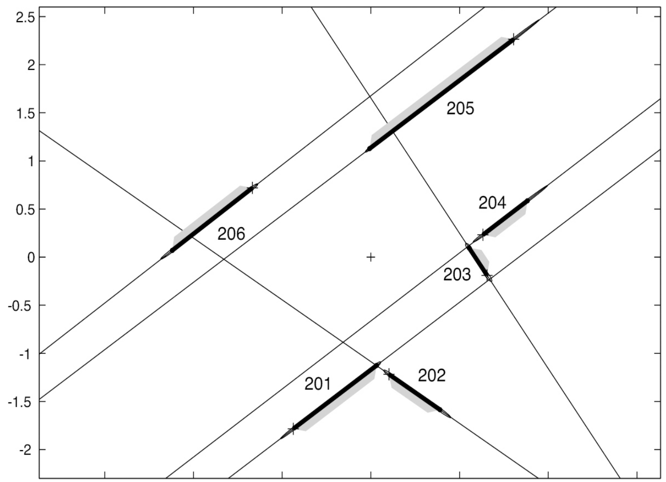


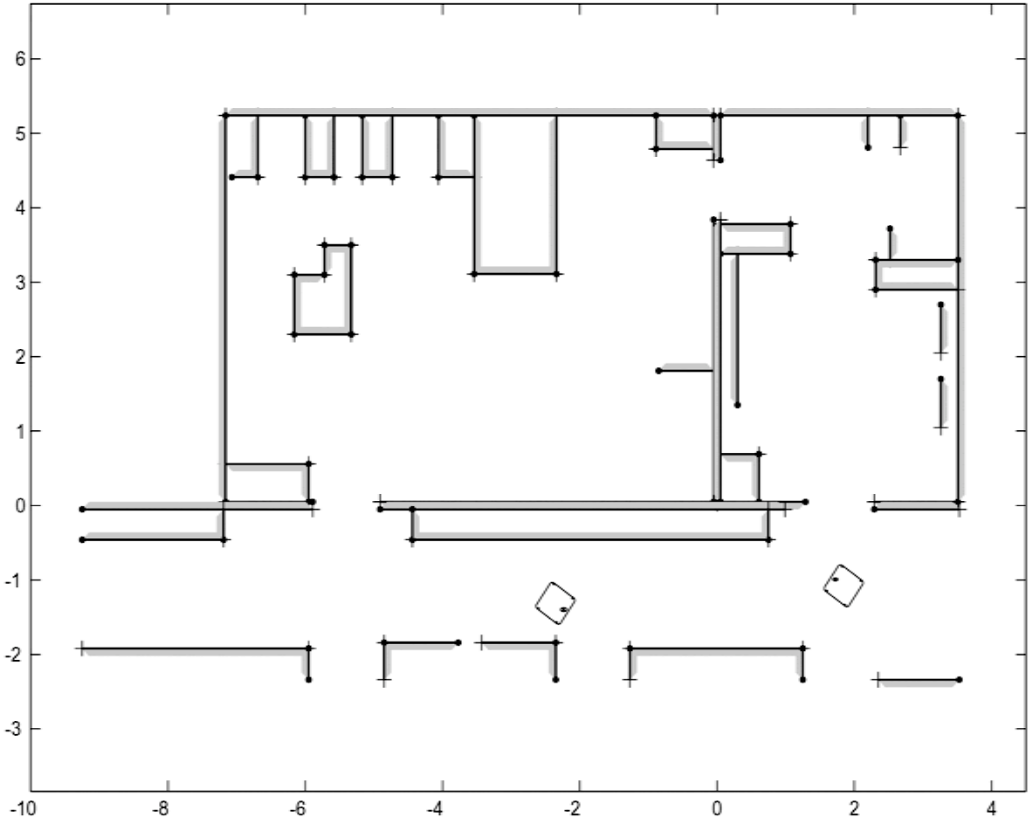


$\alpha = 0.95\ , \quad p = 3$

Global EKF Localization

Pygmalion

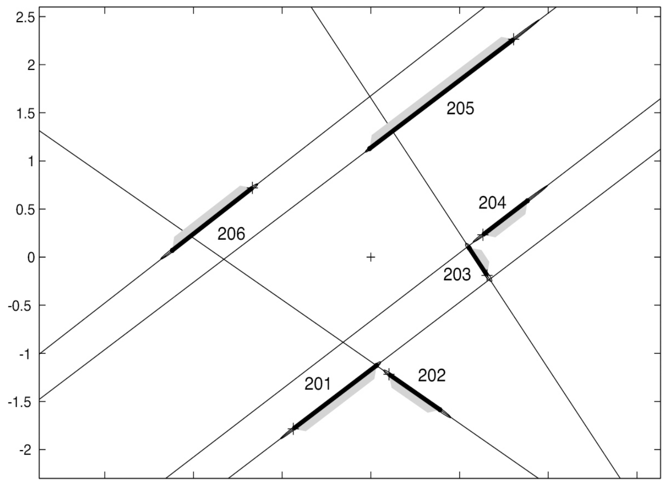


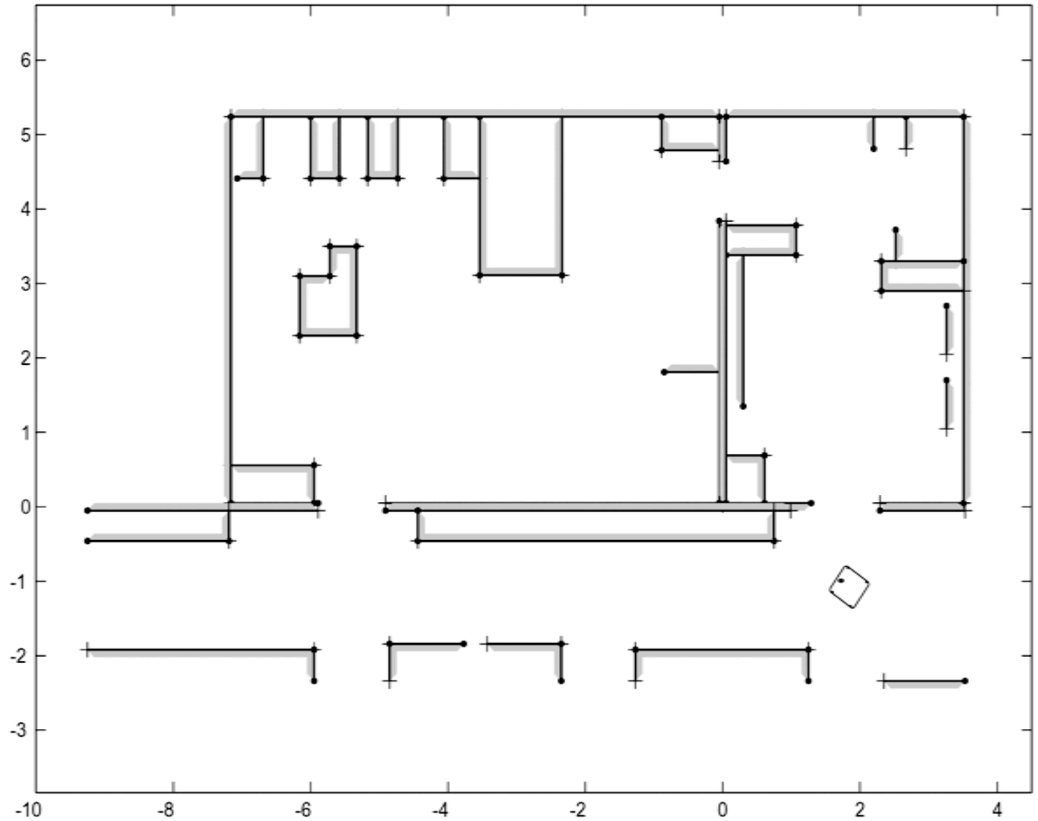


$\alpha = 0.95\ ,\quad p = 4$

Global EKF Localization

Pygmalion





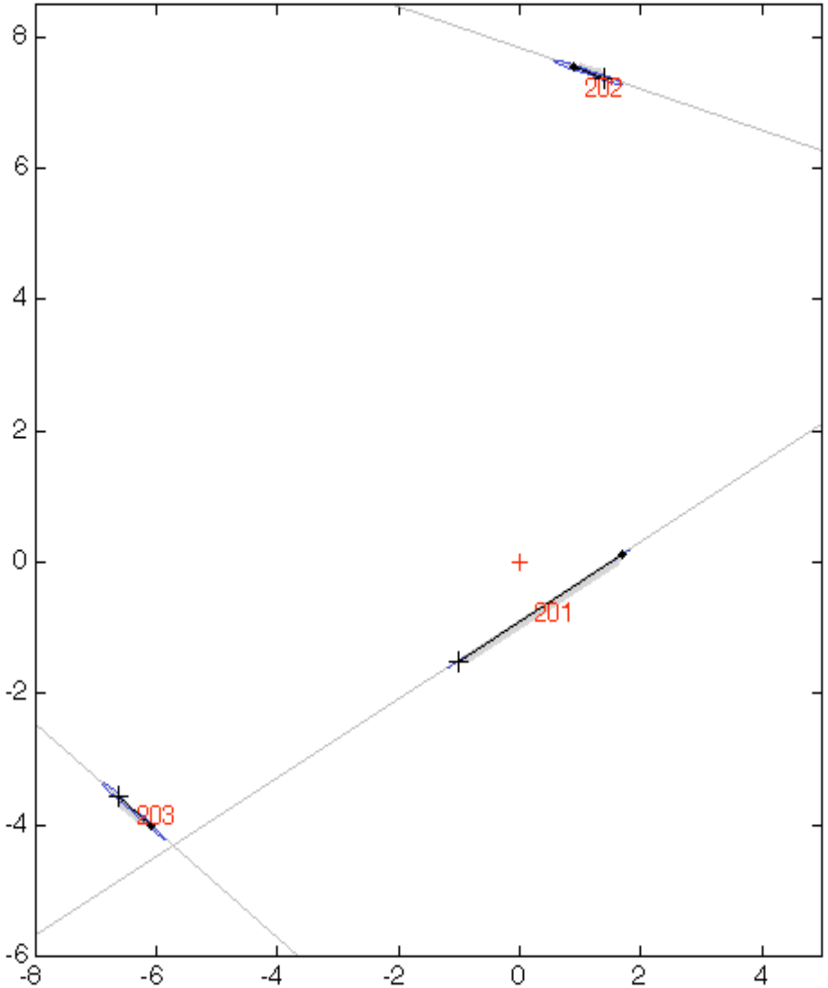
$\alpha = 0.95$, $p = 5$

t_{exe}: **633 ms** (PowerPC at 300 MHz)

Global EKF Localization

At Expo.02

05.07.02, 17.23 h



$\alpha = 0.999$

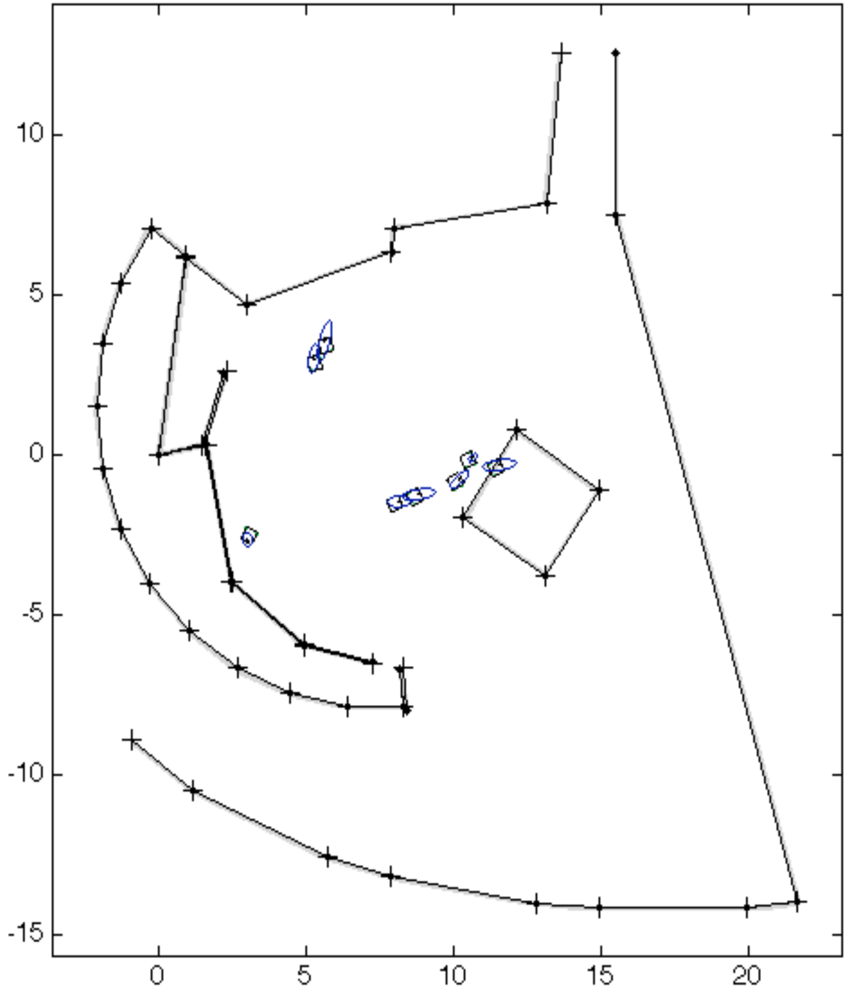
[Arras et al. 03]

Global EKF Localization

At Expo.02

05.07.02, 17.23 h

$\alpha = 0.999$



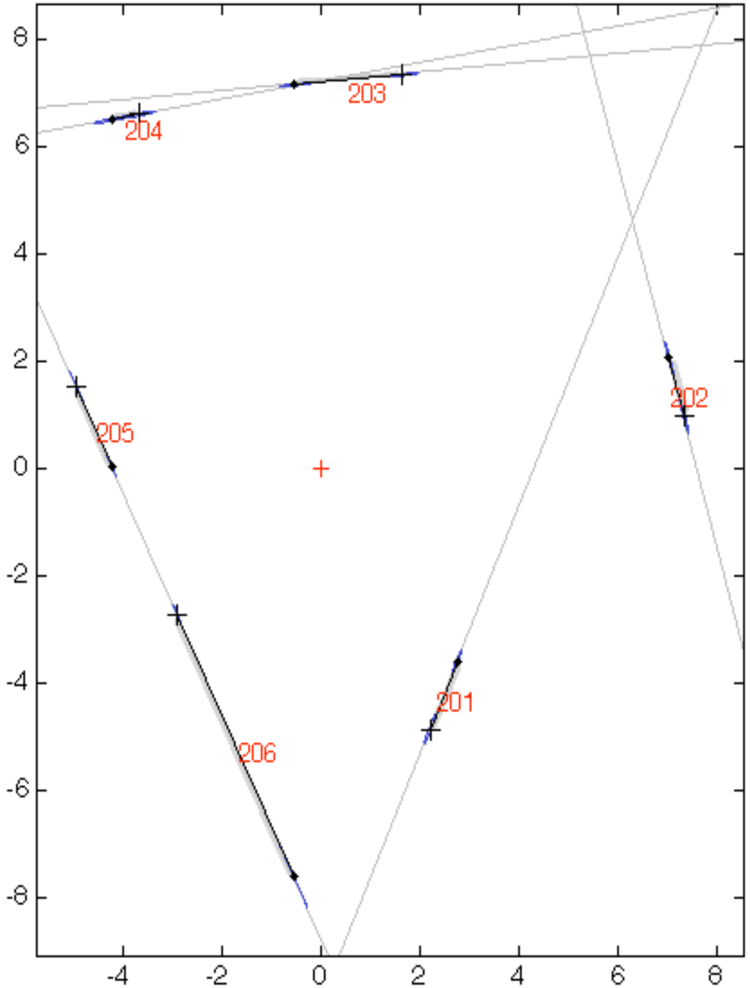
$t_{exe} = 105$ ms

[Arras et al. 03]

Global EKF Localization

At Expo.02

05.07.02, 17.32 h



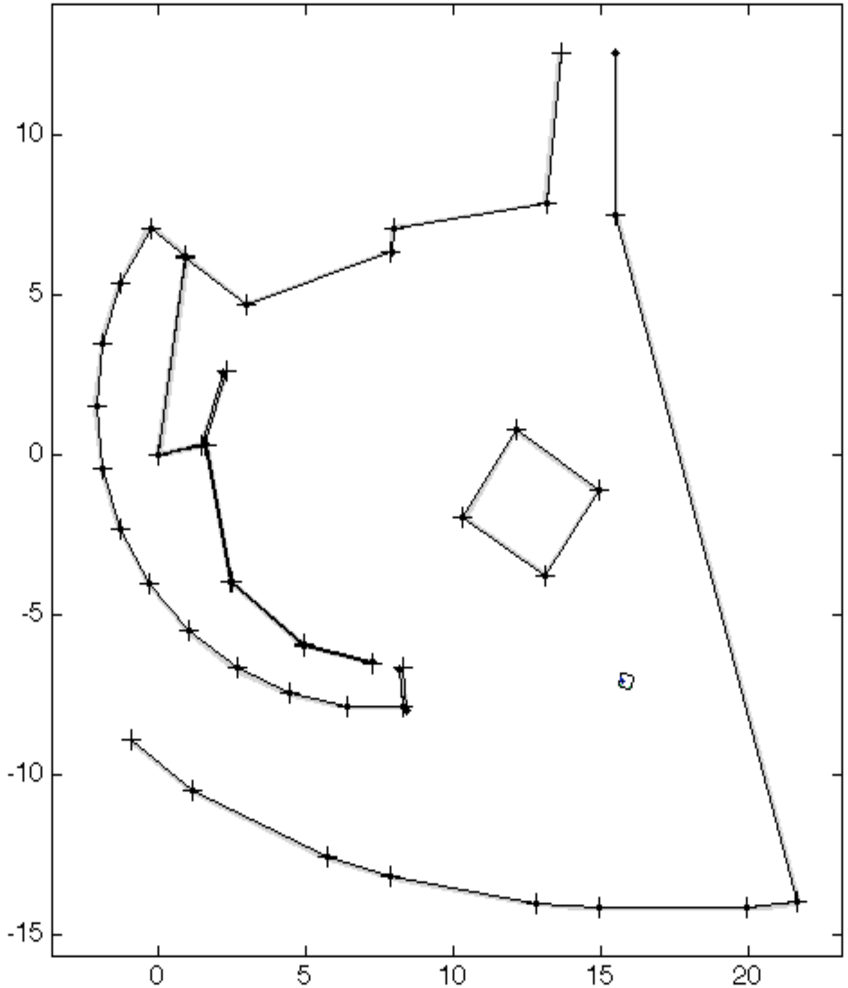
$\alpha = 0.999$

[Arras et al. 03]

Global EKF Localization

At Expo.02

05.07.02, 17.32 h



$\alpha = 0.999$

$t_{exe} = 446$ ms

[Arras et al. 03]

EKF Localization Summary

- **EKF localization** implements **pose tracking**
- Very **efficient** and **accurate** (positioning error down to subcentimeter)
- Filter divergence can cause lost situations from which the EKF **cannot recover**
- Industrial applications

- **Global EKF localization** can be achieved using interpretation tree-based data association
- Worst-case complexity is **exponential**
- **Fast** in practice for **small** maps